13-4 Probabilities of Compound Events

OBJECTIVES
- Find the probability of independent and dependent events.
- Identify mutually exclusive events.
- Find the probability of mutually exclusive and inclusive events.

Real World Application

TRANSPORTATION According to U.S. Department of Transportation statistics, the top ten airlines in the United States arrive on time 80% of the time. During their vacation, the Hiroshi family has direct flights to Washington, D.C., Chicago, Seattle, and San Francisco on different days. What is the probability that all their flights arrived on time?

Since the flights occur on different days, the four flights represent independent events. Let A represent an on-time arrival of an airplane.

$$\begin{aligned} P(\text{all flights on time}) &= \underbrace{P(A)}_{\text{Flight 1}} \cdot \underbrace{P(A)}_{\text{Flight 2}} \cdot \underbrace{P(A)}_{\text{Flight 3}} \cdot \underbrace{P(A)}_{\text{Flight 4}} \\ &= (0.80)^4 \quad A = 0.80 \\ &\approx 0.4096 \text{ or about } 41\% \end{aligned}$$

Thus, the probability of all four flights arriving on time is about 41%.

This problem demonstrates that the probability of more than one independent event is the product of the probabilities of the events.

Probability of Two Independent Events

If two events, A and B, are independent, then the probability of both events occurring is the product of each individual probability.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Example 1 **Using a standard deck of playing cards, find the probability of selecting a face card, replacing it in the deck, and then selecting an ace.**

Let A represent a face card for the first card drawn from the deck, and let B represent the ace in the second selection.

$P(A) = \frac{12}{52}$ or $\frac{3}{13}$ $\quad \frac{\text{12 face cards}}{\text{52 cards in a standard deck}}$

$P(B) = \frac{4}{52}$ or $\frac{1}{13}$ $\quad \frac{\text{4 aces}}{\text{52 cards in a standard deck}}$

The two draws are independent because when the card is returned to the deck, the outcome of the second draw is not affected by the first one.

$$\begin{aligned} P(A \text{ and } B) &= P(A) \cdot P(B) \\ &= \frac{3}{13} \cdot \frac{1}{13} \text{ or } \frac{3}{169} \end{aligned}$$

The probability of selecting a face card first, replacing it, and then selecting an ace is $\frac{3}{169}$.

Example 2 **OCCUPATIONAL HEALTH** **Statistics collected in a particular coal-mining region show that the probability that a miner will develop black lung disease is $\frac{5}{11}$. Also, the probability that a miner will develop arthritis is $\frac{1}{5}$. If one health problem does not affect the other, what is the probability that a randomly-selected miner will not develop black lung disease but will develop arthritis?**

Real World Application

The events are independent since having black lung disease does not affect the existence of arthritis.

$$P(\text{not black lung disease and arthritis}) = [1 - P(\text{black lung disease})] \cdot P(\text{arthritis})$$
$$= \left(1 - \frac{5}{11}\right) \cdot \frac{1}{5} \text{ or } \frac{6}{55}$$

The probability that a randomly-selected miner will not develop black lung disease but will develop arthritis is $\frac{6}{55}$.

What do you think the probability of selecting two face cards would be if the first card drawn were not placed back in the deck? Unlike the situation in Example 1, these events are dependent because the outcome of the first event affects the second event. This probability is also calculated using the product of the probabilities.

first card $\quad P(\text{face card}) = \frac{12}{52}$

second card $\quad P(\text{face card}) = \frac{11}{51}$

$$P(\text{two face cards}) = \frac{12}{52} \cdot \frac{11}{51} \text{ or } \frac{11}{221}$$

Notice that when a face card is removed from the deck, not only is there one less face card, but also one less card in the deck.

Thus, the probability of selecting two face cards from a deck without replacing the cards is $\frac{11}{221}$ or about $\frac{1}{20}$.

Probability of Two Dependent Events

If two events, A and B, are dependent, then the probability of both events occurring is the product of each individual probability.

$$P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$$

Example 3 **Tasha has 3 rock, 4 country, and 2 jazz CDs in her car. One day, before she starts driving, she pulls 2 CDs from her CD carrier without looking.**

a. Determine if the events are independent or dependent.

b. What is the probability that both CDs are rock?

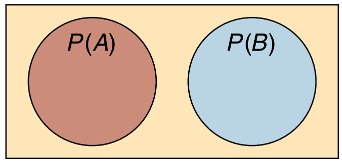
a. The events are dependent. This event is equivalent to selecting one CD, not replacing it, then selecting another CD.

b. Determine the probability.

$$P(\text{rock, rock}) = P(\text{rock}) \cdot P(\text{rock following first rock selection})$$
$$P(\text{rock, rock}) = \frac{3}{9} \cdot \frac{2}{8} \text{ or } \frac{1}{12}$$

The probability that Tasha will select two rock CDs is $\frac{1}{12}$.

There are times when two events cannot happen at the same time. For example, when tossing a number cube, what is the probability of tossing a 2 *or* a 5? In this situation, both events cannot happen at the same time. That is, the events are **mutually exclusive.** The probability of tossing a 2 or a 5 is $P(2) + P(5)$, which is $\frac{1}{6} + \frac{1}{6}$ or $\frac{2}{6}$.



Events A and B are mutually exclusive.

Note that the two events do not overlap, as shown in the Venn diagram. So, the probability of two mutually exclusive events occurring can be represented by the sum of the areas of the circles.

Probability of Mutually Exclusive Events

If two events, A and B, are mutually exclusive, then the probability that either A or B occurs is the sum of their probabilities.

$$P(A \text{ or } B) = P(A) + P(B)$$

Example 4

Lenard is a contestant in a game where if he selects a blue ball or a red ball he gets an all-expenses paid Caribbean cruise. Lenard must select the ball at random from a box containing 2 blue, 3 red, 9 yellow, and 10 green balls. What is the probability that he will win the cruise?

These are mutually exclusive events since Lenard cannot select a blue and a red ball at the same time. Find the sum of the individual probabilities.

$P(\text{blue or red}) = P(\text{blue}) + P(\text{red})$

$= \frac{2}{24} + \frac{3}{24}$ or $\frac{5}{24}$ *$P(\text{blue}) = \frac{2}{24}$, $P(\text{red}) = \frac{3}{24}$*

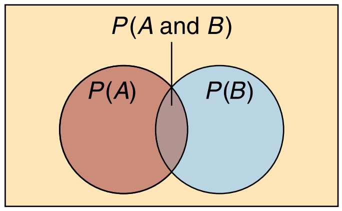
The probability that Lenard will win the cruise is $\frac{5}{24}$.

What is the probability of rolling two number cubes, in which the first number cube shows a 2 or the sum of the number cubes is 6 or 7? Since each number cube can land six different ways, and two number cubes are rolled, the sample space can be represented by making a chart. A **reduced sample space** is the subset of a sample space that contains only those outcomes that satisfy a given condition.

		Second Number Cube					
		1	2	3	4	5	6
First Number Cube	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

It is possible to have the first number cube show a 2 *and* have the sum of the two number cubes be 6 or 7. Therefore, these events are not mutually exclusive. They are called **inclusive events.** In this case, you must adjust the formula for mutually exclusive events.

Note that the circles in the Venn diagram overlap. This area represents the probability of both events occurring at the same time. When the areas of the two circles are added, this overlapping area is counted twice. Therefore, it must be subtracted to find the correct probability of the two events.



Events A and B are inclusive events.

Let A represent the event "the first number cube shows a 2".

Let B represent the event "the sum of the two number cubes is 6 or 7".

$P(A) = \frac{6}{36}$ $\qquad$ $P(B) = \frac{11}{36}$

Note that (2, 4) and (2, 5) are counted twice, both as the first cube showing a 2 and as a sum of 6 or 7. To find the correct probability, you must subtract P(2 and sum of 6 or 7).

$$P(\text{2 or sum of 6 or 7}) = \underbrace{\frac{6}{36}}_{P(2)} + \underbrace{\frac{11}{36}}_{P(\text{sum of 6 or 7})} - \underbrace{\frac{2}{36}}_{P(\text{2 and sum of 6 or 7})} \text{ or } \frac{15}{36}$$

The probability of the first number cube showing a 2 or the sum of the number cubes being 6 or 7 is $\frac{15}{36}$ or $\frac{5}{12}$.

Probability of Inclusive Events

If two events, A and B, are inclusive, then the probability that either A or B occurs is the sum of their probabilities decreased by the probability of both occurring.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Examples

5 **Kerry has read that the probability for a driver's license applicant to pass the road test the first time is $\frac{5}{6}$. He has also read that the probability of passing the written examination on the first attempt is $\frac{9}{10}$. The probability of passing both the road and written examinations on the first attempt is $\frac{4}{5}$.**

a. Determine if the events are mutually exclusive or mutually inclusive.

Since it is possible to pass both the road examination and the written examination, these events are mutually inclusive.

b. What is the probability that Kerry can pass either examination on his first attempt?

$P(\text{passing road exam}) = \frac{5}{6}$ $\qquad$ $P(\text{passing written exam}) = \frac{9}{10}$

$P(\text{passing both exams}) = \frac{4}{5}$

$P(\text{passing either examination}) = \frac{5}{6} + \frac{9}{10} - \frac{4}{5} = \frac{56}{60} \text{ or } \frac{14}{15}$

The probability that Kerry will pass either test on his first attempt is $\frac{14}{15}$.

6 **There are 5 students and 4 teachers on the school publications committee. A group of 5 members is being selected at random to attend a workshop on school newspapers. What is the probability that the group attending the workshop will have at least 3 students?**

At least 3 students means the groups may have 3, 4, or 5 students. It is not possible to select a group of 3 students, a group of 4 students, and a group of 5 students in the same 5-member group. Thus, the events are mutually exclusive.

$$P(\text{at least 3 students}) = P(3 \text{ students}) + P(4 \text{ students}) + P(5 \text{ students})$$

$$= \frac{C(5, 3) \cdot C(4, 2)}{C(9, 5)} + \frac{C(5, 4) \cdot C(4, 1)}{C(9, 5)} + \frac{C(5, 5) \cdot C(4, 0)}{C(9, 5)}$$

$$= \frac{60}{126} + \frac{20}{126} + \frac{1}{126} \text{ or } \frac{9}{14}$$

The probability of at least 3 students going to the workshop is $\frac{9}{14}$.

Check for Understanding

Communicating Mathematics

Read and study the lesson to answer each question.

1. **Describe** the difference between independent and dependent events.
2. a. **Draw** a Venn diagram to illustrate the event of selecting an ace or a diamond from a deck of cards.
   b. Are the events mutually exclusive? Explain why or why not.
   c. **Write** the formula you would use to determine the probability of these events.
3. *Math Journal* **Write** an example of two mutually exclusive events and two mutually inclusive events in your own life. **Explain** why the events are mutually exclusive or inclusive.

Guided Practice

Determine if each event is *independent* or *dependent.* Then determine the probability.

4. the probability of rolling a sum of 7 on the first toss of two number cubes and a sum of 4 on the second toss
5. the probability of randomly selecting two navy socks from a drawer that contains 6 black and 4 navy socks
6. There are 2 bottles of fruit juice and 4 bottles of sports drink in a cooler. Without looking, Desiree chose a bottle for herself and then one for a friend. What is the probability of choosing 2 bottles of the sports drink?

Determine if each event is *mutually exclusive* or *mutually inclusive.* Then determine each probability.

7. the probability of choosing a penny or a dime from 4 pennies, 3 nickels, and 6 dimes
8. the probability of selecting a boy or a blonde-haired person from 12 girls, 5 of whom have blonde hair, and 15 boys, 6 of whom have blonde hair
9. the probability of drawing a king or queen from a standard deck of cards

In a bingo game, balls numbered 1 to 75 are placed in a bin. Balls are randomly drawn and not replaced. Find each probability for the first 5 balls drawn.

10. P(selecting 5 even numbers)

11. P(selecting 5 two digit numbers)

12. P(5 odd numbers or 5 multiples of 4)

13. P(5 even numbers or 5 numbers less than 30)

14. **Business** A furniture importer has ordered 100 grandfather clocks from an overseas manufacturer. Four clocks are damaged in shipment, but the packaging shows no signs of damage. If a dealer buys 6 of the clocks without examining them first, what is the probability that none of the 6 clocks is damaged?

15. **Sports** A baseball team's pitching staff has 5 left-handed and 8 right-handed pitchers. If 2 pitchers are randomly chosen to warm up, what is the probability that at least one of them is right-handed? (*Hint:* Consider the order when selecting one right-handed and one left-handed pitcher.)

EXERCISES

Practice

Determine if each event is *independent* or *dependent.* Then determine the probability.

16. the probability of selecting a blue marble, not replacing it, then a yellow marble from a box of 5 blue marbles and 4 yellow marbles

17. the probability of randomly selecting two oranges from a bowl of 5 oranges and 4 tangerines, if the first selection is replaced

18. A green number cube and a red number cube are tossed. What is the probability that a 4 is shown on the green number cube and a 5 is shown on the red number cube?

19. the probability of randomly taking 2 blue notebooks from a shelf which has 4 blue and 3 black notebooks

20. A bank contains 4 nickels, 4 dimes, and 7 quarters. Three coins are removed in sequence, without replacement. What is the probability of selecting a nickel, a dime, and a quarter in that order?

21. the probability of removing 13 cards from a standard deck of cards and have all of them be red

22. the probability of randomly selecting a knife, a fork, and a spoon in that order from a kitchen drawer containing 8 spoons, 8 forks, and 12 table knives

23. the probability of selecting three different-colored crayons from a box containing 5 red, 4 black, and 7 blue crayons, if each crayon is replaced

24. the probability that a football team will win its next four games if the odds of winning each game are 4 to 3

For Exercises 25-33, determine if each event is *mutually exclusive* or *mutually inclusive.* Then determine each probability.

25. the probability of tossing two number cubes and either one shows a 4

26. the probability of selecting an ace or a red card from a standard deck of cards

27. the probability that if a card is drawn from a standard deck it is red or a face card

28. the probability of randomly picking 5 puppies of which at least 3 are male puppies, from a group of 5 male puppies and 4 female puppies.

29. the probability of two number cubes being tossed and showing a sum of 6 or a sum of 9.

30. the probability that a group of 6 people selected at random from 7 men and 7 women will have at least 3 women

31. the probability of at least 4 tails facing up when 6 coins are dropped on the floor

32. the probability that two cards drawn from a standard deck will both be aces or both will be black

33. from a collection of 6 rock and 5 rap CDs, the probability that at least 2 are rock from 3 randomly selected

Find the probability of each event using a standard deck of cards.

34. P(all red cards) if 5 cards are drawn without replacement

35. P(both kings or both aces) if 2 cards are drawn without replacement

36. P(all diamonds) if 10 cards are selected with replacement

37. P(both red or both queens) if 2 cards are drawn without replacement

There are 5 pennies, 7 nickels, and 9 dimes in an antique coin collection. If two coins are selected at random and the coins are not replaced, find each probability.

38. P(2 pennies)

39. P(2 nickels or 2 silver-colored coins)

40. P(at least 1 nickel)

41. P(2 dimes or 1 penny and 1 nickel)

There are 5 male and 5 female students in the executive council of the Douglas High School honor society. A committee of 4 members is to be selected at random to attend a conference. Find the probability of each group being selected.

42. P(all female)

43. P(all female or all male)

44. P(at least 3 females)

45. P(at least 2 females and at least 1 male)

Applications and Problem Solving

Real World Application

46. Computers A survey of the members of the Piper High School Computer Club shows that $\frac{2}{5}$ of the students who have home computers use them for word processing, $\frac{1}{3}$ use them for playing games, and $\frac{1}{4}$ use them for both word processing and playing games. What is the probability that a student with a home computer uses it for word processing or playing games?

47. Weather A weather forecaster states that the probability of rain is $\frac{3}{5}$, the probability of lightning is $\frac{2}{5}$, and the probability of both is $\frac{1}{5}$. What is the probability that a baseball game will be cancelled due to rain or lightning?

48. Critical Thinking Felicia and Martin are playing a game where the number cards from a single suit are selected. From this group, three cards are then chosen at random. What is the probability that the sum of the value of the cards will be an even number?

49. City Planning There are six women and seven men on a committee for city services improvement. A subcommittee of five members is being selected at random to study the feasibility of modernizing the water treatment facility. What is the probability that the committee will have at least three women?

50. Medicine A study of two doctors finds that the probability of one doctor correctly diagnosing a medical condition is $\frac{93}{100}$ and the probability the second doctor will correctly diagnose a medical condition is $\frac{97}{100}$. What is the probability that at least one of the doctors will make a correct diagnosis?

51. Disaster Relief During the 1999 hurricane season, Hurricanes Dennis, Floyd, and Irene caused extensive flooding and damage in North Carolina. After a relief effort, 2500 people in one supporting community were surveyed to determine if they donated supplies or money. Of the sample, 812 people said they donated supplies and 625 said they donated money. Of these people, 375 people said they donated both. If a member of this community were selected at random, what is the probability that this person donated supplies or money?

52. Critical Thinking If events A and B are inclusive, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

a. Draw a Venn diagram to represent $P(A \text{ or } B \text{ or } C)$.

b. Write a formula to find $P(A \text{ or } B \text{ or } C)$.

53. Product Distribution Ms. Kameko is the shipping manager of an Internet-based audio and video store. Over the past few months, she has determined the following probabilities for items customers might order.

Item	Probability
Action video	$\frac{4}{7}$
Pop/rock CD	$\frac{1}{2}$
Romance DVD	$\frac{5}{11}$
Action video and pop/rock CD	$\frac{2}{9}$
Pop/rock CD and romance DVD	$\frac{1}{7}$
Action video and romance DVD	$\frac{1}{4}$
Action video, pop/rock CD, and romance DVD	$\frac{1}{44}$

What is the probability, rounded to the nearest hundredth, that a customer will order an action video, pop/rock CD, or a romance DVD?

54. Critical Thinking There are 18 students in a classroom. The students are surveyed to determine their birthday (month and day only). Assume that 366 birthdays are possible.

a. What is the probability of any two students in the classroom having the same birthday?

b. Write an inequality that can be used to determine the probability of any two students having the same birthday to be greater than $\frac{1}{2}$.

c. Are there enough students in the classroom to have the probability in part **a** be greater than $\frac{1}{2}$? If not, at least how many more students would there need to be?

55. **Automotive Repairs** An auto club's emergency service has determined that when club members call to report that their cars will not start, the probability that the engine is flooded is $\frac{1}{2}$, the probability that the battery is dead is $\frac{2}{5}$, and the probability that both the engine is flooded and the battery is dead is $\frac{1}{10}$.

a. Are the events mutually exclusive or mutually inclusive?

b. Draw a Venn Diagram to represent the events.

c. What is the probability that the next member to report that a car will not start has a flooded engine or a dead battery?

Mixed Review

56. Two number cubes are tossed and their sum is 6. Find the probability that each cube shows a 3. *(Lesson 13-3)*

57. How many ways can 7 people be seated around a table? *(Lesson 13-2)*

58. **Sports** Ryan plays basketball every weekend. He averages 12 baskets per game out of 20 attempts. He has decided to try to make 15 baskets out of 20 attempts in today's game. How many ways can Ryan make 15 out of 20 baskets? *(Lesson 12-6)*

59. **Ecology** An underground storage container is leaking a toxic chemical. One year after the leak began, the chemical has spread 1200 meters from its source. After two years, the chemical has spread 480 meters more, and by the end of the third year it has reached an additional 192 meters. If this pattern continues, will the spill reach a well dug 2300 meters away? *(Lesson 12-4)*

60. Solve $12^{x+2} = 3^{x-4}$. *(Lesson 11-5)*

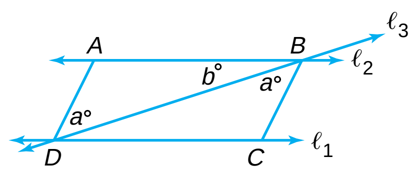
61. **Entertainment** A theater has been staging children's plays during the summer. The average attendance at each performance is 400 people and the cost of a ticket is \$3. Next summer, they would like to increase the cost of the tickets, while maximizing their profits. The director estimates that for every \$1 increase in ticket price, the attendance at each performance will decrease by 20. What price should the director propose to maximize their income, and what maximum income might be expected? *(Lesson 10-5)*

62. **Geology** A drumlin is an elliptical streamlined hill whose shape can be expressed by the equation $r = \ell \cos k\theta$ for $-\frac{\pi}{2k} \le \theta \le \frac{\pi}{2k}$, where ℓ is the length of the drumlin and $k > 1$ is a parameter that is the ratio of the length to the width. Suppose the area of a drumlin is 8270 square yards and the formula for area is $A = \frac{\ell^2\pi}{4k}$. Find the length of a drumlin modeled by $r = \ell \cos 7\theta$. *(Lesson 9-3)*

63. Write a vector equation describing a line passing through $P(1, -5)$ and parallel to $\vec{\mathbf{v}} = \langle -2, -4 \rangle$. *(Lesson 8-6)*

64. Solve $2 \tan x - 4 = 0$ for principal values of x. *(Lesson 7-5)*

65. **SAT/ACT Practice** If $a = 45$, which of the following statements must be true?



I. $\overline{AD} \parallel \overline{BC}$

II. ℓ_3 bisects $\angle ABC$.

III. $b = 45$

A None

B I only

C I and II only

D I and III only

E I, II, and III