

Solving Systems of Equations in Two Variables

OBJECTIVES

- Solve systems of equations graphically.
- Solve systems of equations algebraically.



CONSUMER CHOICES

Madison is thinking about leasing a car for two years. The dealership says that they will lease her the car she has chosen for \$326 per month with only \$200 down. However, if she pays \$1600 down, the lease payment drops to \$226 per month. What is the break-even point when comparing these lease options? Which 2-year lease should she choose if the down payment is not a problem? *This problem will be solved in Example 4.*

Example 4.

The *break-even point* is the point in time at which Madison has paid the same total amount on each lease. After finding that point, you can more easily determine which of these arrangements would be a better deal. The break-even point can be determined by solving a system of equations.

A **system of equations** is a set of two or more equations. To “solve” a system of equations means to find values for the variables in the equations, which make all the equations true at the same time. One way to solve a system of equations is by graphing. The intersection of the graphs represents the point at which the equations have the same x -value and the same y -value. Thus, this ordered pair represents the solution common to both equations. This ordered pair is called the **solution** to the system of equations.

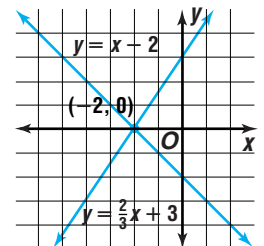
Example 1 Solve the system of equations by graphing.

$$\begin{aligned} 3x - 2y &= -6 \\ x + y &= -2 \end{aligned}$$

First rewrite each equation of the system in slope-intercept form by solving for y .

$$\begin{aligned} 3x - 2y &= -6 && \text{becomes} && y = \frac{3}{2}x + 3 \\ x + y &= -2 && && y = -x - 2 \end{aligned}$$

Since the two lines have different slopes, the graphs of the equations are intersecting lines. The solution to the system is $(-2, 0)$.



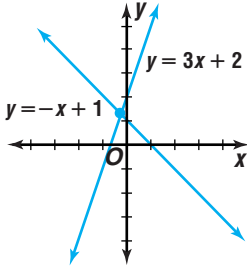
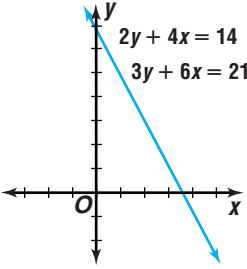
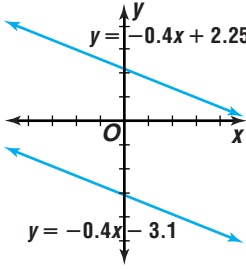
Graphing Calculator Tip

You can estimate the solution to a system of equations by using the **TRACE** function on your graphing calculator.

As you saw in Example 1, when the graphs of two equations intersect there is a solution to the system of equations. However, you may recall that the graphs of two equations may be parallel lines or, in fact, the same line. Each of these situations has a different type of system of linear equations.

A **consistent** system of equations has at least one solution. If there is exactly one solution, the system is **independent**. If there are infinitely many solutions, the system is **dependent**. If there is no solution, the system is **inconsistent**. By rewriting each equation of a system in slope-intercept form, you can more easily determine the type of system you have and what type of solution to expect.

The chart below summarizes the characteristics of these types of systems.

consistent		inconsistent
independent	dependent	
		
$y = 3x + 2$ $y = -x + 1$	$2y + 4x = 14 \rightarrow y = -2x + 7$ $3y + 6x = 21 \rightarrow y = -2x + 7$	$y = -0.4x + 2.25$ $y = -0.4x - 3.1$
different slope	same slope, same intercept	same slope, different intercept
Lines intersect.	Graphs are same line.	Lines are parallel.
one solution	infinitely many solutions	no solution

When graphs result in lines that are the same line, we say the lines coincide.

Often, graphing a system of equations is not the best method of finding its solution. This is especially true when the solution to the system contains non-integer values. Systems of linear equations can also be solved algebraically. Two common ways of solving systems algebraically are the **elimination method** and the **substitution method**. In some cases, one method may be easier to use than the other.

Example 2 Use the elimination method to solve the system of equations.

$$\begin{aligned} 1.5x + 2y &= 20 \\ 2.5x - 5y &= -25 \end{aligned}$$

One way to solve this system is to multiply both sides of the first equation by 5, multiply both sides of the second equation by 2, and add the two equations to eliminate y . Then solve the resulting equation.

$$\begin{array}{r} 5(1.5x + 2y) = 5(20) \\ 2(2.5x - 5y) = 2(-25) \end{array} \quad \rightarrow \quad \begin{array}{r} 7.5x + 10y = 100 \\ 5x - 10y = -50 \\ \hline 12.5x = 50 \\ x = 4 \end{array}$$

Now substitute 4 for x in either of the *original* equations.

$$\begin{aligned} 1.5x + 2y &= 20 \\ 1.5(4) + 2y &= 20 & x = 4 \\ 2y &= 14 \\ y &= 7 \end{aligned}$$

The solution is $(4, 7)$. Check it by substituting into $2.5x - 5y = -25$. If the coordinates make both equations true, then the solution is correct

If one of the equations contains a variable with a coefficient of 1, the system can often be solved more easily by using the substitution method.



Example 3 Use the substitution method to solve the system of equations.

$$\begin{aligned}2x + 3y &= 8 \\ x - y &= 2\end{aligned}$$

You can solve the second equation for either y or x . If you solve for x , the result is $x = y + 2$. Then substitute $y + 2$ for x in the first equation.

$$\begin{aligned}2x + 3y &= 8 \\ 2(y + 2) + 3y &= 8 & x = y + 2 \\ 5y &= 4 \\ y &= \frac{4}{5}\end{aligned}$$

The solution is $(\frac{14}{5}, \frac{4}{5})$.

Now substitute $\frac{4}{5}$ for y in either of the original equations, and solve for x .

$$\begin{aligned}x - y &= 2 \\ x - \frac{4}{5} &= 2 & y = \frac{4}{5} \\ x &= \frac{14}{5}\end{aligned}$$



GRAPHING CALCULATOR EXPLORATION

You can use a graphing calculator to find the solution to an independent system of equations.

- Graph the equations on the same screen.
- Use the **CALC** menu and select **5:intersect** to determine the coordinates of the point of intersection of the two graphs.

TRY THESE

Find the solution to each system.

- $y = 500x - 20$
 $y = -20x + 500$
- $3x - 4y = 320$
 $5x + 2y = 340$

WHAT DO YOU THINK?

3. How accurate are solutions found on the calculator?
4. What type of system do the equations $5x - 7y = 70$ and $10x - 14y = 120$ form? What happens when you try to find the intersection point on the calculator?
5. Graph a system of dependent equations. Find the intersection point. Use the **TRACE** function to move the cursor and find the intersection point again. What pattern do you observe?

You can use a system of equations to solve real-world problems. Choose the best method for solving the system of equations that models the situation.

Example 4 **CONSUMER CHOICES** Refer to the application at the beginning of the lesson.



- a. What is the break-even point in the two lease plans that Madison is considering?
- b. If Madison keeps the lease for 24 months, which lease should she choose?
 - a. First, write an equation to represent the amount she will pay with each plan. Let C represent the total cost and m the number of months she has had the lease.

Lease 1 (\$200 down with monthly payment of \$326): $C = 326m + 200$

Lease 2 (\$1600 down with monthly payment of \$226): $C = 226m + 1600$

Now, solve the system of equations. Since both equations contain C , we can substitute the value of C from one equation into the other.

(continued on the next page)



$$C = 326m + 200$$

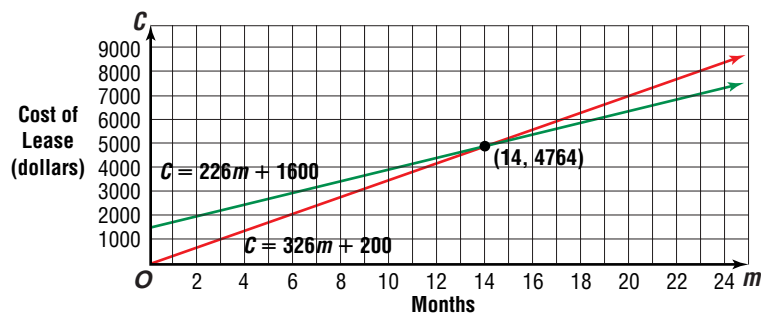
$$226m + 1600 = 326m + 200 \quad C = 226m + 1600$$

$$1400 = 100m$$

$$14 = m$$

With the fourteenth monthly payment, she reaches the break-even point.

- b. The graph of the equations shows that after that point, Lease 1 is more expensive for the 2-year lease. So, Madison should probably choose Lease 2.



CHECK FOR UNDERSTANDING

Communicating Mathematics

Read and study the lesson to answer each question.

- Write a system of equations in which it is easier to use the substitution method to solve the system rather than the elimination method. Explain your choice.
- Refer to the application at the beginning of the lesson. Explain what factors Madison might consider before making a decision on which lease to select.
- Math Journal* Write a description of the three different possibilities that may occur when graphing a system of two linear equations. Include examples and solutions that occur with each possibility.

Guided Practice

- State whether the system $2y + 3x = 6$ and $4y = 16 - 6x$ is *consistent and independent*, *consistent and dependent*, or *inconsistent*. Explain your reasoning.

Solve each system of equations by graphing.

$$5. \begin{cases} y = 5x - 2 \\ y = -2x + 5 \end{cases}$$

$$6. \begin{cases} x - y = 2 \\ 2x = 2y + 10 \end{cases}$$

Solve each system of equations algebraically.

$$7. \begin{cases} 7x + y = 9 \\ 5x - y = 15 \end{cases}$$

$$8. \begin{cases} 3x + 4y = -1 \\ 6x - 2y = 3 \end{cases}$$

$$9. \begin{cases} \frac{1}{3}x - \frac{3}{2}y = -4 \\ 5x - 4y = 14 \end{cases}$$

- Sales** HomePride manufactures solid oak racks for displaying baseball equipment and karate belts. They usually sell six times as many baseball racks as karate-belt racks. The net profit is \$3 from each baseball rack and \$5 from each karate-belt rack. If the company wants a total profit of \$46,000, how many of each type of rack should they sell?

EXERCISES

Practice

State whether each system is *consistent and independent*, *consistent and dependent*, or *inconsistent*.

11. $x + 3y = 18$
 $-x + 2y = 7$

12. $y = 0.5x$
 $2y = x + 4$

13. $-35x + 40y = 55$
 $7x = 8y - 11$

Solve each system of equations by graphing.

14. $x = 5$
 $4x + 5y = 20$

15. $y = -3$
 $2x = 8$

16. $x + y = -2$
 $3x - y = 10$

17. $x + 3y = 0$
 $2x + 6y = 5$

18. $y = x - 2$
 $x - 2y = 4$

19. $3x - 2y = -6$
 $x = 12 - 4y$

20. Determine what type of solution you would expect from the system of equations $3x - 8y = 10$ and $16x - 32y = 75$ without graphing the system. Explain how you determined your answer.

Solve each system of equations algebraically.

21. $5x - y = 16$
 $2x + 3y = 3$

22. $3x - 5y = -8$
 $x + 2y = 1$

23. $y = 6 - x$
 $x = 4.5 + y$

24. $2x + 3y = 3$
 $12x - 15y = -4$

25. $-3x + 10y = 5$
 $2x + 7y = 24$

26. $x = 2y - 8$
 $2x - y = -7$

27. $2x + 5y = 4$
 $3x + 6y = 5$

28. $\frac{3}{5}x - \frac{1}{6}y = 1$
 $\frac{1}{5}x + \frac{5}{6}y = 11$

29. $4x + 5y = -8$
 $3x - 7y = 10$

30. Find the solution to the system of equations $3x - y = -9$ and $4x - 2y = -8$.

31. Explain which method seems most efficient to solve the system of equations $a - b = 0$ and $3a + 2b = -15$. Then solve the system.

Applications and Problem Solving



32. **Sports** Spartan Stadium at San Jose State University in California has a seating capacity of about 30,000. A newspaper article states that the Spartans get four times as many tickets as the visiting team. Suppose S represents the number of tickets for the Spartans and V represents the number of tickets for the visiting team's fans.



a. Which system could be used by a newspaper reader to determine how many tickets each team gets?

A $4S + 4V = 30,000$
 $S = 4V$

B $S - 4V = 0$
 $S + V = 30,000$

C $S + V = 30,000$
 $V - 4S = 0$

b. Solve the system to find how many tickets each team gets.

33. **Geometry** Two triangles have the same perimeter of 20 units. One triangle is an isosceles triangle. The other triangle has a side 6 units long. Its other two sides are the same lengths as the base and leg of the isosceles triangle.

a. What are the dimensions of each triangle?

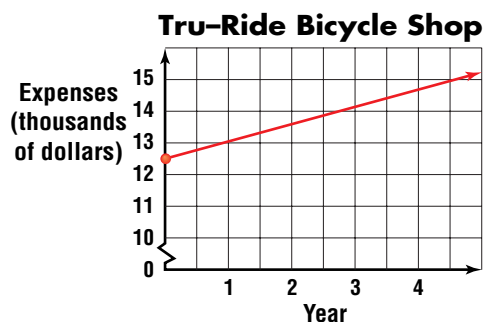
b. What type of triangle is the second triangle?



- 34. Critical Thinking** The solution to a system of two linear equations is $(4, -3)$. One equation has a slope of 4. The slope of the other line is the negative reciprocal of the slope of the first. Find the system of equations.
- 35. Business** The first Earth Day was observed on April 22, 1970. Since then, the week of April 22 has been Earth Week, a time for showing support for environmental causes. Fans Café is offering a reduced refill rate for soft drinks during Earth Week for anyone purchasing a Fans mug. The mug costs \$2.95 filled with 16 ounces of soft drink. The refill price is 50¢. A 16-ounce drink in a disposable cup costs \$0.85.
- What is the approximate break-even point for buying the mug and refills in comparison to buying soft drinks in disposable cups?
 - What does this mean? Which offer do you think is best?
 - How would your decision change if the refillable mug offer was extended for a year?
- 36. Critical Thinking** Determine what must be true of a , b , c , d , e , and f for the system $ax + by = c$ and $dx + ey = f$ to fit each description.
- consistent and independent
 - consistent and dependent
 - inconsistent
- 37. Incentive Plans** As an incentive plan, a company stated that employees who worked for four years with the company would receive \$516 and a laptop computer. Mr. Rodriguez worked for the company for 3.5 years. The company pro-rated the incentive plan, and he still received the laptop computer, but only \$264. What was the value of the laptop computer?
- 38. Ticket Sales** In November 1994, the first live concert on the Internet by a major rock'n'roll band was broadcast. Most fans stand in lines for hours to get tickets for concerts. Suppose you are in line for tickets. There are 200 more people ahead of you than behind you in line. The whole line is three times the number of people behind you. How many people are in line for concert tickets?

Mixed Review

- 39.** Graph $-2x + 7 \geq y$. (Lesson 1-8)
- 40.** Graph $f(x) = 2|x| - 3$. (Lesson 1-7)
- 41.** Write an equation of the line parallel to the graph of $y = 2x + 5$ that passes through the point at $(0, 6)$. (Lesson 1-5)
- 42. Manufacturing** The graph shows the operational expenses for a bicycle shop during its first four years of business. How much was the startup cost of the business? (Lesson 1-3)
- 43.** Find $[f \circ g](x)$ if $f(x) = 3x - 5$ and $g(x) = x + 2$. (Lesson 1-2)
- 44.** State the domain and range of the relation $\{(18, -3), (18, 3)\}$. Is this relation a function? Explain. (Lesson 1-1)



45. SAT/ACT Practice $\sqrt{\frac{\sqrt{25}}{5}} =$

A 1

B $\sqrt{2}$

C 2

D 5

E $5\sqrt{2}$