1-8

Graphing Linear Inequalities

OBJECTIVE
- Graph linear inequalities.

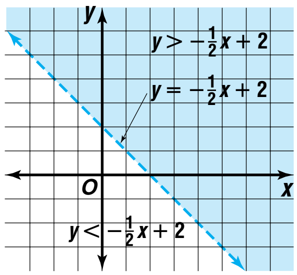
Real World Application

NUTRITION Arctic explorers need endurance and strength. They move sleds weighing more than 1100 pounds each for as much as 12 hours a day. For that reason, Will Steger and members of his exploration team each burn 4000 to 6000 Calories daily!

An *endurance diet* can provide the energy and nutrients necessary for peak performance in the Arctic. An endurance diet has a balance of fat and carbohydrates and protein. Fat is a concentrated energy source that supplies nine calories per gram. Carbohydrates and protein provide four calories per gram and are a quick source of energy. What are some of the combinations of carbohydrates and protein and fat that supply the needed energy for the Arctic explorers? *This problem will be solved in Example 2.*

This situation can be described using a **linear inequality.** A linear inequality is not a function. However, you can use the graphs of linear functions to help you graph linear inequalities.

The graph of $y = -\frac{1}{2}x + 2$ separates the coordinate plane into two regions, called **half planes.** The line described by $y = -\frac{1}{2}x + 2$ is called the **boundary** of each region. If the boundary is part of a graph, it is drawn as a solid line. A boundary that is not part of the graph is drawn as a dashed line. The graph of $y > -\frac{1}{2}x + 2$ is the region above the line. The graph of $y < -\frac{1}{2}x + 2$ is the region below the line.



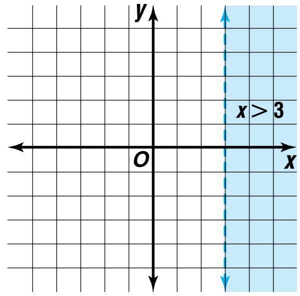
When graphing an inequality, you can determine which half plane to shade by testing a point on either side of the boundary in the original inequality. If it is not on the boundary, the origin (0, 0) is often an easy point to test. If the inequality statement is true for your test point, then shade the half plane that contains the test point. If the inequality statement is false for your test point, then shade the half plane that does not contain the test point.

Example 1 **Graph each inequality.**

a. $x > 3$

The boundary is not included in the graph. So the vertical line $x = 3$ should be a dashed line.

Testing (0, 0) in the inequality yields a false inequality, $0 > 3$. So shade the half plane that does not include (0, 0).

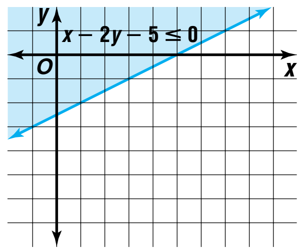


b. $x - 2y - 5 \leq 0$

$$x - 2y - 5 \leq 0$$
$$-2y \leq -x + 5$$
$$y \geq \frac{1}{2}x - \frac{5}{2}$$ *Reverse the inequality when you divide or multiply by a negative.*

The graph does include the boundary. So the line is solid.

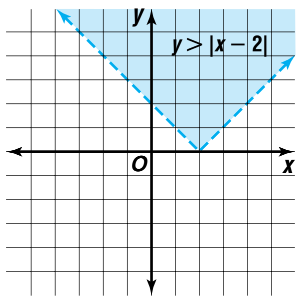
Testing (0, 0) in the inequality yields a true inequality, so shade the half plane that includes (0, 0).



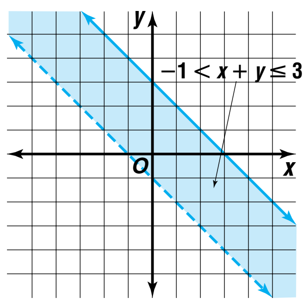
c. $y > |x - 2|$

Graph the equation $y = |x - 2|$ with a dashed boundary.

Testing (0, 0) yields the false inequality $0 > 2$, so shade the region that does not include (0, 0).



You can also graph relations such as $-1 < x + y \leq 3$. The graph of this relation is the intersection of the graph of $-1 < x + y$ and the graph of $x + y \leq 3$. Notice that the boundaries $x + y = 3$ and $x + y = -1$ are parallel lines. The boundary $x + y = 3$ is part of the graph, but $x + y = -1$ is not.



Example **2** **NUTRITION** **Refer to the application at the beginning of the lesson.**

Real World Application

a. Draw a graph that models the combinations of grams of fat and carbohydrates and protein that the arctic team diet may include to satisfy their daily caloric needs.

Let x represent the number of grams of fat and y represent the number of grams of carbohydrates and protein. The team needs at least 4000, but no more than 6000, Calories each day. Write an inequality.

$$4000 \leq 9x + 4y \leq 6000$$

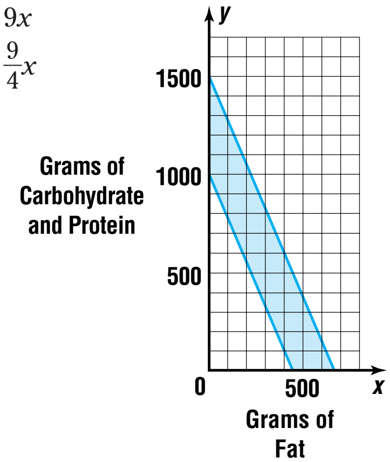
You can write this compound inequality as two inequalities, $4000 \leq 9x + 4y$ and $9x + 4y \leq 6000$. Solve each part for y.

$$4000 \leq 9x + 4y \quad \text{and} \quad 9x + 4y \leq 6000$$

$$4000 - 9x \leq 4y \qquad\qquad 4y \leq 6000 - 9x$$

$$1000 - \frac{9}{4}x \leq y \qquad\qquad y \leq 1500 - \frac{9}{4}x$$

Graph each boundary line and shade the appropriate region. The graph of the compound inequality is the area in which the shading overlaps.



b. Name three combinations of fat or carbohydrates and protein that meet the Calorie requirements.

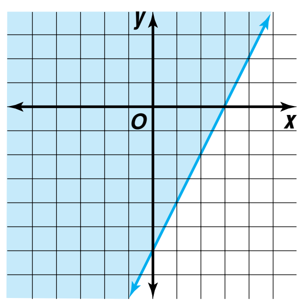
Any point in the shaded region or on the boundary lines meets the requirements. Three possible combinations are (100, 775), (200, 800), and (300, 825). These ordered pairs represent 100 grams of fat and 775 grams of carbohydrate and protein, 200 grams of fat and 800 grams of carbohydrate and protein, and 300 grams of fat and 825 grams of carbohydrate and protein.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Read and study the lesson to answer each question.

1. **Write** the inequality whose graph is shown.
2. **Describe** the process you would use to graph $-3 \leq 2x + y < 7$.
3. **Explain** why you can use a test point to determine which region or regions of the graph of an inequality should be shaded.



Guided Practice Graph each inequality.

4. $x + y < 4$
5. $3x - y \leq 6$
6. $7 < x + y \leq 9$
7. $y < |x + 3|$

8. **Business** Nancy Stone has a small company and has negotiated a special rate for rental cars when she and other employees take business trips. The maximum charge is \$45.00 per day plus \$0.40 per mile. Discounts apply when renting for longer periods of time or during off-peak seasons.
   a. Write a linear inequality that models the total cost of the daily rental $c(m)$ as a function of the total miles driven, m.
   b. Graph the inequality.
   c. Name three combinations of miles and total cost that satisfy the inequality.

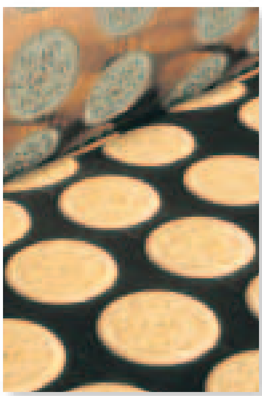
EXERCISES

Practice Graph each inequality.

9. $y < 3$
10. $x - y > -5$
11. $2x + 4y \geq 7$
12. $-y < 2x + 1$
13. $2x - 5y + 19 \leq 0$
14. $-4 \leq x - y \leq 5$
15. $y \geq |x|$
16. $-2 \leq x + 2y \leq 4$
17. $y > |x| + 4$
18. $y < |2x + 3|$
19. $-8 \leq 2x + y < 6$
20. $y - 1 > |x + 3|$
21. Graph the region that satisfies $x \geq 0$ and $y \geq 0$.
22. Graph $2 < |x| \leq 8$.

Applications and Problem Solving

Real World Application



23. **Manufacturing** Many manufacturers use inequalities to solve production problems such as determining how much of each product should be assigned to each machine. Suppose one bakery oven at a cookie manufacturer is being used to bake chocolate cookies and vanilla cookies. A batch of chocolate cookies bakes in 8 minutes, and a batch of vanilla cookies bakes in 10 minutes.
   a. Let x represent the number of batches of chocolate cookies and y represent the number of batches of vanilla cookies. Write a linear inequality for the number of batches of each type of cookie that could be baked in one oven in an 8-hour shift.
   b. Graph the inequality.
   c. Name three combinations of batches of chocolate cookies and vanilla cookies that satisfy the inequality.
   d. Often manufacturers' problems involve as many as 150 products, 218 facilities, 10 plants, and 127 customer zones. Research how problems like this are solved.

24. **Critical Thinking** Graph $|y| \geq x$.

25. Critical Thinking Suppose $xy > 0$.

a. Describe the points whose coordinates are solutions to the inequality.

b. Demonstrate that for points whose coordinates are solutions to the inequality, the equation $|x + y| = |x| + |y|$ holds true.

26. Engineering Mechanics The production cost of a job depends in part on the accuracy required. On most sheet metal jobs, an accuracy of 1, 2, or 0.1 mils is required. A mil is $\frac{1}{1000}$ inch. This means that a dimension must be less than $\frac{1}{1000}$, $\frac{2}{1000}$, or $\frac{1}{10{,}000}$ inch larger or smaller than the blueprint states. Industrial jobs often require a higher degree of accuracy.

a. Write inequalities that models the possible dimensions of a part that is supposed to be 8 inches by $4\frac{1}{4}$ inches if the accuracy required is 2 mils.

b. Graph the region that shows the satisfactory dimensions for the part.

27. Exercise The American College of Sports Medicine recommends that healthy adults exercise at a target level of 60% to 90% of their maximum heart rate. You can estimate your maximum heart rate by subtracting your age from 220.

a. Write a compound inequality that models age, a, and target heart rate, r.

b. Graph the inequality.

Mixed Review

28. Business Gatsby's Automotive Shop charges $55 per hour or any fraction of an hour for labor. *(Lesson 1-7)*

a. What type of function is described?

b. Write the labor charge as a function of the time.

c. Graph the function.

29. The equation of line ℓ is $3x - y = 10$. *(Lesson 1-5)*

a. What is the standard form of the equation of the line that is parallel to ℓ and passes through the point at $(0, -2)$?

b. Write the standard form of the equation of the line that is perpendicular to ℓ and passes through the point at $(0, -2)$.

30. Write the slope-intercept form of the equation of the line through (1, 4) and (5, 7). *(Lesson 1-4)*

31. Temperature The temperature in Indianapolis on January 30 was 23°F at 12:00 A.M. and 48°F at 4:00 P.M. *(Lesson 1-3)*

a. Write two ordered pairs of the form (hours since midnight, temperature) for this date. What is the slope of the line containing these points?

b. What does the slope of the line represent?

32. SAT/ACT Practice Which expression is equivalent to $\frac{9^5 - 9^4}{8}$?

A $\frac{1}{8}$ **B** $\frac{9}{8}$ **C** $\frac{9^3}{8}$

D $\frac{9^9}{8}$ **E** 9^4

Extra Practice See p. A27.