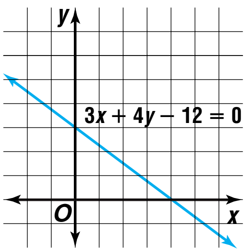
1-3

# Graphing Linear Equations

**OBJECTIVES**
- Graph linear equations.
- Find the *x*- and *y*-intercepts of a line.
- Find the slope of a line through two points.
- Find zeros of linear functions.

Real World Application

**AGRICULTURE** American farmers produce enough food and fiber to meet the needs of our nation and to export huge quantities to countries around the world. In addition to raising grain, cotton and other fibers, fruits, or vegetables, farmers also work on dairy farms, poultry farms, horticultural specialty farms that grow ornamental plants and nursery products, and aquaculture farms that raise fish and shellfish. In 1900, the percent of American workers who were farmers was 37.5%. In 1994, that percent had dropped to just 2.5%. What was the average rate of decline? *This problem will be solved in Example 2.*




The problem above can be solved by using a linear equation. A **linear equation** has the form $Ax + By + C = 0$, where $A$ and $B$ are not both zero. Its graph is a straight line. The graph of the equation $3x + 4y - 12 = 0$ is shown.

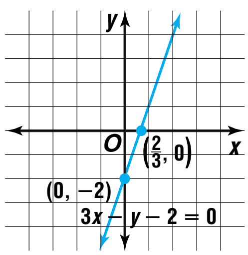
The solutions of a linear equation are the ordered pairs for the points on its graph. An ordered pair corresponds to a point in the coordinate plane. Since two points determine a line, only two points are needed to graph a linear equation. Often the two points that are easiest to find are the ***x*-intercept** and the ***y*-intercept**. The $x$-intercept is the point where the line crosses the $x$-axis, and the $y$-intercept is the point where the graph crosses the $y$-axis. In the graph above, the $x$-intercept is at $(4, 0)$, and the $y$-intercept is at $(0, 3)$. *Usually, the individual coordinates 4 and 3 called the x- and y-intercepts.*

**Example 1** **Graph $3x - y - 2 = 0$ using the $x$-and $y$-intercepts.**

Substitute 0 for $y$ to find the $x$-intercept. Then substitute 0 for $x$ to find the $y$-intercept.

| ***x*-intercept** | ***y*-intercept** |
|---|---|
| $3x - y - 2 = 0$ | $3x - y - 2 = 0$ |
| $3x - (0) - 2 = 0$ | $3(0) - y - 2 = 0$ |
| $3x - 2 = 0$ | $-y - 2 = 0$ |
| $3x = 2$ | $-y = 2$ |
| $x = \frac{2}{3}$ | $y = -2$ |

The line crosses the $x$-axis at $\left(\frac{2}{3}, 0\right)$ and the $y$-axis at $(0, -2)$. Graph the intercepts and draw the line.

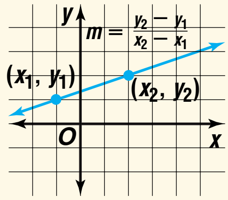


The **slope** of a nonvertical line is the ratio of the change in the ordinates of the points to the corresponding change in the abscissas. The slope of a line is a constant.

**Slope**

The slope, $m$, of the line through $(x_1, y_1)$ and $(x_2, y_2)$ is given by the following equation, if $x_1 \neq x_2$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

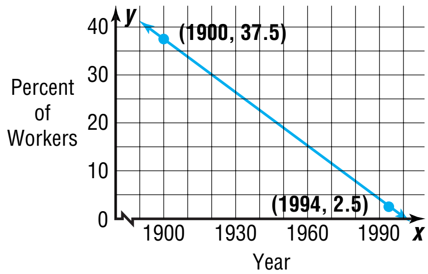



The slope of a line can be interpreted as the rate of change in the $y$-coordinates for each 1-unit increase in the $x$-coordinates.

**Example 2** Real World Application

**AGRICULTURE** **Refer to the application at the beginning of the lesson. What was the average rate of decline in the percent of American workers who were farmers?**

The average rate of change is the slope of the line containing the points at (1900, 37.5) and (1994, 2.5). Find the slope of this line.




$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2.5 - 37.5}{1994 - 1900} \quad \text{Let } x_1 = 1900,\ y_1 = 37.5,\ x_2 = 1994, \text{ and } y_2 = 2.5.$$

$$= \frac{-35}{94} \text{ or about } -0.37$$

On average, the number of American workers who were farmers decreased about 0.37% each year from 1900 to 1994.

A linear equation in the form $Ax + By = C$ where $A$ is positive is written in **standard form.** You can also write a linear equation in **slope-intercept form.** Slope-intercept form is $y = mx + b$, where $m$ is the slope and $b$ is the $y$-intercept of the line. You can graph an equation in slope-intercept form by graphing the $y$-intercept and then finding a second point on the line using the slope.

**Slope-Intercept Form**

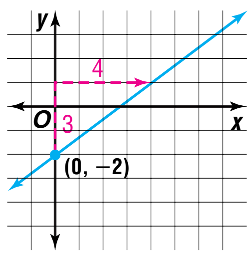
If a line has slope $m$ and $y$-intercept $b$, the slope-intercept form of the equation of the line can be written as follows.

$$y = mx + b$$

**Example 3** **Graph each equation using the $y$-intercept and the slope.**

**a.** $y = \frac{3}{4}x - 2$

The $y$-intercept is $-2$. Graph $(0, -2)$.
Use the slope to graph a second point.
Connect the points to graph the line.



**Graphing Calculator Appendix**

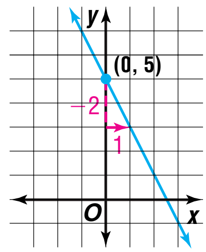
For keystroke instruction on how to graph linear equations, see page A5.

**b.** $2x + y = 5$

Rewrite the equation in slope-intercept form.

$2x + y = 5 \rightarrow y = -2x + 5$

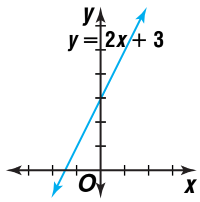
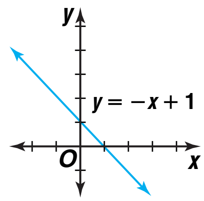
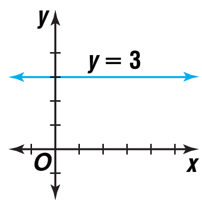
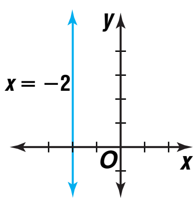
The $y$-intercept is 5. Graph (0, 5). Then use the slope to graph a second point. Connect the points to graph the line.




There are four different types of slope for a line. The table below shows a graph with each type of slope.

*A line with undefined slope is sometimes described as having "no slope."*

**Types of Slope**

| positive slope | negative slope | 0 slope | undefined slope |
|---|---|---|---|
|   |   |   |   |

Notice from the graphs that not all linear equations represent functions. A linear function is defined as follows. *When is a linear equation not a function?*

| **Linear Functions** | A linear function is defined by $f(x) = mx + b$, where $m$ and $b$ are real numbers. |
|---|---|

Values of $x$ for which $f(x) = 0$ are called **zeros of the function $f$.** For a linear function, the zeros can be found by solving the equation $mx + b = 0$. If $m \neq 0$, then $-\frac{b}{m}$ is the only zero of the function. The zeros of a function are the $x$-intercepts. Thus, for a linear function, the $x$-intercept has coordinates $\left(-\frac{b}{m}, 0\right)$.

In the case where $m = 0$, we have $f(x) = b$. This function is called a **constant function** and its graph is a horizontal line. The constant function $f(x) = b$ has no zeros when $b \neq 0$ or every value of $x$ is a zero if $b = 0$.

**Example 4** **Find the zero of each function. Then graph the function.**

**Graphing Calculator Appendix**

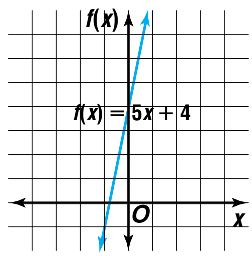
For keystroke instruction on how to find the zeros of a linear function using the **CALC** menu, see page A11.

**a.** $f(x) = 5x + 4$

To find the zeros of $f(x)$, set $f(x)$ equal to 0 and solve for $x$.

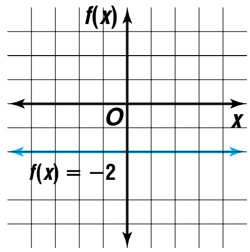
$5x + 4 = 0 \Rightarrow x = -\frac{4}{5}$

$-\frac{4}{5}$ is a zero of the function. So the coordinates of one point on the graph are $\left(-\frac{4}{5}, 0\right)$. Find the coordinates of a second point. When $x = 0$, $f(x) = 5(0) + 4$, or 4. Thus, the coordinates of a second point are (0, 4).



**b.** $f(x) = -2$

Since $m = 0$ and $b = -2$, this function has no $x$-intercept, and therefore no zeros. The graph of the function is a horizontal line 2 units below the $x$-axis.

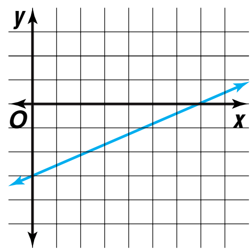



## CHECK FOR UNDERSTANDING

**Communicating Mathematics**

**Read and study the lesson to answer each question.**

1. **Explain** the significance of $m$ and $b$ in $y = mx + b$.
2. **Name** the zero of the function whose graph is shown at the right. Explain how you found the zero.
3. **Describe** the process you would use to graph a line with a $y$-intercept of 2 and a slope of $-4$.
4. **Compare and contrast** the graphs of $y = 5x + 8$ and $y = -5x + 8$.




**Guided Practice**

**Graph each equation using the *x*- and *y*-intercepts.**

5. $3x - 4y + 2 = 0$
6. $x + 2y - 5 = 0$

**Graph each equation using the *y*-intercept and the slope.**

7. $y = x + 7$
8. $y = 5$

**Find the zero of each function. If no zero exists, write *none.* Then graph the function.**

9. $f(x) = \frac{1}{2}x + 6$
10. $f(x) = 19$

11. **Archaeology** Archaeologists use bones and other artifacts found at historical sites to study a culture. One analysis they perform is to use a function to determine the height of the person from a tibia bone. Typically a man whose tibia is 38.500 centimeters long is 173 centimeters tall. A man with a 44.125-centimeter tibia is 188 centimeters tall.
    a. Write two ordered pairs that represent the function.
    b. Determine the slope of the line through the two points.
    c. Explain the meaning of the slope in this context.

## EXERCISES

**Practice**

**Graph each equation.**

**12.** $y = 4x - 9$  **13.** $y = 3$  **14.** $2x - 3y + 15 = 0$

**15.** $x - 4 = 0$  **16.** $y = 6x - 1$  **17.** $y = 5 - 2x$

**18.** $y + 8 = 0$  **19.** $2x + y = 0$  **20.** $y = \frac{2}{3}x - 4$

**21.** $y = 25x + 150$  **22.** $2x + 5y = 8$  **23.** $3x - y = 7$

**Find the zero of each function. If no zero exists, write *none.* Then graph the function.**

**24.** $f(x) = 9x + 5$  **25.** $f(x) = 4x - 12$  **26.** $f(x) = 3x + 1$

**27.** $f(x) = 14x$  **28.** $f(x) = 12$  **29.** $f(x) = 5x - 8$

**30.** Find the zero for the function $f(x) = 5x - 2$.

**31.** Graph $y = -\frac{3}{2}x + 3$. What is the zero of the function $f(x) = -\frac{3}{2}x + 3$?

**32.** Write a linear function that has no zero. Then write a linear function that has infinitely many zeros.

**Applications and Problem Solving**

**33.** **Electronics** The voltage $V$ in volts produced by a battery is a linear function of the current $i$ in amperes drawn from it. The opposite of the slope of the line represents the battery's effective resistance $R$ in ohms. For a certain battery, $V = 12.0$ when $i = 1.0$ and $V = 8.4$ when $i = 10.0$.

**a.** What is the effective resistance of the battery?

**b.** Find the voltage that the battery would produce when the current is 25.0 amperes.

**34.** **Critical Thinking** A line passes through $A(3, 7)$ and $B(-4, 9)$. Find the value of $a$ if $C(a, 1)$ is on the line.

**35.** **Chemistry** According to Charles' Law, the pressure $P$ in pascals of a fixed volume of a gas is linearly related to the temperature $T$ in degrees Celsius. In an experiment, it was found that when $T = 40$, $P = 90$ and when $T = 80$, $P = 100$.

**a.** What is the slope of the line containing these points?

**b.** Explain the meaning of the slope in this context.

**c.** Graph the function.

**36.** **Critical Thinking** The product of the slopes of two non-vertical perpendicular lines is always $-1$. Is it possible for two perpendicular lines to both have positive slope? Explain.

**37.** **Accounting** A business's capital costs are expenses for things that last more than one year and lose value or wear out over time. Examples include equipment, buildings, and patents. The value of these items declines, or depreciates over time. One way to calculate depreciation is the straight-line method, using the value and the estimated life of the asset. Suppose $v(t) = 10{,}440 - 290t$ describes the value $v(t)$ of a piece of software after $t$ months.

**a.** Find the zero of the function. What does the zero represent?

**b.** Find the slope of the function. What does the slope represent?

**c.** Graph the function.

38. **Critical Thinking** How is the slope of a linear function related to the number of zeros for the function?

39. **Economics** Economists call the relationship between a nation's disposable income and personal consumption expenditures the marginal propensity to consume or MPC. An MPC of 0.7 means that for each \$1 increase in disposable income, consumption increases \$0.70. That is, 70% of each additional dollar earned is spent and 30% is saved.

   a. Suppose a nation's disposable income, $x$, and personal consumption expenditures, $y$, are shown in the table at the right. Find the MPC.

   b. If disposable income were to increase \$1805 in a year, how many additional dollars would the average family spend?

| $x$ (billions of dollars) | $y$ (billions of dollars) |
|---|---|
| 56 | 50 |
| 76 | 67.2 |

   c. The marginal propensity to save, MPS, is 1 − MPC. Find the MPS.

   d. If disposable income were to increase \$1805 in a year, how many additional dollars would the average family save?

**Mixed Review**

40. Given $f(x) = 2x$ and $g(x) = x^2 - 4$, find $(f + g)(x)$ and $(f - g)(x)$. *(Lesson 1-2)*

41. **Business** Computer Depot offers a 12% discount on computers sold Labor Day weekend. There is also a \$100 rebate available. *(Lesson 1-2)*

   a. Write a function for the price after the discount $d(p)$ if $p$ represents the original price of a computer.

   b. Write a function for the price after the rebate $r(d)$ if $d$ represents the discounted price.

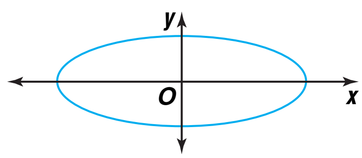
   c. Use composition of functions to write a function to relate the selling price to the original price of a computer.

   d. Find the selling prices of computers with original prices of \$799.99, \$999.99, and \$1499.99.

42. Find $[f \circ g](-3)$ and $[g \circ f](-3)$ if $f(x) = x^2 - 4x + 5$ and $g(x) = x - 2$. *(Lesson 1-2)*

43. Given $f(x) = 4 + 6x - x^3$, find $f(9)$. *(Lesson 1-1)*

44. Determine whether the graph at the right represents a function. Explain. *(Lesson 1-1)*




45. Given that $x$ is an integer, state the relation representing $y = 11 - x$ and $3 \leq x \leq 0$ by listing a set of ordered pairs. Then state whether the relation is a function. *(Lesson 1-1)*

46. **SAT/ACT Practice** What is the sum of four integers whose average is 15?

   **A** 3.75

   **B** 15

   **C** 30

   **D** 60

   **E** cannot be determined

**Extra Practice** See p. A26.

GRAPHING CALCULATOR EXPLORATION

# 1-3B Analyzing Families of Linear Graphs

*An Extension of Lesson 1-3*

**OBJECTIVE**

- Investigate the effect of changing the value of $m$ or $b$ in $y = mx + b$.

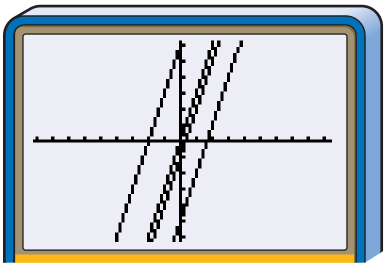
A **family of graphs** is a group of graphs that displays one or more similar characteristics. For linear functions, there are two types of families of graphs. Using the slope-intercept form of the equation, one family is characterized by having the same slope $m$ in $y = mx + b$. The other type of family has the same $y$-intercept $b$ in $y = mx + b$.

You can investigate families of linear graphs by graphing several equations on the same graphing calculator screen.

**Example**

**Graph $y = 3x - 5$, $y = 3x - 1$, $y = 3x$, and $y = 3x + 6$. Describe the similarities and differences among the graphs.**

Graph all of the equations on the same screen. Use the viewing window, $[-9.4, 9.4]$ by $[-6.2, 6.2]$.



[−9.4, 9.4] scl:1 by [−6.2, 6.2] scl:1

Notice that the graphs appear to be parallel lines with the same positive slope. They are in the family of lines that have the slope 3.

The slope of each line is the same, but the lines have different $y$-intercepts. Each of the other three lines are the graph of $y = 3x$ shifted either up or down.

| equation | slope | $y$-intercept | relationship to graph of $y = 3x$ |
|---|---|---|---|
| $y = 3x - 5$ | 3 | −5 | shifted 5 units down |
| $y = 3x - 1$ | 3 | −1 | shifted 1 unit down |
| $y = 3x$ | 3 | 0 | same |
| $y = 3x + 6$ | 3 | 6 | shifted 6 units up |

**TRY THESE**

1. Graph $y = 4x - 2$, $y = 2x - 2$, $y = -2$, $y = -x - 2$, and $y = -6x - 2$ on the same graphing calculator screen. Describe how the graphs are similar and different.

**WHAT DO YOU THINK?**

2. Use the results of the Example and Exercise 1 to predict what the graph of $y = 3x - 2$ will look like.
3. Write a paragraph explaining the effect of different values of $m$ and $b$ on the graph of $y = mx + b$. Include sketches to illustrate your explanation.