

OBJECTIVE

- Write linear equations.

Writing Linear Equations



ECONOMICS Each year, the U.S. Department of Commerce publishes its *Survey of Current*

Business. Included in the report is the average personal income of U.S. workers.

Personal income is one indicator of the health of the U.S. economy. How could you use the data on average personal income for 1980 to 1997 to predict the average personal income in 2010? *This problem will be solved in Example 3.*

Years since 1980	Average Personal Income (\$)
0	9916
5	13,895
10	18,477
11	19,100
12	19,802
13	20,810
14	21,846
15	23,233
16	24,457
17	25,660



A mathematical **model** may be an equation used to approximate a real-world set of data. Often when you work with real-world data, you know information about a line without knowing its equation. You can use characteristics of the graph of the data to write an equation for a line. This equation is a model of the data. Writing an equation of a line may be done in a variety of ways depending upon the information you are given. If one point and the slope of a line are known, the slope-intercept form can be used to write the equation.

Example 1 Write an equation in slope-intercept form for each line described.

a. a slope of $-\frac{3}{4}$ and a y-intercept of 7

Substitute $-\frac{3}{4}$ for m and 7 for b in the general slope-intercept form.

$$y = mx + b \rightarrow y = -\frac{3}{4}x + 7$$

The slope-intercept form of the equation of the line is $y = -\frac{3}{4}x + 7$.

b. a slope of -6 and passes through the point at $(1, -3)$

Substitute the slope and coordinates of the point in the general slope-intercept form of a linear equation. Then solve for b .

$$y = mx + b$$

$$-3 = -6(1) + b \quad \text{Substitute } -3 \text{ for } y, 1 \text{ for } x, \text{ and } -6 \text{ for } m.$$

$$3 = b \quad \text{Add 6 to each side of the equation.}$$

The y-intercept is 3. Thus, the equation for the line is $y = -6x + 3$.

Example

2 BUSINESS Alvin Hawkins is opening a home-based business. He determined that he will need \$6000 to buy a computer and supplies to start. He expects expenses for each following month to be \$700. Write an equation that models the total expense y after x months.

The initial cost is the y -intercept of the graph. Because the total expense rises \$700 each month, the slope is 700.

$$y = mx + b$$

$$y = 700x + 6000 \quad \text{Substitute 700 for } m \text{ and 6000 for } b.$$

The total expense can be modeled by $y = 700x + 6000$.

When you know the slope and a point on a line, you can also write an equation for the line in **point-slope form**. Using the definition of slope for points (x, y) and (x_1, y_1) , if $\frac{y - y_1}{x - x_1} = m$, then $y - y_1 = m(x - x_1)$.

Point-Slope Form

If the point with coordinates (x_1, y_1) lies on a line having slope m , the point-slope form of the equation of the line can be written as follows.

$$y - y_1 = m(x - x_1)$$

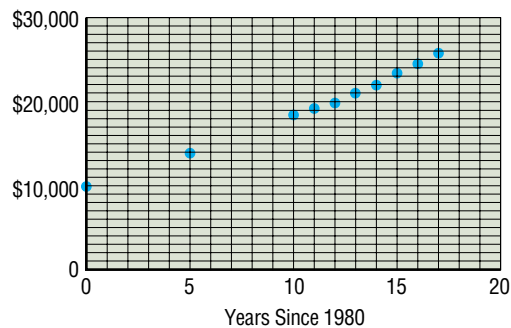
If you know the coordinates of two points on a line, you can find the slope of the line. Then the equation of the line can be written using either the slope-intercept or the point-slope form.

Example

3 ECONOMICS Refer to the application at the beginning of the lesson.

- Find a linear equation that can be used as a model to predict the average personal income for any year.
- Assume that the rate of growth of personal income remains constant over time and use the equation to predict the average personal income for individuals in the year 2010.
- Evaluate the prediction.

- Graph the data. Then select two points to represent the data set and draw a line that might approximate the data. Suppose we chose $(0, 9916)$ and $(17, 25,660)$. Use the coordinates of those points to find the slope of the line you drew.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{25,660 - 9916}{17 - 0} \quad x_1 = 0, y_1 = 9916, x_2 = 17, y_2 = 25,660$$

$$\approx 926$$

Thus for each 1-year increase, average personal income increases \$926.



Use point-slope form.

$$y - y_1 = m(x - x_1)$$

$$y - 9916 = 926(x - 0) \quad \text{Substitute } 0 \text{ for } x_1, 9916 \text{ for } y_1, \text{ and } 926 \text{ for } m.$$

$$y = 926x + 9916$$

The slope-intercept form of the model equation is $y = 926x + 9916$.

- b.** Evaluate the equation for $x = 2010$ to predict the average personal income for that year. The years since 1980 will be $2010 - 1980$ or 30. So $x = 30$.

$$y = 926x + 9916$$

$$y = 926(30) + 9916 \quad \text{Substitute } 30 \text{ for } x.$$

$$y = 37,696$$

The predicted average personal income is about \$37,696 for the year 2010.

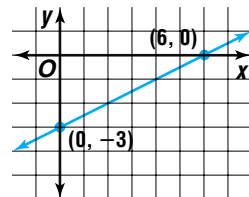
- c.** Most of the actual data points are close to the graph of the model equation. Thus, the equation and the prediction are probably reliable.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Read and study the lesson to answer each question.

- 1. List** all the different sets of information that are sufficient to write the equation of a line.
- 2. Demonstrate** two different ways to find the equation of the line with a slope of $\frac{1}{4}$ passing through the point at $(3, -4)$.
- 3. Explain** what 55 and 49 represent in the equation $c = 55h + 49$, which represents the cost c of a plumber's service call lasting h hours.
- 4. Write** an equation for the line whose graph is shown at the right.
- 5. Math Journal** Write a sentence or two to describe when it is easier to use the point-slope form to write the equation of a line and when it is easier to use the slope-intercept form.



Guided Practice

Write an equation in slope-intercept form for each line described.

6. slope = $-\frac{1}{4}$, y -intercept = -10
7. slope = 4, passes through $(3, 2)$
8. passes through $(5, 2)$ and $(7, 9)$
9. horizontal and passes through $(-9, 2)$
10. **Botany** Do you feel like every time you cut the grass it needs to be cut again right away? Be grateful you aren't cutting the Bermuda grass that grows in Africa and Asia. It can grow at a rate of 5.9 inches per day! Suppose you cut a Bermuda grass plant to a length of 2 inches.
 - a. Write an equation that models the length of the plant y after x days.
 - b. If you didn't cut it again, how long would the plant be in one week?
 - c. Can this rate of growth be maintained indefinitely? Explain.



EXERCISES

Practice

Write an equation in slope-intercept form for each line described.

11. slope = 5, y-intercept = -2
 12. slope = 8, passes through (-7, 5)
 13. slope = $-\frac{3}{4}$, y-intercept = 0
 14. slope = -12, y-intercept = $\frac{1}{2}$
 15. passes through A(4, 5), slope = 6
 16. no slope and passes through (12, -9)
 17. passes through A(1, 5) and B(-8, 9)
 18. x-intercept = -8, y-intercept = 5
 19. passes through A(8, 1) and B(-3, 1)
 20. vertical and passes through (-4, -2)
 21. the y-axis
 22. slope = 0.25, x-intercept = 24
23. Line ℓ passes through A(-2, -4) and has a slope of $-\frac{1}{2}$. What is the standard form of the equation for line ℓ ?

24. Line m passes through C(-2, 0) and D(1, -3). Write the equation of line m in standard form.

25. **Sports** Skiers, hikers, and climbers often experience altitude sickness as they reach elevations of 8000 feet and more. A good rule of thumb for the amount of time that it takes to become acclimated to high elevations is 2 weeks for the first 7000 feet. After that, it will take 1 week more for each additional 2000 feet of altitude.

- Write an equation for the time t to acclimate to an altitude of f feet.
- Mt. Whitney in California is the highest peak in the contiguous 48 states. It is located in Eastern Sierra Nevada, on the border between Sequoia National Park and Inyo National Forest. About how many weeks would it take a person to acclimate to Mt. Whitney's elevation of 14,494 feet?

26. **Critical Thinking** Write an expression for the slope of a line whose equation is $Ax + By + C = 0$.

27. **Transportation** The mileage in miles per gallon (mpg) for city and highway driving of several 1999 models are given in the chart.

Model	City (mpg)	Highway (mpg)
A	24	32
B	20	29
C	20	29
D	20	28
E	23	30
F	24	30
G	27	37
H	22	28



- Find a linear equation that can be used to find a car's highway mileage based on its city mileage.
- Model J's city mileage is 19 mpg. Use your equation to predict its highway mileage.
- Highway mileage for Model J is 26 mpg. How well did your equation predict the mileage? Explain.

Applications and Problem Solving



- 28. Economics** Research the average personal income for the current year.
- Find the value that the equation in Example 2 predicts.
 - Is the average personal income equal to the prediction? Explain any difference.
- 29. Critical Thinking** Determine whether the points at (5, 9), (-3, 3), and (1, 6) are collinear. Justify your answer.

Mixed Review

- 30.** Graph $3x - 2y - 5 = 0$. (Lesson 1-3)
- 31. Business** In 1995, retail sales of apparel in the United States were \$70,583 billion. Apparel sales were \$82,805 billion in 1997. (Lesson 1-3)
- Assuming a linear relationship, find the average annual rate of increase.
 - Explain how the rate is related to the graph of the line.
- 32.** If $f(x) = x^3$ and $g(x) = 3x$, find $g[f(-2)]$. (Lesson 1-2)
- 33.** Find $(f \cdot g)(x)$ and $\left(\frac{f}{g}\right)(x)$ for $f(x) = x^3$ and $g(x) = x^2 - 3x + 7$. (Lesson 1-2)
- 34.** Given that x is an integer, state the relation representing $y = x^2$ and $-4 \leq x \leq -2$ by listing a set of ordered pairs. Then state whether this relation is a function. (Lesson 1-1)
- 35. SAT/ACT Practice** If $xy = 1$, then x is the reciprocal of y . Which of the following is the arithmetic mean of x and y ?
- A $\frac{y^2 + 1}{2y}$ B $\frac{y + 1}{2y}$ C $\frac{y^2 + 2}{2y}$
 D $\frac{y^2 + 1}{y}$ E $\frac{x^2 + 1}{y}$

MID-CHAPTER QUIZ

- What are the domain and the range of the relation $\{(-2, -3), (-2, 3), (4, 7), (2, -8), (4, 3)\}$? Is the relation a function? Explain. (Lesson 1-1)
- Find $f(4)$ for $f(x) = 7 - x^2$. (Lesson 1-1)
- If $g(x) = \frac{3}{x-1}$, what is $g(n+2)$? (Lesson 1-1)
- Retail** Amparo bought a jacket with a gift certificate she received as a birthday present. The jacket was marked 33% off, and the sales tax in her area is 5.5%. If she paid \$45.95 for the jacket, use composition of functions to determine the original price of the jacket. (Lesson 1-2)
- If $f(x) = \frac{1}{x-1}$ and $g(x) = x + 1$, find $[f \circ g](x)$ and $[g \circ f](x)$. (Lesson 1-2)

Graph each equation. (Lesson 1-3)

- $2x - 4y = 8$ 7. $3x = 2y$
- Find the zero of $f(x) = 5x - 3$. (Lesson 1-3)
- Points $A(2, 5)$ and $B(7, 8)$ lie on line ℓ . What is the standard form of the equation of line ℓ ? (Lesson 1-4)
- Demographics** In July 1990, the population of Georgia was 6,506,416. By July 1997, the population had grown to 7,486,242. (Lesson 1-4)
 - If x represents the year and y represents the population, find the average annual rate of increase of the population.
 - Write an equation to model the population change.