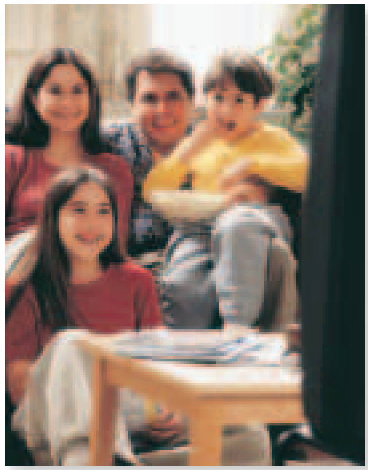
13-3

Probability and Odds

OBJECTIVES
- Find the probability of an event.
- Find the odds for the success and failure of an event.

Real World Application

MARKET RESEARCH To determine television ratings, Nielsen Media Research estimates how many people are watching any given television program. This is done by selecting a sample audience, having them record their viewing habits in a journal, and then counting the number of viewers for each program. There are about 100 million households in the U.S., and only 5000 are selected for the sample group. What is the probability of any one household being selected to participate? *This problem will be solved in Example 1.*



When we are uncertain about the occurrence of an event, we can measure the chances of its happening with **probability.** For example, there are 52 possible outcomes when selecting a card at random from a standard deck of playing cards. The set of all outcomes of an event is called the **sample space.** A desired outcome, drawing the king of hearts for example, is called a **success.** Any other outcome is called a **failure.** The probability of an event is the ratio of the number of ways an event can happen to the total number of outcomes in the sample space, which is the sum of successes and failures. There is one way to draw a king of hearts, and there are a total of 52 outcomes when selecting a card from a standard deck. So, the probability of selecting the king of hearts is $\frac{1}{52}$.

Probability of Success and of Failure

If an event can succeed in s ways and fail in f ways, then the probability of success $P(s)$ and the probability of failure $P(f)$ are as follows.

$$P(s) = \frac{s}{s+f} \qquad P(f) = \frac{f}{s+f}$$

Example 1 Real World Application

MARKET RESEARCH **What is the probability of any one household being chosen to participate for the Nielsen Media Research group?**

Use the probability formula. Since 5000 households are selected to participate $s = 5000$. The denominator, $s + f$, represents the total number of households, those selected, s, and those not selected, f. So, $s + f = 100{,}000{,}000$.

$$P(5000) = \frac{5000}{100{,}000{,}000} \text{ or } \frac{1}{20{,}000} \qquad P(s) = \frac{s}{s+f}$$

The probability of any one household being selected is $\frac{1}{20{,}000}$ or 0.005%.

An event that cannot fail has a probability of 1. An event that cannot succeed has a probability of 0. Thus, the probability of success $P(s)$ is always between 0 and 1 inclusive. That is, $0 \leq P(s) \leq 1$.

Example 2 **A bag contains 5 yellow, 6 blue, and 4 white marbles.**

a. What is the probability that a marble selected at random will be yellow?

b. What is the probability that a marble selected at random will *not* be white?

a. The probability of selecting a yellow marble is written $P(\text{yellow})$. There are 5 ways to select a yellow marble from the bag, and 6 + 4 or 10 ways not to select a yellow marble. So, $s = 5$ and $f = 10$.

$$P(\text{yellow}) = \frac{5}{5 + 10} \text{ or } \frac{1}{3} \qquad P(s) = \frac{s}{s + f}$$

The probability of selecting a yellow marble is $\frac{1}{3}$

b. There are 4 ways to select a white marble. So there are 11 ways not to select a white marble.

$$P(\text{not white}) = \frac{11}{4 + 11} \text{ or } \frac{11}{15}$$

The probability of *not* selecting a white marble is $\frac{11}{15}$.

The counting methods you used for permutations and combinations are often used in determining probability.

Example 3 **A circuit board with 20 computer chips contains 4 chips that are defective. If 3 chips are selected at random, what is the probability that all 3 are defective?**

There are $C(4, 3)$ ways to select 3 out of 4 defective chips, and $C(20, 3)$ ways to select 3 out of 20 chips.

$$P(3 \text{ defective chips}) = \frac{C(4, 3)}{C(20, 3)} \begin{array}{l} \leftarrow \textit{ways of selecting 3 defective chips} \\ \leftarrow \textit{ways of selecting 3 chips} \end{array}$$

$$= \frac{\frac{4!}{1!\,3!}}{\frac{20!}{17!\,3!}} \text{ or } \frac{1}{285}$$

The probability of selecting three defective computer chips is $\frac{1}{285}$.

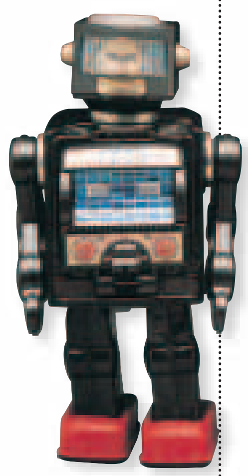
The sum of the probability of success and the probability of failure for any event is always equal to 1.

$$P(s) + P(f) = \frac{s}{s + f} + \frac{f}{s + f}$$

$$= \frac{s + f}{s + f} \text{ or } 1$$

This property is often used in finding the probability of events. For example, the probability of drawing a king of hearts is $P(s) = \frac{1}{52}$, so the probability of not drawing the king of hearts is $P(f) = 1 - \frac{1}{52}$ or $\frac{51}{52}$. Because their sum is 1, $P(s)$ and $P(f)$ are called **complements.**

Example 4 **The CyberToy Company has determined that out of a production run of 50 toys, 17 are defective. If 5 toys are chosen at random, what is the probability that at least 1 is defective?**



The complement of selecting at least 1 defective toy is selecting no defective toys. That is, P(at least 1 defective toy) = 1 − P(no defective toys).

$$P(\text{at least 1 defective toy}) = 1 - P(\text{no defective toys}).$$

$$= 1 - \frac{C(33, 5)}{C(50, 5)} \quad \begin{array}{l} \leftarrow \textit{ways of selecting 5 defective toys} \\ \leftarrow \textit{ways of selecting 5 toys} \end{array}$$

$$= 1 - \frac{237{,}336}{2{,}118{,}760}$$

$$\approx 0.8879835375 \quad \textit{Use a calculator.}$$

The probability of selecting at least 1 defective toy is about 89%.

Another way to measure the chance of an event occurring is with **odds.** The probability of success of an event and its complement are used when computing the odds of an event.

Odds

The odds of the successful outcome of an event is the ratio of the probability of its success to the probability of its failure.

$$\text{Odds} = \frac{P(s)}{P(f)}$$

Example 5 **Katrina must select at random a chip from a box to determine which question she will receive in a mathematics contest. There are 6 blue and 4 red chips in the box. If she selects a blue chip, she will have to solve a trigonometry problem. If the chip is red, she will have to write a geometry proof.**

a. What is the probability that Katrina will draw a red chip?

b. What are the odds that Katrina will have to write a geometry proof?

a. The probability that Katrina will select a red chip is $\frac{4}{10}$ or $\frac{2}{5}$.

b. To find the odds that Katrina will have to write a geometry proof, you need to know the probability of a successful outcome and of a failing outcome.

Let s represent selecting a red chip and f represent not selecting a red chip.

$P(s) = \frac{2}{5}$ $\qquad$ $P(f) = 1 - \frac{2}{5}$ or $\frac{3}{5}$

Now find the odds.

$$\frac{P(s)}{P(f)} = \frac{\frac{2}{5}}{\frac{3}{5}} \text{ or } \frac{2}{3}$$

The odds that Katrina will choose a red chip and thus have to write a geometry proof is $\frac{2}{3}$. *The ratio $\frac{2}{3}$ is read "2 to 3."*

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Sometimes when computing odds, you must find the sample space first. This can involve finding permutations and combinations.

Example 6 **Twelve male and 16 female students have been selected as equal qualifiers for 6 college scholarships. If the awarded recipients are to be chosen at random, what are the odds that 3 will be male and 3 will be female?**

First, determine the total number of possible groups.

$C(12, 3)$ *number of groups of 3 males*
$C(16, 3)$ *number of groups of 3 females*

Using the Basic Counting Principle we can find the number of possible groups of 3 males and 3 females.

$C(12, 3) \cdot C(16, 3) = \frac{12!}{9!\,3!} \cdot \frac{16!}{13!\,3!}$ or 123,200 possible groups

The total number of groups of 6 recipients out of the 28 who qualified is $C(28, 6)$ or 376,740. So, the number of groups that do not have 3 males and 3 females is $376{,}740 - 123{,}200$ or 253,540.

Finally, determine the odds.

$P(s) = \frac{123{,}200}{376{,}740}$ $P(f) = \frac{253{,}540}{376{,}740}$

$$\text{odds} = \frac{\frac{123{,}200}{376{,}740}}{\frac{253{,}540}{376{,}740}} \text{ or } \frac{880}{1811}$$

Thus, the odds of selecting a group of 3 males and 3 females are $\frac{880}{1811}$ or close to $\frac{1}{2}$.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Read and study the lesson to answer each question.

1. **Explain** how you would interpret $P(E) = \frac{1}{2}$.
2. Find two examples of the use of probability in newspapers or magazines. **Describe** how probability concepts are applied.
3. **Write** about the difference between the probability of the successful outcome of an event and the odds of the successful outcome of an event.
4. **You Decide** Mika has figured that his odds of winning the student council election are 3 to 2. Geraldo tells him that, based on those odds, the probability of his winning is 60%. Mika disagreed. Who is correct? Explain your answer.

Guided Practice

A box contains 3 tennis balls, 7 softballs, and 11 baseballs. One ball is chosen at random. Find each probability.

5. P(softball)
6. P(not a baseball)
7. P(golf ball)
8. In an office, there are 7 women and 4 men. If one person is randomly called on the phone, find the probability the person is a woman.

Of 7 kittens in a litter, 4 have stripes. Three kittens are picked at random. Find the odds of each event.

9. All three have stripes. **10.** Only 1 has stripes. **11.** One is not striped.

12. Meteorology A local weather forecast states that the probability of rain on Saturday is 80%. What are the odds that it will not rain Saturday? (*Hint*: Rewrite the percent as a fraction.)

EXERCISES

Practice

Using a standard deck of 52 cards, find each probability. *The face cards include kings, queens, and jacks.*

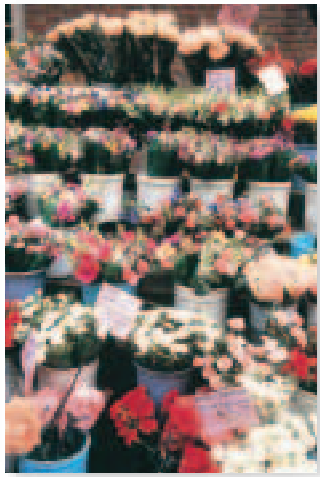
13. *P*(face card) **14.** *P*(a card of 6 or less)

15. *P*(a black, non-face card) **16.** *P*(not a face card)

One flower is randomly taken from a vase containing 5 red flowers, 2 white flowers, and 3 pink flowers. Find each probability.

17. *P*(red) **18.** *P*(white)

19. *P*(not pink) **20.** *P*(red or pink)



Jacob has 10 rap, 18 rock, 8 country, and 4 pop CDs in his music collection. Two are selected at random. Find each probability.

21. *P*(2 pop) **22.** *P*(2 country)

23. *P*(1 rap and 1 rock) **24.** *P*(not rock)

25. A number cube is thrown two times. What is the probability of rolling 2 fives?

A box contains 1 green, 2 yellow, and 3 red marbles. Two marbles are drawn at random without replacement. What are the odds of each event occurring?

26. drawing 2 red marbles **27.** not drawing yellow marbles

28. drawing 1 green and 1 red **29.** drawing two different colors

Of 27 students in a class, 11 have blue eyes, 13 have brown eyes, and 3 have green eyes. If 3 students are chosen at random what are the odds of each event occurring?

30. all three have blue eyes **31.** 2 have brown and 1 has blue eyes

32. no one has brown eyes **33.** only 1 has green eyes

34. The odds of winning a prize in a raffle with one raffle ticket are $\frac{1}{249}$. What is the probability of winning with one ticket?

35. The probability of being accepted to attend a state university is $\frac{4}{5}$. What are the odds of being accepted to this university?

36. From a deck of 52 cards, 5 cards are drawn. What are the odds of having three cards of one suit and the other two cards be another suit?

Applications and Problem Solving

Real World Application

37. Weather During a particular hurricane, hurricane trackers determine that the odds of it hitting the South Carolina coast are 1 to 4. What is the probability of this happening?

38. Baseball At one point in the 1999 season, Ken Griffey, Jr. had a batting average of 0.325. What are the odds that he would hit the ball the next time he came to bat?

39. Security Kim uses a combination lock on her locker that has 3 wheels, each labeled with 10 digits from 0 to 9. The combination is a particular sequence with no digits repeating.

a. What is the probability of someone guessing the correct combination?

b. If the digits can be repeated, what are the odds against someone guessing the combination?

40. Critical Thinking Spencer is carrying out a survey of the bear population at Yellowstone National Park. He spots two bears—one has a light colored coat and the other has a dark coat.

a. Assume that there are equal numbers of male and female bears in the park. What is the probability that both bears are male?

b. If the lighter colored bear is male, what are the odds that both are male?

41. Testing Ms. Robinson gives her precalculus class 20 study problems. She will select 10 to answer on an upcoming test. Carl can solve 15 of the problems.

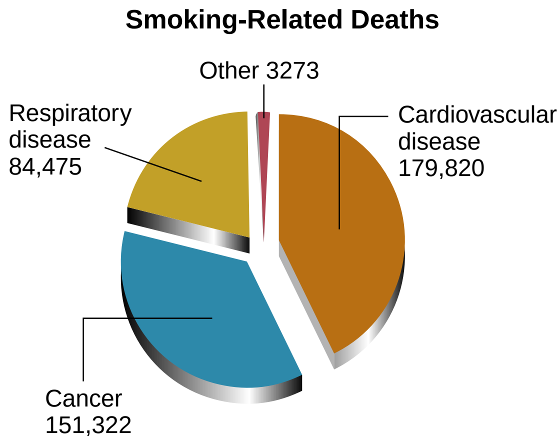
a. Find the probability that Carl can solve all 10 problems on the test.

b. Find the odds that Carl will know how to solve 8 of the problems.

42. Mortality Rate During 1990, smoking was linked to 418,890 deaths in the United States. The graph shows the diseases that caused these smoking-related deaths.

a. Find the probability that a smoking-related death was the result of either cardiovascular disease or cancer.

b. Determine the odds against a smoking-related death being caused by cancer.



43. Critical Thinking A plumber cuts a pipe in two pieces at a point selected at random. What is the probability that the length of the longer piece of pipe is at least 8 times the length of the shorter piece of pipe?

Mixed Review

44. A food vending machine has 6 different items on a revolving tray. How many different ways can the items be arranged on the tray? *(Lesson 13-2)*

45. The Foxtrail Condominium Association is electing board members. How many groups of 4 can be chosen from the 10 candidates who are running? *(Lesson 13-1)*

46. Find S_{14} for the arithmetic series for which $a_1 = 3.2$ and $d = 1.5$. *(Lesson 12-1)*

47. Simplify $7^{\log_7 2x}$. *(Lesson 11-4)*

48. **Landscaping** Carolina bought a new sprinkler to water her lawn. The sprinkler rotates 360° while spraying a stream of water. Carolina places the sprinkler in her yard so the ordered pair that represents its location is (7, 2), and the sprinkler sends out water that just barely reaches the point at (10, −8). Find an equation representing the farthestmost points the water can reach. *(Lesson 10-2)*

49. Find the product $3(\cos \pi + \boldsymbol{i} \sin \pi) \cdot 2\left(\cos \frac{\pi}{4} + \boldsymbol{i} \sin \frac{\pi}{4}\right)$. Then express it in rectangular form. *(Lesson 9-7)*

50. Find an ordered pair to represent $\vec{\mathbf{u}}$ if $\vec{\mathbf{u}} = \vec{\mathbf{v}} + \vec{\mathbf{w}}$, if $\vec{\mathbf{v}} = \langle 3, -5 \rangle$ and $\vec{\mathbf{w}} = \langle -4, 2 \rangle$. *(Lesson 8-2)*

51. **SAT Practice** What is the area of an equilateral triangle with sides $2s$ units long?

A s^2 units^2

B $\sqrt{3}s^2$ units^2

C $2s^2$ units^2

D $4s^2$ units^2

E $6s^2$ units^2

MID-CHAPTER QUIZ

Find each value. (Lesson 13-1)

1. $P(15, 5)$.

2. $C(20, 9)$.

3. Regular license plates in Ohio have three letters followed by four digits. How many different license plate arrangements are possible? (Lesson 13-1)

4. Suppose there are 12 runners competing in the finals of a track event. Awards are given to the top five finishers. How many top-five arrangements are possible? (Lesson 13-1)

5. An ice cream shop has 18 different flavors of ice cream, which can be ordered in a cup, sugar cone, or waffle cone. There is also a choice of six toppings. How many two-scoop servings with a topping are possible? (Lesson 13-1)

6. How many nine-letter patterns can be formed from the letters in the word *quadratic*? (Lesson 13-2)

7. How many different arrangements can be made with ten pieces of silverware laid in a row if three are identical spoons, four are identical forks, and three are identical knives? (Lesson 13-2)

8. Eight children are riding a merry-go-round. How many ways can they be seated? (Lesson 13-2)

9. Two cards are drawn at random from a standard deck of 52 cards. What is the probability that both are hearts? (Lesson 13-3)

10. A bowl contains four apples, three bananas, three oranges, and two pears. If two pieces of fruit are selected at random, what are the odds of selecting an orange and a banana? (Lesson 13-3)

Extra Practice See p. A51.