

Applying Trigonometric Functions

OBJECTIVE

- Use trigonometry to find the measures of the sides of right triangles.



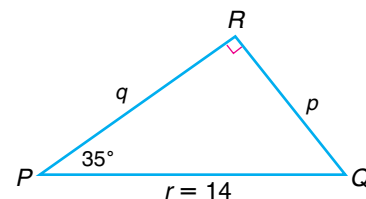
ENTERTAINMENT

The circus has arrived and the roustabouts must put up the main tent in a field near town. A tab is located on the side of the tent 40 feet above the ground. A rope is tied to the tent at this point and then the rope is placed around a stake on the ground. If the angle that the rope makes with the level ground is $50^\circ 15'$, how long is the rope? What is the distance between the bottom of the tent and the stake? *This problem will be solved in Example 2.*

Trigonometric functions can be used to solve problems involving right triangles. The most common functions used are the sine, cosine, and tangent.

Examples 1 If $P = 35^\circ$ and $r = 14$, find q .

From the art at the right, you know the measures of an angle and the hypotenuse. You want to know the measure of the side adjacent to the given angle. The cosine function relates the side adjacent to the angle and the hypotenuse.



$$\cos P = \frac{q}{r} \quad \cos = \frac{\text{side adjacent}}{\text{hypotenuse}}$$

$$\cos 35^\circ = \frac{q}{14} \quad \text{Substitute } 35^\circ \text{ for } P \text{ and } 14 \text{ for } r.$$

$$14 \cos 35^\circ = q \quad \text{Multiply each side by } 14.$$

$$11.46812862 \approx q \quad \text{Use a calculator.}$$

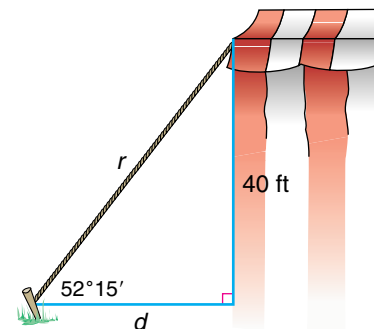
Therefore, q is about 11.5.

2 **ENTERTAINMENT** Refer to the application above.

a. If the angle that the rope makes with the level ground is $52^\circ 15'$, how long is the rope?

b. What is the distance between the bottom of the tent and the stake?

- a. You know the measures of an angle and the side opposite the angle. To find the length of the rope, you need to know the measure of the hypotenuse. In this case, use the sine function.



(continued on the next page)



$$\sin 52^\circ 15' = \frac{40}{r}$$

$$r \sin 52^\circ 15' = 40$$

$$r = \frac{40}{\sin 52^\circ 15'}$$

$$r \approx 50.58875357$$

$\sin = \frac{\text{side opposite}}{\text{hypotenuse}}$
Multiply each side by r.
Divide each side by $\sin 52^\circ 15'$.
Use a calculator.

The rope is about 50.6 feet long.

- b. To find the distance between the bottom of the tent and the stake, you need to know the length of the side adjacent to the known angle. Use the tangent function.

$$\tan 52^\circ 15' = \frac{40}{d}$$

$$d \tan 52^\circ 15' = 40$$

$$d = \frac{40}{\tan 52^\circ 15'}$$

$$d \approx 30.97130911$$

$\tan = \frac{\text{side opposite}}{\text{side adjacent}}$
Multiply each side by d.
Divide each side by $\tan 52^\circ 15'$.
Use a calculator.

The distance between the bottom of the tent and the stake is about 31.0 feet.

You can use right triangle trigonometry to solve problems involving other geometric figures.

Example 3 GEOMETRY A regular pentagon is inscribed in a circle with diameter 8.34 centimeters. The *apothem* of a regular polygon is the measure of a line segment from the center of the polygon to the midpoint of one of its sides. Find the apothem of the pentagon.

First, draw a diagram. If the diameter of the circle is 8.34 centimeters, the radius is $8.34 \div 2$ or 4.17 centimeters. The measure of α is $360^\circ \div 10$ or 36° .

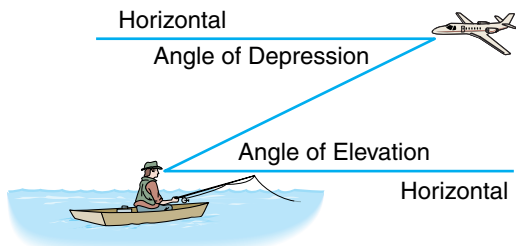
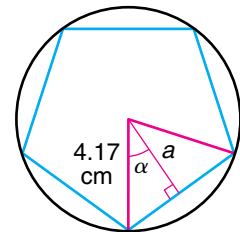
$$\cos 36^\circ = \frac{a}{4.17}$$

$$4.17 \cos 36^\circ = a$$

$$3.373600867 \approx a$$

$\cos = \frac{\text{side adjacent}}{\text{hypotenuse}}$
Multiply each side by 4.17.
Use a calculator.

The apothem is about 3.37 centimeters.



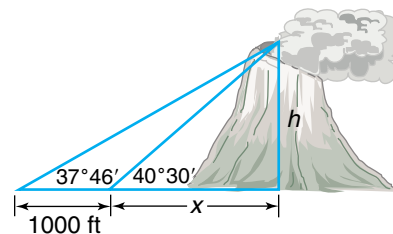
There are many other applications that require trigonometric solutions. For example, surveyors use special instruments to find the measures of **angles of elevation** and **angles of depression**. An angle of elevation is the angle between a horizontal line and the line of sight from an observer to an object at a higher level. An angle of depression is the angle between a horizontal line and the line of sight from the observer to an object at a lower level. The angle of elevation and the angle of depression are equal in measure because they are alternate interior angles.

Example



4 SURVEYING On May 18, 1980, Mount Saint Helens, a volcano in Washington, erupted with such force that the top of the mountain was blown off. To determine the new height at the summit of Mount Saint Helens, a surveyor measured the angle of elevation to the top of the volcano to be $37^\circ 46'$. The surveyor then moved 1000 feet closer to the volcano and measured the angle of elevation to be $40^\circ 30'$. Determine the new height of Mount Saint Helens.

Draw a diagram to model the situation. Let h represent the height of the volcano and x represent the distance from the surveyor's second position to the center of the base of the volcano. Write two equations involving the tangent function.



$$\tan 37^\circ 46' = \frac{h}{1000 + x}$$

$$(1000 + x)\tan 37^\circ 46' = h$$

$$\tan 40^\circ 30' = \frac{h}{x}$$

$$x \tan 40^\circ 30' = h$$

Therefore, $(1000 + x)\tan 37^\circ 46' = x \tan 40^\circ 30'$. Solve this equation for x .

$$(1000 + x)\tan 37^\circ 46' = x \tan 40^\circ 30'$$

$$1000 \tan 37^\circ 46' + x \tan 37^\circ 46' = x \tan 40^\circ 30'$$

$$1000 \tan 37^\circ 46' = x \tan 40^\circ 30' - x \tan 37^\circ 46'$$

$$1000 \tan 37^\circ 46' = x(\tan 40^\circ 30' - \tan 37^\circ 46')$$

$$\frac{1000 \tan 37^\circ 46'}{\tan 40^\circ 30' - \tan 37^\circ 46'} = x$$

$$9765.826092 \approx x \quad \text{Use a calculator.}$$

Use this value for x and the equation $x \tan 40^\circ 30' = h$ to find the height of the volcano.

$$x \tan 40^\circ 30' = h$$

$$9765.826092 \tan 40^\circ 30' \approx h$$

$$8340.803443 \approx h \quad \text{Use a calculator.}$$

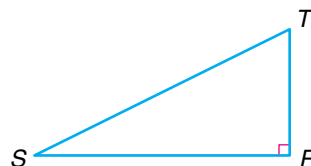
The new height of Mount Saint Helens is about 8341 feet.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Read and study the lesson to answer each question.

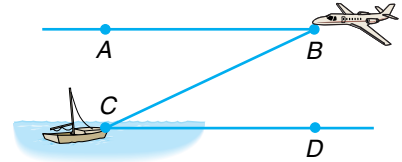
- State which trigonometric function you would use to solve each problem.
 - If $S = 42^\circ$ and $ST = 8$, find RS .
 - If $T = 55^\circ$ and $RT = 5$, find RS .
 - If $S = 27^\circ$ and $TR = 7$, find TS .



- Write a problem that could be solved using the tangent function.

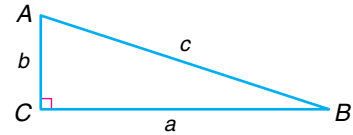


3. **Name** the angle of elevation and the angle of depression in the figure at the right. Compare the measures of these angles. Explain.
4. **Describe** a way to use trigonometry to determine the height of the building where you live.



Guided Practice Solve each problem. Round to the nearest tenth.

5. If $b = 13$ and $A = 76^\circ$, find a .
6. If $B = 26^\circ$ and $b = 18$, find c .
7. If $B = 16^\circ 45'$ and $c = 13$, find a .
8. **Geometry** Each base angle of an isosceles triangle measures $55^\circ 30'$. Each of the congruent sides is 10 centimeters long.
- Find the altitude of the triangle.
 - What is the length of the base?
 - Find the area of the triangle.
9. **Boating** The Ponce de Leon lighthouse in St. Augustine, Florida, is the second tallest brick tower in the United States. It was built in 1887 and rises 175 feet above sea level. How far from the shore is a motorboat if the angle of depression from the top of the lighthouse is $13^\circ 15'$?

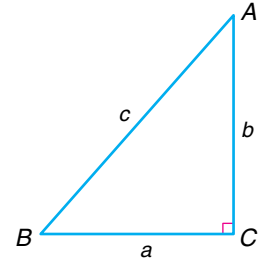


EXERCISES

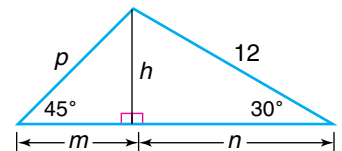
Practice

Solve each problem. Round to the nearest tenth.

10. If $A = 37^\circ$ and $b = 6$, find a .
11. If $c = 16$ and $B = 67^\circ$, find a .
12. If $B = 62^\circ$ and $c = 24$, find b .
13. If $A = 29^\circ$ and $a = 4.6$, find c .
14. If $a = 17.3$ and $B = 77^\circ$, find c .
15. If $b = 33.2$ and $B = 61^\circ$, find a .
16. If $B = 49^\circ 13'$ and $b = 10$, find a .
17. If $A = 16^\circ 55'$ and $c = 13.7$, find a .
18. If $a = 22.3$ and $B = 47^\circ 18'$, find c .
19. Find h , n , m , and p . Round to the nearest tenth.
20. **Geometry** The apothem of a regular pentagon is 10.8 centimeters.
- Find the radius of the circumscribed circle.
 - What is the length of a side of the pentagon?
 - Find the perimeter of the pentagon.
21. **Geometry** Each base angle of an isosceles triangle measures $42^\circ 30'$. The base is 14.6 meters long.
- Find the length of a leg of the triangle.
 - Find the altitude of the triangle.
 - What is the area of the triangle?



Exercises 10–18



Exercise 19

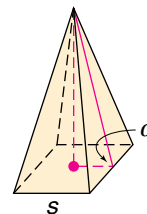
**Applications
and Problem
Solving**



22. **Geometry** A regular hexagon is inscribed in a circle with diameter 6.4 centimeters.
- What is the apothem of the hexagon?
 - Find the length of a side of the hexagon.
 - Find the perimeter of the hexagon.
 - The area of a regular polygon equals one half times the perimeter of the polygon times the apothem. Find the area of the polygon.

23. **Engineering** The escalator at St. Petersburg Metro in Russia has a vertical rise of 195.8 feet. If the angle of elevation of the escalator is $10^\circ 21' 36''$, find the length of the escalator.

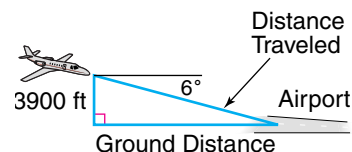
24. **Critical Thinking** Write a formula for the volume of the regular pyramid at the right in terms of α and s the length of each side of the base.



25. **Fire Fighting** The longest truck-mounted ladder used by the Dallas Fire Department is 108 feet long and consists of four hydraulic sections. Gerald Travis, aerial expert for the department, indicates that the optimum operating angle of this ladder is 60° . The fire fighters find they need to reach the roof of an 84-foot burning building. Assume the ladder is mounted 8 feet above the ground.

- Draw a labeled diagram of the situation.
- How far from the building should the base of the ladder be placed to achieve the optimum operating angle?
- How far should the ladder be extended to reach the roof?

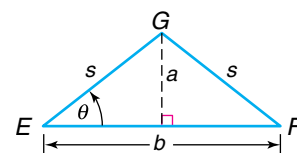
26. **Aviation** When a 757 passenger jet begins its descent to the Ronald Reagan International Airport in Washington, D.C., it is 3900 feet from the ground. Its angle of descent is 6° .



- What is the plane's ground distance to the airport?
- How far must the plane fly to reach the runway?

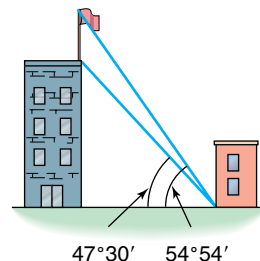
27. **Boat Safety** The Cape Hatteras lighthouse on the North Carolina coast was built in 1870 and rises 208 feet above sea level. From the top of the lighthouse, the lighthouse keeper observes a yacht and a barge along the same line of sight. The angle of depression for the yacht is 20° , and the angle of depression for the barge is $12^\circ 30'$. For safety purposes, the keeper thinks that the two sea vessels should be at least 300 feet apart. If they are less than 300 feet, she plans to sound the horn. How far apart are these vessels? Does the keeper have to sound the horn?

28. **Critical Thinking** Derive two formulas for the length of the altitude a of the triangle shown at the right, given that b , s , and θ are known. Justify each of the steps you take in your reasoning.



29. **Recreation** Latasha and Markisha are flying kites on a windy spring day. Latasha has released 250 feet of string, and Markisha has released 225 feet of string. The angle that Latasha's kite string makes with the horizontal is 35° . The angle that Markisha's kite string makes with the horizontal is 42° . Which kite is higher and by how much?

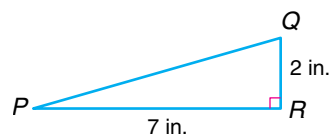
30. **Architecture** A flagpole 40 feet high stands on top of the Wentworth Building. From a point in front of Bailey's Drugstore, the angle of elevation for the top of the pole is $54^\circ 54'$, and the angle of elevation for the bottom of the pole is $47^\circ 30'$. How high is the building?



Mixed Review

31. Find the values of the six trigonometric functions for a 120° angle using the unit circle. (Lesson 5-3)

32. Find the sine, cosine, and tangent ratios for $\angle P$. (Lesson 5-2)



33. Write $43^\circ 15' 35''$ as a decimal to the nearest thousandth. (Lesson 5-1)

34. Graph $y \leq |x + 2|$. (Lesson 3-3)

35. **Consumerism** Kareem and Erin went shopping for school supplies. Kareem bought 3 notebooks and 2 packages of pencils for \$5.80. Erin bought 4 notebooks and 1 package of pencils for \$6.20. What is the cost of one notebook? What is the cost of one package of pencils? (Lesson 2-1)

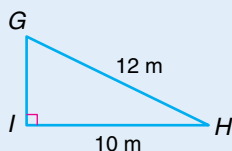
36. **SAT/ACT Practice** An automobile travels m miles in h hours. At this rate, how far will it travel in x hours?

- A $\frac{m}{x}$ B $\frac{m}{xh}$ C $\frac{m}{h}$ D $\frac{mh}{x}$ E $\frac{mx}{h}$

MID-CHAPTER QUIZ

1. Change 34.605° to degrees, minutes, and seconds. (Lesson 5-1)
2. If a -400° angle is in standard position, determine a coterminal angle that is between 0° and 360° . State the quadrant in which the terminal side lies. (Lesson 5-1)

3. Find the six trigonometric functions for $\angle G$. (Lesson 5-2)



4. Find the values of the six trigonometric functions for angle θ in standard position if a point with coordinates $(2, -5)$ lies on its terminal side. (Lesson 5-3)

5. **National Landmarks** Suppose the angle of elevation of the sun is 27.8° . Find the length of the shadow made by the Washington Monument, which is 550 feet tall. (Lesson 5-4)