1-1

Relations and Functions

OBJECTIVES

- Determine whether a given relation is a function.
- Identify the domain and range of a relation or function.
- Evaluate functions.

METEOROLOGY Have you ever wished that you could change the weather? One of the technologies used in weather management is cloud seeding. In cloud seeding, microscopic particles are released in a cloud to bring about rainfall. The data in the table show the number of acre-feet of rain from pairs of similar unseeded and seeded clouds.

An acre-foot is a unit of volume equivalent to one foot of water covering an area of one acre. An acre-foot contains 43,560 cubic feet or about 27,154 gallons.

Acre-Feet of Rain	
Unseeded Clouds	**Seeded Clouds**
1.0	4.1
4.9	17.5
4.9	7.7
11.5	31.4
17.3	32.7
21.7	40.6
24.4	92.4
26.1	115.3
26.3	118.3
28.6	119.0

Source: Wadsworth International Group

We can write the values in the table as a set of ordered pairs. A pairing of elements of one set with elements of a second set is called a **relation.** The first element of an ordered pair is the *abscissa.* The set of abscissas is called the **domain** of the relation. The second element of an ordered pair is the *ordinate.* The set of ordinates is called the **range** of the relation. *Sets D and R are often used to represent domain and range.*

Relation, Domain, and Range

A relation is a set of ordered pairs. The domain is the set of all abscissas of the ordered pairs. The range is the set of all ordinates of the ordered pairs.

Example 1

METEOROLOGY **State the relation of the rain data above as a set of ordered pairs. Also state the domain and range of the relation.**

Relation: {(28.6, 119.0), (26.3, 118.3), (26.1, 115.3), (24.4, 92.4), (21.7, 40.6), (17.3, 32.7), (11.5, 31.4), (4.9, 17.5), (4.9, 7.7), (1.0, 4.1)}

Domain: {1.0, 4.9, 11.5, 17.3, 21.7, 24.4, 26.1, 26.3, 28.6}

Range: {4.1, 7.7, 31.4, 17.5, 32.7, 40.6, 92.4, 115.3, 118.3, 119.0}

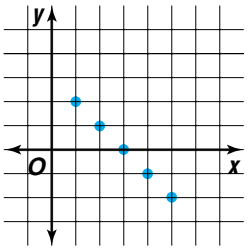
There are multiple representations for each relation. You have seen that a relation can be expressed as a set of ordered pairs. Those ordered pairs can also be expressed as a table of values. The ordered pairs can be graphed for a pictorial representation of the relation. Some relations can also be described by a rule or equation relating the first and second coordinates of each ordered pair.

Example 2 **The domain of a relation is all positive integers less than 6. The range y of the relation is 3 less x, where x is a member of the domain. Write the relation as a table of values and as an equation. Then graph the relation.**

Table:

x	y
1	2
2	1
3	0
4	−1
5	−2

Graph:

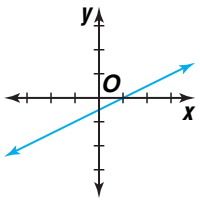


Equation: $y = 3 - x$

You can use the graph of a relation to determine its domain and range.

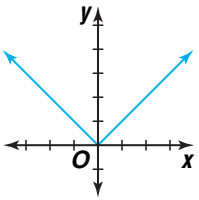
Example 3 **State the domain and range of each relation.**

a.



It appears from the graph that all real numbers are included in the domain and range of the relation.

b.



It appears from the graph that all real numbers are included in the domain. The range includes the non-negative real numbers.

The relations in Example 3 are a special type of relation called a **function.**

Function A function is a relation in which each element of the domain is paired with exactly one element in the range.

Example 4 **State the domain and range of each relation. Then state whether the relation is a function.**

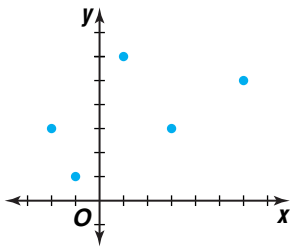
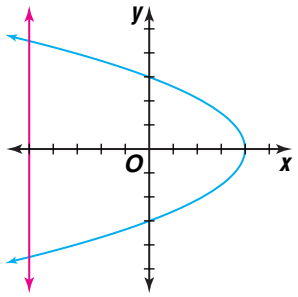
a. {(−3, 0), (4, −2), (2, −6)}

The domain is {−3, 2, 4}, and the range is {−6, −2, 0}. Each element of the domain is paired with exactly one element of the range, so this relation is a function.

b. {(4, −2), (4, 2), (9, −3), (−9, 3)}

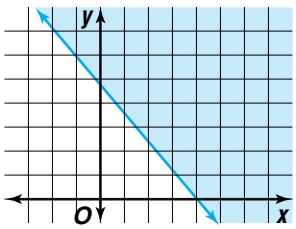
For this relation, the domain is {−9, 4, 9}, and the range is {−3, −2, 2, 3}. In the domain, 4 is paired with two elements of the range, −2 and 2. Therefore, this relation is *not* a function.

An alternate definition of a function is a set of ordered pairs in which no two pairs have the same first element. This definition can be applied when a relation is represented by a graph. If every vertical line drawn on the graph of a relation passes through no more than one point of the graph, then the relation is a function. This is called the **vertical line test.**

a relation that is a function	a relation that is not a function
	

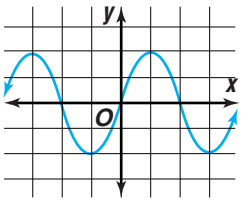
Example 5 **Determine if the graph of each relation represents a function. Explain.**

a.



No, the graph does not represent a function. A vertical line at $x = 1$ would pass through infinitely many points.

b.



Every element of the domain is paired with exactly one element of the range. Thus, the graph represents a function.

x is called the independent variable, and y is called the dependent variable.

Any letter may be used to denote a function. In **function notation,** the symbol $f(x)$ is read "f of x" and should be interpreted as the value of the function f at x. Similarly, $h(t)$ is the value of function h at t. The expression $y = f(x)$ indicates that for each element in the domain that replaces x, the function assigns one and only one replacement for y. The ordered pairs of a function can be written in the form (x, y) or $(x, f(x))$.

Every function can be evaluated for each value in its domain. For example, to find $f(-4)$ if $f(x) = 3x^3 - 7x^2 - 2x$, evaluate the expression $3x^3 - 7x^2 - 2x$ for $x = -4$.

Example 6 **Evaluate each function for the given value.**

a. $f(-4)$ **if** $f(x) = 3x^3 - 7x^2 - 2x$

$$\begin{aligned} f(-4) &= 3(-4)^3 - 7(-4)^2 - 2(-4) \\ &= -192 - 112 - (-8) \text{ or } -296 \end{aligned}$$

b. $g(9)$ **if** $g(x) = |6x - 77|$

$$\begin{aligned} g(9) &= |6(9) - 77| \\ &= |-23| \text{ or } 23 \end{aligned}$$

Functions can also be evaluated for another variable or an expression.

Example 7 **Evaluate each function for the given value.**

a. $h(a)$ if $h(x) = 3x^7 - 10x^4 + 3x - 11$

$$\begin{aligned} h(a) &= 3(a)^7 - 10(a)^4 + 3(a) - 11 \quad x = a \\ &= 3a^7 - 10a^4 + 3a - 11 \end{aligned}$$

b. $j(c - 5)$ if $j(x) = x^2 - 7x + 4$

$$\begin{aligned} j(c - 5) &= (c - 5)^2 - 7(c - 5) + 4 \quad x = c - 5 \\ &= c^2 - 10c + 25 - 7c + 35 + 4 \\ &= c^2 - 17c + 64 \end{aligned}$$

When you are given the equation of a function but the domain is not specified, the domain is all real numbers for which the corresponding values in the range are also real numbers.

Example 8 **State the domain of each function.**

a. $f(x) = \dfrac{x^3 + 5x}{x^2 - 4x}$

Any value that makes the denominator equal to zero must be excluded from the domain of f since division by zero is undefined. To determine the excluded values, let $x^2 - 4x = 0$ and solve.

$$x^2 - 4x = 0$$
$$x(x - 4) = 0$$
$$x = 0 \text{ or } x = 4$$

Therefore, the domain includes all real numbers except 0 and 4.

b. $g(x) = \dfrac{1}{\sqrt{x - 4}}$

Any value that makes the radicand negative must be excluded from the domain of g since the square root of a negative number is not a real number. Also, the denominator cannot be zero. Let $x - 4 \leq 0$ and solve for the excluded values.

$$x - 4 \leq 0$$
$$x \leq 4$$

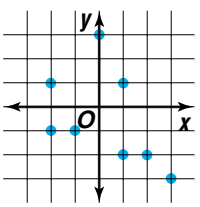
The domain excludes numbers less than or equal to 4. The domain is written as $\{x \mid x > 4\}$, which is read "the set of all x such that x is greater than 4."

CHECK FOR UNDERSTANDING

Communicating Mathematics

Read and study the lesson to answer each question.

1. **Represent** the relation $\{(-4, 2), (6, 1), (0, 5), (8, -4), (2, 2), (-4, 0)\}$ in two other ways.
2. **Draw** the graph of a relation that is not a function.
3. **Describe** how to use the vertical line test to determine whether the graph at the right represents a function.



CONTENTS

4. **You Decide** Keisha says that all functions are relations but not all relations are functions. Kevin says that all relations are functions but not all functions are relations. Who is correct and why?

Guided Practice

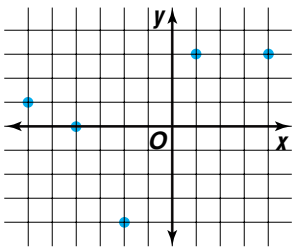
5. The domain of a relation is all positive integers less than 8. The range y of the relation is x less 4, where x is a member of the domain. Write the relation as a table of values and as an equation. Then graph the relation.

State each relation as a set of ordered pairs. Then state the domain and range.

6.

x	y
−3	4
0	0
3	−4
6	−8

7.



Given that x is an integer, state the relation representing each equation by making a table of values. Then graph the ordered pairs of the relation.

8. $y = 3x + 5$ and $-4 \leq x \leq 4$

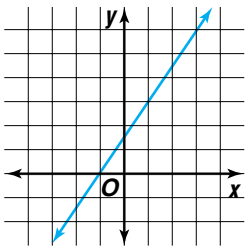
9. $y = -5$ and $1 \leq x \leq 8$

State the domain and range of each relation. Then state whether the relation is a function. Write *yes* or *no*. Explain.

10. $\{(1, 2), (2, 4), (-3, -6), (0, 0)\}$

11. $\{(6, -2), (3, 4), (6, -6), (-3, 0)\}$

12. Study the graph at the right.
   a. State the domain and range of the relation.
   b. State whether the graph represents a function. Explain.



Evaluate each function for the given value.

13. $f(-3)$ if $f(x) = 4x^3 + x^2 - 5x$

14. $g(m + 1)$ if $g(x) = 2x^2 - 4x + 2$

15. State the domain of $f(x) = \sqrt{x + 1}$.

16. **Sports** The table shows the heights and weights of members of the Los Angeles Lakers basketball team during a certain year.
   a. State the relation of the data as a set of ordered pairs. Also state the domain and range of the relation.
   b. Graph the relation.
   c. Determine whether the relation is a function.

Los Angeles Lakers

Height (in.)	Weight (lb)
83	240
81	220
82	245
78	200
83	255
73	200
80	215
77	210
78	190
73	180
86	300
77	220
82	260

Source: Preview Sports

EXERCISES

Practice

Write each relation as a table of values and as an equation. Graph the relation.

17. the domain is all positive integers less than 10, the range is 3 times x, where x is a member of the domain

18. the domain is all negative integers greater than -7, the range is x less 5, where x is a member of the domain

19. the domain is all integers greater than -5 and less than or equal to 4, the range is 8 more than x, where x is a member of the domain

State each relation as a set of ordered pairs. Then state the domain and range.

20.

x	y
−5	−5
−3	−3
−1	−1
1	1

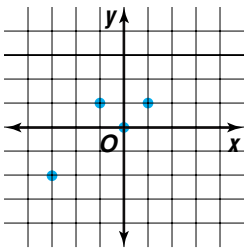
21.

x	y
−10	0
−5	0
0	0
5	0

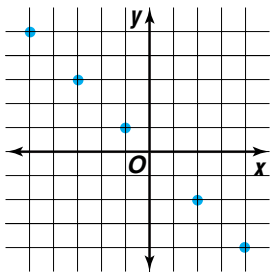
22.

x	y
4	0
5	1
8	0
13	1

23.



24.



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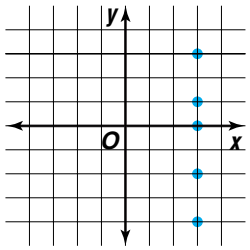


internet CONNECTION

Graphing Calculator Programs
For a graphing calculator program that plots points in a relation, visit **www.amc.glencoe.com**

Given that x is an integer, state the relation representing each equation by making a table of values. Then graph the ordered pairs of the relation.

26. $y = x - 5$ and $-4 \le x \le 1$

27. $y = -x$ and $1 \le x < 7$

28. $y = |x|$ and $-5 \le x \le 1$

29. $y = 3x - 3$ and $0 < x < 6$

30. $y^2 = x - 2$ and $x = 11$

31. $|2y| = x$ and $x = 4$

State the domain and range of each relation. Then state whether the relation is a function. Write *yes* or *no*. Explain.

32. $\{(4, 4), (5, 4), (6, 4)\}$

33. $\{(1, -2), (1, 4), (1, -6), (1, 0)\}$

34. $\{(4, -2), (4, 2), (1, -1), (1, 1), (0, 0)\}$

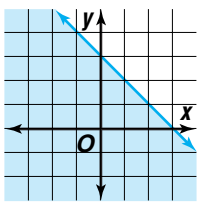
35. $\{(0, 0), (2, 2), (2, -2), (5, 8), (5, -8)\}$

36. $\{(-1.1, -2), (-0.4, -1), (-0.1, -1)\}$

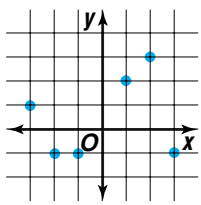
37. $\{(2, -3), (9, 0), (8, -3), (-9, 8)\}$

For each graph, state the domain and range of the relation. Then explain whether the graph represents a function.

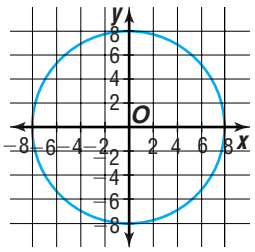
38.



39.



40.



Evaluate each function for the given value.

41. $f(3)$ if $f(x) = 2x + 3$

42. $g(-2)$ if $g(x) = 5x^2 + 3x - 2$

43. $h(0.5)$ if $h(x) = \frac{1}{x}$

44. $j(2a)$ if $j(x) = 1 - 4x^3$

45. $f(n - 1)$ if $f(x) = 2x^2 - x + 9$

46. $g(b^2 + 1)$ if $g(x) = \frac{3 - x}{5 + x}$

47. Find $f(5m)$ if $f(x) = |x^2 - 13|$.

State the domain of each function.

48. $f(x) = \frac{3x}{x^2 - 5}$ **49.** $g(x) = \sqrt{x^2 - 9}$ **50.** $h(x) = \frac{x + 2}{\sqrt{x^2 - 7}}$

Graphing Calculator

51. You can use the table feature of a graphing calculator to find the domain of a function. Enter the function into the Y= list. Then observe the y-values in the table. An error indicates that an x-value is excluded from the domain. Determine the domain of each function.

a. $f(x) = \frac{3}{x - 1}$ **b.** $g(x) = \frac{3 - x}{5 + x}$ **c.** $h(x) = \frac{x^2 - 12}{x^2 - 4}$

Applications and Problem Solving

52. Education The table shows the number of students who applied and the number of students attending selected universities.

a. State the relation of the data as a set of ordered pairs. Also state the domain and range of the relation.

b. Graph the relation.

c. Determine whether the relation is a function. Explain.

University	Number Applied	Number Attending
Auburn University	9244	3166
University of California, Davis	18,584	3697
University of Illinois-Champaign-Urbana	18,140	5805
University of Maryland	16,182	3999
State University of New York – Stony Brook	13,589	2136
The Ohio State University	18,912	5950
Texas A&M University	13,877	6233

Source: *Newsweek, "How to get into college, 1998"*

53. Critical Thinking If $f(2m + 1) = 24m^3 + 36m^2 + 26m$, what is $f(x)$? (*Hint:* Begin by solving $x = 2m + 1$ for m.)

54. Aviation The temperature of the atmosphere decreases about 5°F for every 1000 feet that an airplane ascends. Thus, if the ground-level temperature is 95°F, the temperature can be found using the function $t(d) = 95 - 0.005d$, where $t(d)$ is the temperature at a height of d feet. Find the temperature outside of an airplane at each height.

a. 500 ft **b.** 750 ft **c.** 1000 ft **d.** 5000 ft **e.** 30,000 ft

55. Geography A global positioning system, GPS, uses satellites to allow a user to determine his or her position on Earth. The system depends on satellite signals that are reflected to and from a hand-held transmitter. The time that the signal takes to reflect is used to determine the transmitter's position. Radio waves travel through air at a speed of 299,792,458 meters per second. Thus, the function $d(t) = 299{,}792{,}458t$ relates the time t in seconds to the distance traveled $d(t)$ in meters.

a. Find the distance a sound wave will travel in 0.05, 0.2, 1.4, and 5.9 seconds.

b. If a signal from a GPS satellite is received at a transmitter in 0.08 seconds, how far from the transmitter is the satellite?

Extra Practice See p. A26.

56. Critical Thinking $P(x)$ is a function for which $P(1) = 1, P(2) = 2, P(3) = 3,$ and $P(x + 1) = \frac{P(x - 2)\,P(x - 1) + 1}{P(x)}$ for $x \geq 3$. Find the value of $P(6)$.

57. SAT Practice What is the value of $7^2 - (3^2 + 4^2)$?

A 56
B 24
C 0
D -24
E -56

CAREER CHOICES

Veterinary Medicine

If you like working with animals and have a strong interest in science, you may want to consider a career in veterinary medicine. Many veterinarians work with small animals, such as pets, maintaining their good health and treating illnesses and injuries. Some veterinarians work with large animals, such as farm animals, to ensure the health of animals that we depend upon for food. Still other veterinarians work to control diseases in wildlife.

Duties of veterinarians can include administering medications to the animals, performing surgeries, instructing people in the care of animals, and researching genetics, prevention of disease, and better animal nutrition.

Many veterinarians work in private practice, but jobs are also available in industry and governmental agencies.

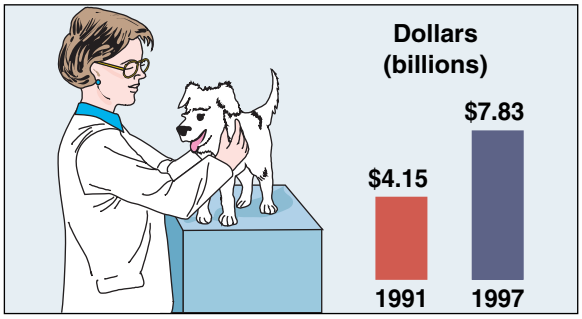
Career Overview

Degree Preferred:
D.V.M. (doctor of veterinary medicine) consisting of six years of college

Related Courses:
biology, chemistry, mathematics

Outlook:
number of jobs expected to increase through 2006

Spending on Pet Health Care



Source: American Veterinary Medical Association

internet CONNECTION For more information on careers in veterinary medicine, visit: **www.amc.glencoe.com**