2-5

Determinants and Multiplicative Inverses of Matrices

OBJECTIVES
- Evaluate determinants.
- Find inverses of matrices.
- Solve systems of equations by using inverses of matrices.

Real World Application

INVESTMENTS Marshall plans to invest $10,500 into two different bonds in order to spread out his risk. The first bond has an annual return of 10%, and the second bond has an annual return of 6%. If Marshall expects an 8.5% return from the two bonds, how much should he invest into each bond? *This problem will be solved in Example 5.*

This situation can be described by a system of equations represented by a matrix. You can solve the system by writing and solving a matrix equation.

The term determinant is often used to mean the value of the determinant.

Each square matrix has a **determinant.** The determinant of $\begin{bmatrix} 8 & 4 \\ 7 & 6 \end{bmatrix}$ is a number denoted by $\begin{vmatrix} 8 & 4 \\ 7 & 6 \end{vmatrix}$ or $\det \begin{bmatrix} 8 & 4 \\ 7 & 6 \end{bmatrix}$. The value of a second-order determinant is defined as follows. *A matrix that has a nonzero determinant is called nonsingular.*

Second-Order Determinant	The value of $\det \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$, or $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, is $a_1b_2 - a_2b_1$.

Example 1 **Find the value of $\begin{vmatrix} 8 & 4 \\ 7 & 6 \end{vmatrix}$.**

$$\begin{vmatrix} 8 & 4 \\ 7 & 6 \end{vmatrix} = 8(6) - 7(4) \text{ or } 20$$

The **minor** of an element of any nth-order determinant is a determinant of order $(n - 1)$. This minor can be found by deleting the row and column containing the element.

$$\begin{vmatrix} \cancel{a_1} & \cancel{b_1} & \cancel{c_1} \\ \cancel{a_2} & b_2 & c_2 \\ \cancel{a_3} & b_3 & c_3 \end{vmatrix} \quad \textit{The minor of } a_1 \textit{ is } \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}.$$

One method of evaluating an nth-order determinant is expanding the determinant by minors. The first step is choosing a row, any row, in the matrix. At each position in the row, multiply the *element* times its *minor* times its *position sign,* and then add the results together for the whole row. The position signs in a matrix are alternating positives and negatives, beginning with a positive in the first row, first column.

$$\begin{bmatrix} + & - & + & \cdots \\ - & + & - & \cdots \\ + & - & + & \cdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

Third-Order Determinant

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

Example 2 **Find the value of** $\begin{vmatrix} -4 & -6 & 2 \\ 5 & -1 & 3 \\ -2 & 4 & -3 \end{vmatrix}$.

$$\begin{vmatrix} -4 & -6 & 2 \\ 5 & -1 & 3 \\ -2 & 4 & -3 \end{vmatrix} = -4 \begin{vmatrix} -1 & 3 \\ 4 & -3 \end{vmatrix} - (-6) \begin{vmatrix} 5 & 3 \\ -2 & -3 \end{vmatrix} + 2 \begin{vmatrix} 5 & -1 \\ -2 & 4 \end{vmatrix}$$

$$= -4(-9) + 6(-9) + 2(18)$$
$$= 18$$

Graphing Calculator Tip

You can use the **det(** option in the **MATH** listings of the **MATRX** menu to find a determinant.

For any $m \times m$ matrix, the identity matrix, I, must also be an $m \times m$ matrix.

The **identity matrix for multiplication** for any square matrix A is the matrix I, such that $IA = A$ and $AI = A$. A second-order matrix can be represented by $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$. Since $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$, the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity matrix for multiplication for any second-order matrix.

Identity Matrix for Multiplication

The identity matrix of nth order, I_n, is the square matrix whose elements in the main diagonal, from upper left to lower right, are 1s, while all other elements are 0s.

Multiplicative inverses exist for some matrices. Suppose A is equal to $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$, a nonzero matrix of second order.

The term inverse matrix generally implies the multiplicative inverse of a matrix.

The **inverse matrix** A^{-1} can be designated as $\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix}$. The product of a matrix A and its inverse A^{-1} must equal the identity matrix, I, for multiplication.

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a_1x_1 + b_1x_2 & a_1y_1 + b_1y_2 \\ a_2x_1 + b_2x_2 & a_2y_1 + b_2y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

From the previous matrix equation, two systems of linear equations can be written as follows.

$$a_1x_1 + b_1x_2 = 1 \qquad a_1y_1 + b_1y_2 = 0$$
$$a_2x_1 + b_2x_2 = 0 \qquad a_2y_1 + b_2y_2 = 1$$

By solving each system of equations, values for x_1, x_2, y_1, and y_2 can be obtained.

$$x_1 = \frac{b_2}{a_1b_2 - a_2b_1} \qquad y_1 = \frac{-b_1}{a_1b_2 - a_2b_1}$$
$$x_2 = \frac{-a_2}{a_1b_2 - a_2b_1} \qquad y_2 = \frac{a_1}{a_1b_2 - a_2b_1}$$

If a matrix A has a determinant of 0 then A^{-1} does not exist.

The denominator $a_1b_2 - a_2b_1$ is equal to the determinant of A. If the determinant of A is not equal to 0, the inverse exists and can be defined as follows.

Inverse of a Second-Order Matrix

If $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$ and $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0$, then $A^{-1} = \dfrac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \begin{bmatrix} b_2 & -b_1 \\ -a_2 & a_1 \end{bmatrix}$.

$A \cdot A^{-1} = A^{-1} \cdot A = I$, where I is the identity matrix.

Example 3 **Find the inverse of the matrix $\begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$.**

Graphing Calculator Appendix

For keystroke instruction on how to find the inverse of a matrix, see pages A16-A17.

First, find the determinant of $\begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$.

$$\begin{vmatrix} 2 & -3 \\ 4 & 4 \end{vmatrix} = 2(4) - 4(-3) \text{ or } 20$$

The inverse is $\frac{1}{20}\begin{bmatrix} 4 & 3 \\ -4 & 2 \end{bmatrix}$ or $\begin{bmatrix} \frac{1}{5} & \frac{3}{20} \\ -\frac{1}{5} & \frac{1}{10} \end{bmatrix}$. *Check to see if $A \cdot A^{-1} = A^{-1} \cdot A = 1$.*

Just as you can use the multiplicative inverse of 3 to solve $3x = -27$, you can use a matrix inverse to solve a matrix equation in the form $AX = B$. To solve this equation for X, multiply each side of the equation by the inverse of A. *When you multiply each side of a matrix equation by the same number or matrix, be sure to place the number or matrix on the left or on the right on each side of the equation to maintain equality.*

$$\begin{aligned} AX &= B \\ A^{-1}AX &= A^{-1}B && \textit{Multiply each side of the equation by } A^{-1}. \\ IX &= A^{-1}B && A^{-1} \cdot A = I \\ X &= A^{-1}B && IX = X \end{aligned}$$

Example 4 **Solve the system of equations by using matrix equations.**

$$\begin{aligned} 2x + 3y &= -17 \\ x - y &= 4 \end{aligned}$$

Write the system as a matrix equation.

$$\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -17 \\ 4 \end{bmatrix}$$

To solve the matrix equation, first find the inverse of the coefficient matrix.

$$\frac{1}{\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}} \begin{bmatrix} -1 & -3 \\ -1 & 2 \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -1 & -3 \\ -1 & 2 \end{bmatrix} \qquad \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = 2(-1) - (1)(3) \text{ or } -5$$

Now multiply each side of the matrix equation by the inverse and solve.

$$-\frac{1}{5}\begin{bmatrix}-1 & -3\\ -1 & 2\end{bmatrix}\cdot\begin{bmatrix}2 & 3\\ 1 & -1\end{bmatrix}\cdot\begin{bmatrix}x\\ y\end{bmatrix} = -\frac{1}{5}\begin{bmatrix}-1 & -3\\ -1 & 2\end{bmatrix}\cdot\begin{bmatrix}-17\\ 4\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}-1\\ -5\end{bmatrix}$$

The solution is $(-1, -5)$.

Example 5 Real World Application

INVESTMENTS **Refer to the application at the beginning of the lesson. How should Marshall divide his \$10,500 investment between the bond with a 10% annual return and a bond with a 6% annual return so that he has a combined annual return on his investments of 8.5%?**

First, let x represent the amount to invest in the bond with an annual return of 10%, and let y represent the amount to invest in the bond with a 6% annual return. So, $x + y = 10{,}500$ since Marshall is investing \$10,500.

Write an equation in standard form that represents the amounts invested in both bonds and the combined annual return of 8.5%. That is, the amount of interest earned from the two bonds is the same as if the total were invested in a bond that earns 8.5%.

$$10\%x + 6\%y = 8.5\%(x + y) \quad \textit{Interest on 10\% bond = 10\%x}$$
$$0.10x + 0.06y = 0.085(x + y) \quad \textit{Interest on 6\% bond = 6\%y}$$
$$0.10x + 0.06y = 0.085x + 0.085y \quad \textit{Distributive Property}$$
$$0.015x - 0.025y = 0$$
$$3x - 5y = 0 \quad \textit{Multiply by 200 to simplify the coefficients.}$$

Now solve the system of equations $x + y = 10{,}500$ and $3x - 5y = 0$. Write the system as a matrix equation and solve.

$$\begin{aligned} x + y &= 10{,}500\\ 3x - 5y &= 0\end{aligned} \longrightarrow \begin{bmatrix}1 & 1\\ 3 & -5\end{bmatrix}\cdot\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}10{,}500\\ 0\end{bmatrix}$$

Multiply each side of the equation by the inverse of the coefficient matrix.

$$-\frac{1}{8}\begin{bmatrix}-5 & -1\\ -3 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 1\\ 3 & -5\end{bmatrix}\cdot\begin{bmatrix}x\\ y\end{bmatrix} = -\frac{1}{8}\begin{bmatrix}-5 & -1\\ -3 & 1\end{bmatrix}\cdot\begin{bmatrix}10{,}500\\ 0\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}6562.5\\ 3937.5\end{bmatrix}$$

The solution is (6562.5, 3937.5). So, Marshall should invest \$6562.50 in the bond with a 10% annual return and \$3937.50 in the bond with a 6% annual return.

CHECK FOR UNDERSTANDING

Read and study the lesson to answer each question.

1. **Describe** the types of matrices that are considered to be nonsingular.
2. **Explain** why the matrix $\begin{bmatrix} 3 & 2 & 0 \\ 4 & -3 & 5 \end{bmatrix}$ does not have a determinant. Give another example of a matrix that does not have a determinant.
3. **Describe** the identity matrix under multiplication for a fourth-order matrix.
4. **Write** an explanation as to how you can decide whether the system of equations, $ax + cy = e$ and $bx + dy = f$, has a solution.

Guided Practice

Find the value of each determinant.

5. $\begin{vmatrix} 4 & -1 \\ -2 & 3 \end{vmatrix}$

6. $\begin{vmatrix} 12 & -26 \\ -15 & 32 \end{vmatrix}$

7. $\begin{vmatrix} 4 & 1 & 0 \\ 5 & -15 & -1 \\ -2 & 10 & 7 \end{vmatrix}$

8. $\begin{vmatrix} 6 & 4 & -1 \\ 0 & 3 & 3 \\ -9 & 0 & 0 \end{vmatrix}$

Find the inverse of each matrix, if it exists.

9. $\begin{bmatrix} -2 & 3 \\ 5 & 7 \end{bmatrix}$

10. $\begin{bmatrix} 4 & 6 \\ 6 & 9 \end{bmatrix}$

Solve each system of equations by using a matrix equation.

11. $5x + 4y = -3$
 $-3x - 5y = -24$

12. $6x - 3y = 63$
 $5x - 9y = 85$

13. **Metallurgy** Aluminum alloy is used in airplane construction because it is strong and lightweight. A metallurgist wants to make 20 kilograms of aluminum alloy with 70% aluminum by using two metals with 55% and 80% aluminum content. How much of each metal should she use?

EXERCISES

Practice

Find the value of each determinant.

14. $\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}$

15. $\begin{vmatrix} -4 & -1 \\ 0 & -1 \end{vmatrix}$

16. $\begin{vmatrix} 9 & 12 \\ 12 & 16 \end{vmatrix}$

17. $\begin{vmatrix} -2 & 3 \\ -2 & 1 \end{vmatrix}$

18. $\begin{vmatrix} 13 & 7 \\ -5 & -8 \end{vmatrix}$

19. $\begin{vmatrix} -6 & 5 \\ 0 & -8 \end{vmatrix}$

20. $\begin{vmatrix} 4 & -1 & -2 \\ 0 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix}$

21. $\begin{vmatrix} 2 & -1 & 3 \\ 3 & 0 & -2 \\ 1 & -3 & 0 \end{vmatrix}$

22. $\begin{vmatrix} 8 & 9 & 3 \\ 3 & 5 & 7 \\ -1 & 2 & 4 \end{vmatrix}$

23. $\begin{vmatrix} 4 & 6 & 7 \\ 3 & -2 & -4 \\ 1 & 1 & 1 \end{vmatrix}$

24. $\begin{vmatrix} 25 & 36 & 15 \\ 31 & -12 & -2 \\ 17 & 15 & 9 \end{vmatrix}$

25. $\begin{vmatrix} 1.5 & -3.6 & 2.3 \\ 4.3 & 0.5 & 2.2 \\ -1.6 & 8.2 & 6.6 \end{vmatrix}$

26. Find det A if $A = \begin{bmatrix} 0 & 1 & -4 \\ 3 & 2 & 3 \\ 8 & -3 & 4 \end{bmatrix}$.

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CONTENTS

Find the inverse of each matrix, if it exists.

27. $\begin{bmatrix} 2 & -3 \\ -2 & -2 \end{bmatrix}$ **28.** $\begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}$ **29.** $\begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}$

30. $\begin{bmatrix} 6 & 7 \\ -6 & 7 \end{bmatrix}$ **31.** $\begin{bmatrix} -4 & 6 \\ 8 & -12 \end{bmatrix}$ **32.** $\begin{bmatrix} 9 & 13 \\ 27 & 36 \end{bmatrix}$

33. What is the inverse of $\begin{bmatrix} \frac{3}{4} & -\frac{1}{8} \\ 5 & \frac{1}{2} \end{bmatrix}$?

Solve each system by using a matrix equation.

34. $4x - y = 1$
$x + 2y = 7$

35. $9x - 6y = 12$
$4x + 6y = -12$

36. $x + 5y = 26$
$3x - 2y = -41$

37. $4x + 8y = 7$
$3x - 3y = 0$

38. $3x - 5y = -24$
$5x + 4y = -3$

39. $9x + 3y = 1$
$5x + y = 1$

Solve each matrix equation. The inverse of the coefficient matrix is given.

40. $\begin{bmatrix} 3 & -2 & 3 \\ 1 & 2 & 2 \\ -2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$, if the inverse is $\frac{1}{9}\begin{bmatrix} -4 & 1 & -10 \\ -3 & 3 & -3 \\ 5 & 1 & 8 \end{bmatrix}$.

41. $\begin{bmatrix} -6 & 5 & 3 \\ 9 & -2 & -1 \\ 3 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -9 \\ 5 \\ -1 \end{bmatrix}$, if the inverse is $-\frac{1}{9}\begin{bmatrix} -1 & -2 & 1 \\ -12 & -15 & 21 \\ 15 & 21 & -33 \end{bmatrix}$.

Graphing Calculator

Use a graphing calculator to find the value of each determinant.

42. $\begin{vmatrix} -2 & -4 & 2 & -3 \\ 2 & 3 & 6 & 0 \\ 0 & 9 & 4 & -5 \\ 4 & -7 & 1 & 8 \end{vmatrix}$

43. $\begin{vmatrix} 2 & -9 & 1 & 8 & 4 \\ -10 & -1 & 2 & 7 & 0 \\ 0 & 4 & -6 & 1 & -8 \\ 6 & -14 & 11 & 0 & 3 \\ 5 & 1 & -3 & 2 & -1 \end{vmatrix}$

Use the algebraic methods you learned in this lesson and a graphing calculator to solve each system of equations.

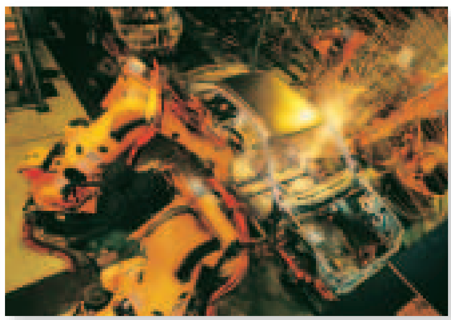
44. $0.3x + 0.5y = 4.74$
$12x - 6.5y = -1.2$

45. $x - 2y + z = 7$
$6x + 2y - 2z = 4$
$4x + 6y + 4z = 14$

Applications and Problem Solving

Real World Application

46. Industry The Flat Rock auto assembly plant in Detroit, Michigan, produces three different makes of automobiles. In 1994 and 1995, the plant constructed a total of 390,000 cars. If 90,000 more cars were made in 1994 than in 1995, how many cars were made in each year?



47. Critical Thinking Demonstrate that the expression for A^{-1} is the multiplicative inverse of A for any nonsingular second-order matrix.

48. Chemistry How many gallons of 10% alcohol solution and 25% alcohol solution should be combined to make 12 gallons of a 15% alcohol solution?

49. **Critical Thinking** If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, does $(A^2)^{-1} = (A^{-1})^2$? Explain.

50. **Geometry** The area of a triangle with vertices at (a, b), (c, d), and (e, f) can be determined using the equation $A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$. What is the area of a triangle with vertices at $(1, -3)$, $(0, 4)$, and $(3, 0)$? (*Hint:* You may need to use the absolute value of the determinant to avoid a negative area.)

51. **Retail** Suppose that on the first day of a sale, a store sold 38 complete computer systems and 53 printers. During the second day, 22 complete systems and 44 printers were sold. On day three of the sale, the store sold 21 systems and 26 printers. Total sales for these items for the three days were \$49,109, \$31,614, and \$26,353 respectively. What was the unit cost of each of these two selected items?

52. **Education** The following type of problem often appears on placement tests or college entrance exams.

Jessi has a total of 179 points on her last two history tests. The second test score is a 7-point improvement from the first score. What are her scores for the two tests?

Mixed Review

53. **Geometry** The vertices of a square are $H(8, 5)$, $I(4, 1)$, $J(0, 5)$, and $K(4, 9)$. Use matrices to determine the coordinates of the square translated 3 units left and 4 units up. *(Lesson 2-4)*

54. Multiply $\begin{bmatrix} 8 & -7 \\ -4 & 0 \end{bmatrix}$ by $\frac{3}{4}$. *(Lesson 2-3)*

55. Solve the system $x - 3y + 2z = 6$, $4x + y - z = 8$, and $-7x - 5y + 4z = -10$. *(Lesson 2-2)*

56. Graph $g(x) = -2[\![x + 5]\!]$. *(Lesson 1-7)*

57. Write the standard form of the equation of the line that is perpendicular to $y = -2x + 5$ and passes through the point at $(2, 5)$. *(Lesson 1-5)*

58. Write the point-slope form of the equation of the line that passes through the points at $(1, 5)$ and $(2, 3)$. Then write the equation in slope-intercept form. *(Lesson 1-4)*

59. **Safety** In 1990, the Americans with Disabilities Act (ADA) went into effect. This act made provisions for public places to be accessible to all individuals, regardless of their physical challenges. One of the provisions of the ADA is that ramps should not be steeper than a rise of 1 foot for every 12 feet of horizontal distance. *(Lesson 1-3)*

   a. What is the slope of such a ramp?

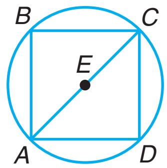
   b. What would be the maximum height of a ramp 18 feet long?

60. Find $[f \circ g](x)$ and $[g \circ f](x)$ if $f(x) = x^2 + 3x + 2$ and $g(x) = x - 1$. *(Lesson 1-2)*

Extra Practice See p. A29.

61. Determine if the set of points whose coordinates are (2, 3), (−3, 4), (6, 3), (2, 4), and (−3, 3) represent a function. Explain. *(Lesson 1-1)*

62. **SAT Practice** The radius of circle E is 3. Square $ABCD$ is inscribed in circle E. What is the best approximation for the difference between the circumference of circle E and the perimeter of square $ABCD$?



A 3
B 2
C 1
D 0.5
E 0

CAREER CHOICES

Agricultural Manager

When you hear the word *agriculture,* you may think of a quaint little farmhouse with chickens and cows running around like in the storybooks of your childhood, but today Old McDonald's farm is big business. Agricultural managers guide and assist farmers and ranchers in maximizing their profits by overseeing the day-to-day activities. Their duties are as varied as there are types of farms and ranches.

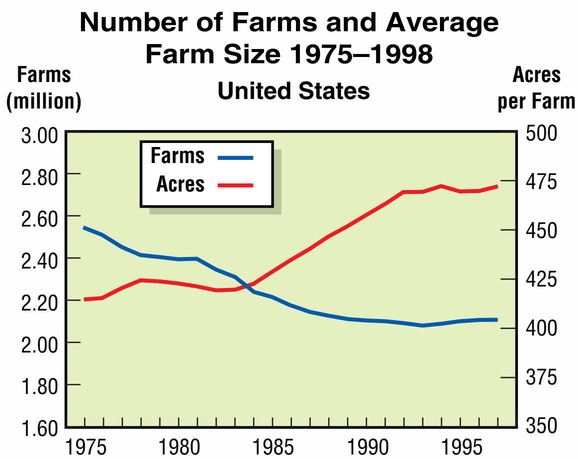
An agricultural manager may oversee one aspect of the farm, as in feeding livestock on a large dairy farm, or tackle all of the activities on a smaller farm. They also may hire and supervise workers and oversee the purchase and maintenance of farm equipment essential to the farm's operation.

Career Overview

Degree Preferred:
Bachelor's degree in agriculture

Related Courses:
mathematics, science, finance

Outlook:
number of jobs expected to decline through 2006



Source: NASS, Livestock & Economics Branch

interNET CONNECTION For more information on careers in agriculture, visit: **www.amc.glencoe.com**

GRAPHING CALCULATOR EXPLORATION

2-5B Augmented Matrices and Reduced Row-Echelon Form

An Extension of Lesson 2-5

OBJECTIVE
- Find reduced row-echelon form of an augmented matrix to solve systems of equations.

Another way to use matrices to solve a system of equations is to use an *augmented matrix.* An augmented matrix is composed of columns representing the coefficients of each variable and the constant term.

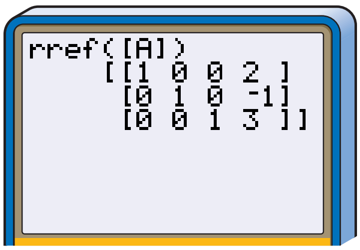
system of equations	*Identify the coefficients and constants.*	**augmented matrix**
$x - 2y + z = 7$	$\mathbf{1}x - \mathbf{2}y + \mathbf{1}z = \mathbf{7}$	$\begin{bmatrix} 1 & -2 & 1 & 7 \\ 3 & 1 & -1 & 2 \\ 2 & 3 & 2 & 7 \end{bmatrix}$
$3x + y - z = 2$	$\mathbf{3}x + \mathbf{1}y - \mathbf{1}z = \mathbf{2}$	
$2x + 3y + 2z = 7$	$\mathbf{2}x + \mathbf{3}y + \mathbf{2}z = \mathbf{7}$	

Each equation is always written with the constant term on the right.

A line is often drawn to separate the constants column.

Through a series of calculations that simulate the elimination methods you used in algebraically solving a system in multiple unknowns, you can find the *reduced row-echelon form* of the matrix, which is $\begin{bmatrix} 1 & 0 & 0 & c_1 \\ 0 & 1 & 0 & c_2 \\ 0 & 0 & 1 & c_3 \end{bmatrix}$, where c_1, c_2, and c_3 represent constants. The graphing calculator has a function **rref(** that will calculate this form once you have entered the augmented matrix. It is located in the **MATH** submenu when **MATRX** menu is accessed.

For example, if the augmented matrix above is stored as matrix A, you would enter the matrix name after the parenthesis and then insert a closing parenthesis before pressing ENTER. The result is shown at the right.



Use the following exercises to discover how this matrix is related to the solution of the system.

TRY THESE

Write an augmented matrix for each system of equations. Then find the reduced row-echelon form.

1. $2x + y - 2z = 7$
$x - 2y - 5z = -1$
$4x + y + z = -1$

2. $x + y + z - 6 = 0$
$2x - 3y + 4z - 3 = 0$
$4x - 8y + 4z - 12 = 0$

3. $w + x + y + z = 0$
$2w + x - y - z = 1$
$-w - x + y + z = 0$
$2x + y = 0$

WHAT DO YOU THINK?

4. Write the equations represented by each reduced row-echelon form of the matrix in Exercises 1-3. How do these equations related to the original system?

5. What would you expect to see on the graphing calculator screen if the constants were irrational or repeating decimals?