5-1

Angles and Degree Measure

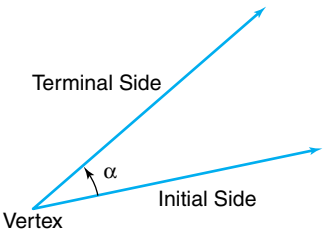
OBJECTIVES

- Convert decimal degree measures to degrees, minutes, and seconds and vice versa.
- Find the number of degrees in a given number of rotations.
- Identify angles that are coterminal with a given angle.

Real World Application

NAVIGATION The sextant is an optical instrument invented around 1730. It is used to measure the angular elevation of stars, so that a navigator can determine the ship's current latitude. Suppose a navigator determines a ship in the Pacific Ocean to be located at north latitude 15.735°. How can this be written as degrees, minutes, and seconds? *This problem will be solved in Example 1.*

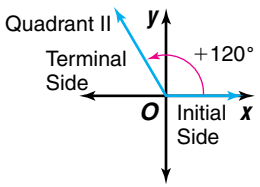
An angle may be generated by rotating one of two rays that share a fixed endpoint known as the **vertex.** One of the rays is fixed to form the **initial side** of the angle, and the second ray rotates to form the **terminal side.**

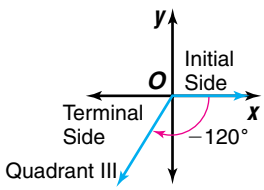


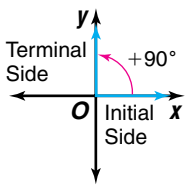
The measure of an angle provides us with information concerning the direction of the rotation and the amount of the rotation necessary to move from the initial side of the angle to the terminal side.

- If the rotation is in a counterclockwise direction, the angle formed is a *positive angle.*
- If the rotation is clockwise, it is a *negative angle.*

An angle with its vertex at the origin and its initial side along the positive x-axis is said to be in **standard position.** In the figures below, all of the angles are in standard position.

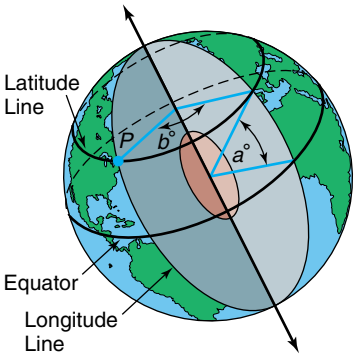






The most common unit used to measure angles is the **degree.** The concept of degree measurement is rooted in the ancient Babylonian culture. The Babylonians based their numeration system on 60 rather than 10 as we do today. In an equilateral triangle, they assigned the measure of each angle to be 60. Therefore, one sixtieth $\left(\frac{1}{60}\right)$ of the measure of the angle of an equilateral triangle was equivalent to one unit or degree (1°). The degree is subdivided into 60 equal parts known as **minutes** (1′), and the minute is subdivided into 60 equal parts known as **seconds** (1″).

Angles are used in a variety of real-world situations. For example, in order to locate every point on Earth, cartographers use a grid that contains circles through the poles, called *longitude lines,* and circles parallel to the equator, called *latitude lines.* Point P is located by traveling north from the equator through a central angle of $a°$ to a circle of latitude and then west along that circle through an angle of $b°$. Latitude and longitude can be expressed in degrees as a decimal value or in degrees, minutes, and seconds.



Example 1 **NAVIGATION** **Refer to the application at the beginning of the lesson.**

a. Change north latitude 15.735° to degrees, minutes, and seconds.

$15.735° = 15° + (0.735 \cdot 60)'$ *Multiply the decimal portion of the degree measure by 60 to find the number of minutes.*

$= 15° + 44.1'$

$= 15° + 44' + (0.1 \cdot 60)''$ *Multiply the decimal portion of the minute measure by 60 to find the number of seconds.*

$= 15° + 44' + 6''$

$15.735°$ can be written as $15° 44' 6''$.

b. Write north latitude 39° 5′ 34″ as a decimal rounded to the nearest thousandth.

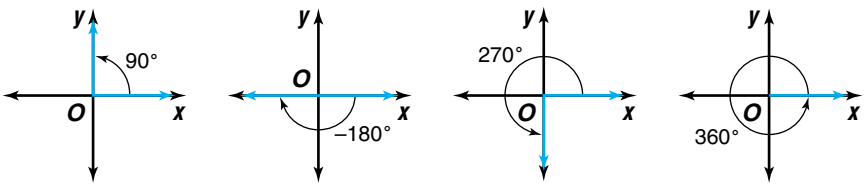
$39° 5' 34'' = 39° + 5'\left(\frac{1°}{60'}\right) + 34''\left(\frac{1°}{3600''}\right)$ or about $39.093°$

$39° 5' 34''$ can be written as $39.093°$.

Graphing Calculator Tip

▶**DMS** on the **[ANGLE]** menu allows you to convert decimal degree values to degrees, minutes, and seconds.

If the terminal side of an angle that is in standard position coincides with one of the axes, the angle is called a **quadrantal angle.** In the figures below, all of the angles are quadrantal.



A full rotation around a circle is 360°. Measures of more than 360° represent multiple rotations.

Example 2 **Give the angle measure represented by each rotation.**

a. 5.5 rotations clockwise

$5.5 \times -360 = -1980$ *Clockwise rotations have negative measures.*

The angle measure of 5.5 clockwise rotations is $-1980°$.

b. 3.3 rotations counterclockwise

$3.3 \times 360 = 1188$ *Counterclockwise rotations have positive measures.*

The angle measure of 3.3 counterclockwise rotations is $1188°$.

Two angles in standard position are called **coterminal angles** if they have the same terminal side. Since angles differing in degree measure by multiples of 360° are equivalent, every angle has infinitely many coterminal angles.

Coterminal Angles

If α is the degree measure of an angle, then all angles measuring $\alpha + 360k°$, where k is an integer, are coterminal with α.

The symbol α is the lowercase Greek letter alpha.

Any angle coterminal with an angle of 75° can be written as $75° + 360k°$, where k is the number of rotations around the circle. The value of k is a positive integer if the rotations are counterclockwise and a negative integer if the rotations are clockwise.

Examples

3 **Identify all angles that are coterminal with each angle. Then find one positive angle and one negative angle that are coterminal with the angle.**

a. 45°

All angles having a measure of $45° + 360k°$, where k is an integer, are coterminal with 45°. A positive angle is $45° + 360°(1)$ or 405°. A negative angle is $45° + 360°(-2)$ or −675°.

b. 225°

All angles having a measure of $225° + 360k°$, where k is an integer, are coterminal with 225°. A positive angle is $225° + 360(2)°$ or 945°. A negative angle is $225° + 360(-1)°$ or −135°.

4 **If each angle is in standard position, determine a coterminal angle that is between 0° and 360°. State the quadrant in which the terminal side lies.**

a. 775°

In $\alpha + 360k°$, you need to find the value of α. First, determine the number of complete rotations (k) by dividing 775 by 360.

$\frac{775}{360} \approx 2.152777778$

Then, determine the number of remaining degrees (α).

Method 1

$\alpha \approx 0.152777778 \text{ rotations} \cdot 360°$
$\approx 55°$

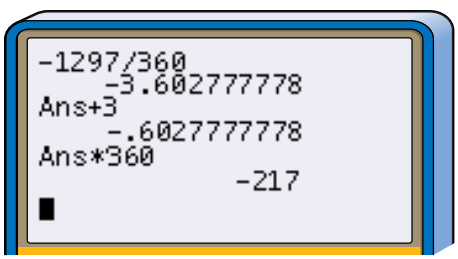
Method 2

$\alpha + 360(2)° = 775°$
$\alpha + 720° = 775°$
$\alpha = 55°$

The coterminal angle (α) is 55°. Its terminal side lies in the first quadrant.

b. −1297°

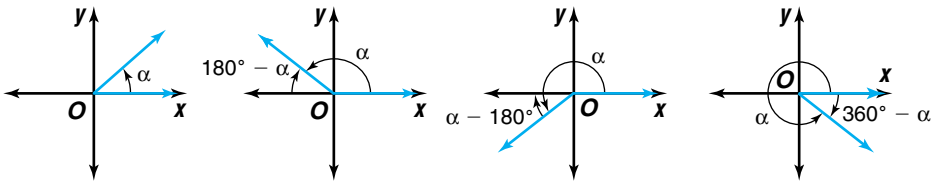
Use a calculator.



The angle is −217°, but the coterminal angle needs to be positive.
$360° - 217° = 143°$

The coterminal angle (α) is 143°.
Its terminal side lies in the second quadrant.

If α is a nonquadrantal angle in standard position, its **reference angle** is defined as the acute angle formed by the terminal side of the given angle and the x-axis. You can use the figures and the rule below to find the reference angle for any angle α where $0° < \alpha < 360°$. If the measure of α is greater than $360°$ or less than $0°$, it can be associated with a coterminal angle of positive measure between $0°$ and $360°$.



Reference Angle Rule

For any angle α, $0° < \alpha < 360°$, its reference angle α' is defined by
a. α, when the terminal side is in Quadrant I,
b. $180° - \alpha$, when the terminal side is in Quadrant II,
c. $\alpha - 180°$, when the terminal side is in Quadrant III, and
d. $360° - \alpha$, when the terminal side is in Quadrant IV.

Example 5 **Find the measure of the reference angle for each angle.**

a. 120°

Since $120°$ is between $90°$ and $180°$, the terminal side of the angle is in the second quadrant.

$180° - 120° = 60°$

The reference angle is $60°$.

b. −135°

A coterminal angle of -135 is $360 - 135$ or 225. Since 225 is between $180°$ and $270°$, the terminal side of the angle is in the third quadrant.

$225° - 180° = 45°$

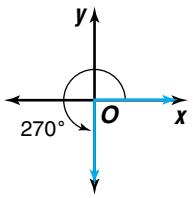
The reference angle is $45°$.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Read and study the lesson to answer each question.

1. **Describe** the difference between an angle with a positive measure and an angle with a negative measure.
2. **Explain** how to write $29° 45' 26''$ as a decimal degree measure.
3. **Write** an expression for the measures of all angles that are coterminal with the angle shown.
4. **Sketch** an angle represented by 3.5 counterclockwise rotations. Give the angle measure represented by this rotation.



Guided Practice

Change each measure to degrees, minutes, and seconds.

5. $34.95°$
6. $-72.775°$

Write each measure as a decimal to the nearest thousandth.

7. $-128° 30' 45''$
8. $29° 6' 6''$

Give the angle measure represented by each rotation.

9. 2 rotations clockwise

10. 4.5 rotations counterclockwise

Identify all angles that are coterminal with each angle. Then find one positive angle and one negative angle that are coterminal with each angle.

11. 22°

12. −170°

If each angle is in standard position, determine a coterminal angle that is between 0° and 360°. State the quadrant in which the terminal side lies.

13. 453°

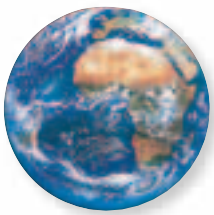
14. −798°

Find the measure of the reference angle for each angle.

15. 227°

16. −210°

17. Geography Earth rotates once on its axis approximately every 24 hours. About how many degrees does a point on the equator travel through in one hour? in one minute? in one second?



EXERCISES

Practice

Change each measure to degrees, minutes, and seconds.

18. −16.75°

19. 168.35°

20. −183.47°

21. 286.88°

22. 27.465°

23. 246.876°

Write each measure as a decimal to the nearest thousandth.

24. 23° 14′ 30″

25. −14° 5′ 20″

26. 233° 25′ 15″

27. 173° 24′ 35″

28. −405° 16′ 18″

29. 1002° 30′ 30″

Give the angle measure represented by each rotation.

30. 3 rotations clockwise

31. 2 rotations counterclockwise

32. 1.5 rotations counterclockwise

33. 7.5 rotations clockwise

34. 2.25 rotations counterclockwise

35. 5.75 rotations clockwise

36. How many degrees are represented by 4 counterclockwise revolutions?

Identify all angles that are coterminal with each angle. Then find one positive angle and one negative angle that are coterminal with each angle.

37. 30°

38. −45°

39. 113°

40. 217°

41. −199°

42. −305°

43. Determine the angle between 0° and 360° that is coterminal with all angles represented by $310° + 360k°$, where k is any integer.

44. Find the angle that is two counterclockwise rotations from 60°. Then find the angle that is three clockwise rotations from 60°.

If each angle is in standard position, determine a coterminal angle that is between 0° and 360°. State the quadrant in which the terminal side lies.

45. 400°

46. −280°

47. 940°

48. 1059°

49. −624°

50. −989°

51. In what quadrant is the terminal side of a 1275° angle located?

Find the measure of the reference angle for each angle.

52. 327° **53.** 148° **54.** 563° **55.** −420° **56.** −197° **57.** 1045°

58. Name four angles between 0° and 360° with a reference angle of 20°.

Applications and Problem Solving

59. Technology A computer's hard disk is spinning at 12.5 revolutions per second. Through how many degrees does it travel in a second? in a minute?

60. Critical Thinking Write an expression that represents all quadrantal angles.

61. Biking During the winter, a competitive bike rider trains on a stationary bike. Her trainer wants her to warm up for 5 to 10 minutes by pedaling slowly. Then she is to increase the pace to 95 revolutions per minute for 30 seconds. Through how many degrees will a point on the outside of the tire travel during the 30 seconds of the faster pace?

62. Flywheels A high-performance composite flywheel rotor can spin anywhere between 30,000 and 100,000 revolutions per minute. What is the range of degrees through which the composite flywheel can travel in a minute? Write your answer in scientific notation.

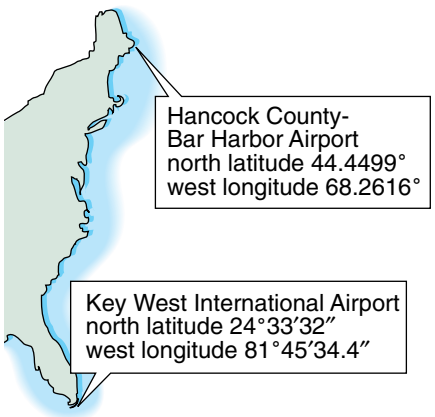
63. Astronomy On January 28, 1998, an x-ray satellite spotted a neutron star that spins at a rate of 62 times per second. Through how many degrees does this neutron star rotate in a second? in a minute? in an hour? in a day?

64. Critical Thinking Write an expression that represents any angle that is coterminal with a 25° angle, a 145° angle, and a 265° angle.

65. Aviation The locations of two airports are indicated on the map.

a. Write the latitude and longitude of the Hancock County–Bar Harbor airport in Bar Harbor, Maine, as degrees, minutes, and seconds.

b. Write the latitude and longitude of the Key West International Airport in Key West, Florida, as a decimal to the nearest thousandth.



66. Entertainment A tower restaurant in Sydney, Australia, is 300 meters above sea level and provides a 360° panoramic view of the city as it rotates every 70 minutes. A tower restaurant in San Antonio, Texas, is 750 feet tall. It revolves at a rate of one revolution per hour.

a. In a day, how many more revolutions does the restaurant in San Antonio make than the one in Sydney?

b. In a week, how many more degrees does a speck of dirt on the window of the restaurant in San Antonio revolve than a speck of dirt on the window of the restaurant in Sydney?

Mixed Review

internet CONNECTION

Data Update For the latest information about motor vehicle production, visit our website at **www.amc.glencoe.com**

67. Manufacturing The percent of the motor vehicles produced in the United States since 1950 is depicted in the table at the right. *(Lesson 4-8)*

a. Write an equation to model the percent of the motor vehicles produced in the United States as a function of the number of years since 1950.

b. According to the equation, what percent of motor vehicles will be produced in the United States in the year 2010?

Motor Vehicle Production in the United States

Year	Percent
1950	75.7
1960	47.9
1970	28.2
1980	20.8
1990	20.1
1992	20.2
1993	23.3
1994	24.8
1997	22.7

Source: American Automobile Manufacturers Association

68. Solve $\sqrt[3]{6n + 5} - 15 = -10$. *(Lesson 4-7)*

69. Solve $\frac{x + 3}{x+2} = 2 - \frac{3}{x^2 + 5x + 6}$. *(Lesson 4-6)*

70. Use the Remainder Theorem to find the remainder if $x^3 + 8x + 1$ is divided by $x - 2$. *(Lesson 4-3)*

71. Write a polynomial equation of least degree with roots -5, -6, and 10. *(Lesson 4-1)*

72. If r varies inversely as t and $r = 18$ when $t = -3$, find r when $t = -11$. *(Lesson 3-8)*

73. Determine whether the graph of $y = \frac{x^2-1}{x + 1}$ has infinite discontinuity, jump discontinuity, or point discontinuity, or is continuous. Then graph the function. *(Lesson 3-7)*

74. Graph $f(x) = |(x + 1)^2 + 2|$. Determine the interval(s) for which the function is increasing and the interval(s) for which the function is decreasing. *(Lesson 3-5)*

75. Use the graph of the parent function $f(x) = \frac{1}{x}$ to describe the graph of the related function $g(x) = \frac{3}{x} - 2$. *(Lesson 3-2)*

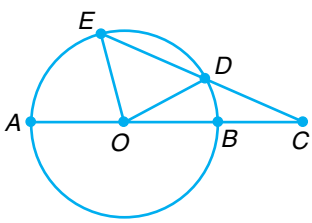
76. Solve the system of inequalities $y \leq 5$, $3y \geq 2x + 9$, and $-3y \leq 6x - 9$ by graphing. Name the coordinates of the vertices of the convex set. *(Lesson 2-6)*

77. Find $[f \cdot g](x)$ if $f(x) = x - 0.2x$ and $g(x) = x - 0.3x$. *(Lesson 1-2)*

78. SAT/ACT Practice $\overline{AB}$ is a diameter of circle O, and $m\angle BOD = 15°$. If $m\angle EOA = 85°$, find $m\angle ECA$.

A 85° **B** 50° **C** 70°

D 35° **E** 45°



Extra Practice See p. A34.