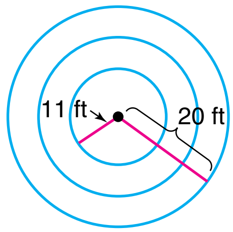
6-2

Linear and Angular Velocity

OBJECTIVE
- Find linear and angular velocity.

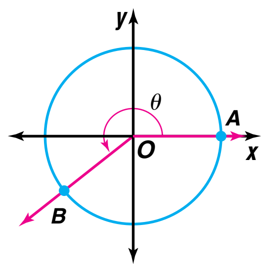
Real World Application

ENTERTAINMENT The Children's Museum in Indianapolis, Indiana, houses an antique carousel. The carousel contains three concentric circles of animals. The inner circle of animals is approximately 11 feet from the center, and the outer circle of animals is approximately 20 feet from the center. The carousel makes $2\frac{5}{8}$ rotations per minute. Determine the angular and linear velocities of someone riding an animal in the inner circle and of someone riding an animal in the same row in the outer circle. *This problem will be solved in Examples 3 and 5.*



The carousel is a circular object that turns about an axis through its center. Other examples of objects that rotate about a central axis include Ferris wheels, gears, tires, and compact discs. As the carousel or any other circular object rotates counterclockwise about its center, an object at the edge moves through an angle relative to its starting position known as the **angular displacement,** or angle of rotation.

Consider a circle with its center at the origin of a rectangular coordinate system and point B on the circle rotating counterclockwise. Let the positive x-axis, or $\overrightarrow{OA}$, be the initial side of the central angle. The terminal side of the central angle is $\overrightarrow{OB}$. The angular displacement is θ. The measure of θ changes as B moves around the circle. All points on $\overrightarrow{OB}$ move through the same angle per unit of time.



Example 1 **Determine the angular displacement in radians of 4.5 revolutions. Round to the nearest tenth.**

Each revolution equals 2π radians. For 4.5 revolutions, the number of radians is $4.5 \times 2\pi$ or 9π. 9π radians equals about 28.3 radians.

The ratio of the change in the central angle to the time required for the change is known as **angular velocity.** Angular velocity is usually represented by the lowercase Greek letter ω (omega).

Angular Velocity

If an object moves along a circle during a time of t units, then the angular velocity, ω, is given by

$$\omega = \frac{\theta}{t},$$

where θ is the angular displacement in radians.

Notice that the angular velocity of a point on a rotating object is not dependent upon the distance from the center of the rotating object.

Example 2 **Determine the angular velocity if 7.3 revolutions are completed in 5 seconds. Round to the nearest tenth.**

The angular displacement is $7.3 \times 2\pi$ or 14.6π radians.

$$\omega = \frac{\theta}{t}$$

$$\omega = \frac{14.6\pi}{5} \qquad \theta = 14.6\pi,\ t = 5$$

$$\omega \approx 9.173450548 \qquad \text{Use a calculator.}$$

The angular velocity is about 9.2 radians per second.

To avoid mistakes when computing with units of measure, you can use a procedure called **dimensional analysis.** In dimensional analyses, unit labels are treated as mathematical factors and can be divided out.

Example 3 Real World Application

ENTERTAINMENT **Refer to the application at the beginning of the lesson. Determine the angular velocity for each rider in radians per second.**

The carousel makes $2\frac{5}{8}$ or 2.625 revolutions per minute. Convert revolutions per minute to radians per second.

$$\frac{2.625\ \cancel{\text{revolutions}}}{1\ \cancel{\text{minute}}} \times \frac{1\ \cancel{\text{minute}}}{60\ \text{seconds}} \times \frac{2\pi\ \text{radians}}{1\ \cancel{\text{revolution}}} \approx 0.275 \text{ radian per second}$$

Each rider has an angular velocity of about 0.275 radian per second.

The carousel riders have the same angular velocity. However, the rider in the outer circle must travel a greater distance than the one in the inner circle. The arc length formula can be used to find the relationship between the linear and angular velocities of an object moving in a circular path. If the object moves with constant **linear velocity** (v) for a period of time (t), the distance (s) it travels is given by the formula $s = vt$. Thus, the linear velocity is $v = \frac{s}{t}$.

As the object moves along the circular path, the radius r forms a central angle of measure θ. Since the length of the arc is $s = r\theta$, the following is true.

$$s = r\theta$$

$$\frac{s}{t} = \frac{r\theta}{t} \qquad \text{Divide each side by } t.$$

$$v = r\frac{\theta}{t} \qquad \text{Replace } \frac{s}{t} \text{ with } v.$$

Linear Velocity

If an object moves along a circle of radius of r units, then its linear velocity, v is given by

$$v = r\frac{\theta}{t},$$

where $\frac{\theta}{t}$ represents the angular velocity in radians per unit of time.

Since $\omega = \frac{\theta}{t}$, the formula for linear velocity can also be written as $v = r\omega$.

Examples

4 **Determine the linear velocity of a point rotating at an angular velocity of 17π radians per second at a distance of 5 centimeters from the center of the rotating object. Round to the nearest tenth.**

$v = r\omega$

$v = 5(17\pi)$ *$r = 5$, $\omega = 17\pi$*

$v \approx 267.0353756$ *Use a calculator.*

The linear velocity is about 267.0 centimeters per second.

5 **ENTERTAINMENT** **Refer to the application at the beginning of the lesson. Determine the linear velocity for each rider.**

Real World Application

From Example 3, you know that the angular velocity is about 0.275 radian per second. Use this number to find the linear velocity for each rider.

Rider on the Inner Circle

$v = r\omega$

$v \approx 11(0.275)$ *$r = 11$, $\omega = 0.275$*

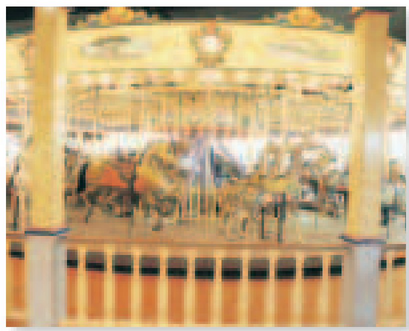
$v \approx 3.025$

Rider on the Outer Circle

$v = r\omega$

$v \approx 20(0.275)$ *$r = 20$, $\omega = 0.275$*

$v \approx 5.5$



The linear velocity of the rider on the inner circle is about 3.025 feet per second, and the linear velocity of the rider on the outer circle is about 5.5 feet per second.

6 **CAR RACING** **The tires on a race car have a diameter of 30 inches. If the tires are turning at a rate of 2000 revolutions per minute, determine the race car's speed in miles per hour (mph).**

Real World Application

If the diameter is 30 inches, the radius is $\frac{1}{2} \times 30$ or 15 inches. This measure needs to be written in miles. The rate needs to be written in hours.

$$v = \underbrace{15\ \cancel{\text{in.}} \times \frac{1\ \cancel{\text{ft}}}{12\ \cancel{\text{in.}}} \times \frac{1\ \text{mi}}{5280\ \cancel{\text{ft}}}}_{r} \times \underbrace{\frac{2000\ \cancel{\text{rev}}}{1\ \cancel{\text{min}}} \times \frac{2\pi}{1\ \cancel{\text{rev}}} \times \frac{60\ \cancel{\text{min}}}{1\ \text{h}}}_{\omega}$$

$v \approx 178.4995826$ mph   *Use a calculator.*

The speed of the race car is about 178.5 miles per hour.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Read and study the lesson to answer each question.

1. **Draw** a circle and represent an angular displacement of 3π radians.
2. **Write** an expression that could be used to change 5 revolutions per minute to radians per second.
3. **Compare and contrast** linear and angular velocity.
4. **Explain** how two people on a rotating carousel can have the same angular velocity but different linear velocity.
5. **Show** that when the radius of a circle is doubled, the angular velocity remains the same and the linear velocity of a point on the circle is doubled.

Guided Practice

Determine each angular displacement in radians. Round to the nearest tenth.

6. 5.8 revolutions
7. 710 revolutions

Determine each angular velocity. Round to the nearest tenth.

8. 3.2 revolutions in 7 seconds
9. 700 revolutions in 15 minutes

Determine the linear velocity of a point rotating at the given angular velocity at a distance r from the center of the rotating object. Round to the nearest tenth.

10. $\omega = 36$ radians per second, $r = 12$ inches
11. $\omega = 5\pi$ radians per minute, $r = 7$ meters

12. **Space** A geosynchronous equatorial orbiting (GEO) satellite orbits 22,300 miles above the equator of Earth. It completes one full revolution each 24 hours. Assume Earth's radius is 3960 miles.
    a. How far will the GEO satellite travel in one day?
    b. What is the satellite's linear velocity in miles per hour?

EXERCISES

Practice

Determine each angular displacement in radians. Round to the nearest tenth.

13. 3 revolutions
14. 2.7 revolutions
15. 13.2 revolutions
16. 15.4 revolutions
17. 60.7 revolutions
18. 3900 revolutions

Determine each angular velocity. Round to the nearest tenth.

19. 1.8 revolutions in 9 seconds
20. 3.5 revolutions in 3 minutes
21. 17.2 revolutions in 12 seconds
22. 28.4 revolutions in 19 seconds
23. 100 revolutions in 16 minutes
24. 122.6 revolutions in 27 minutes

25. A Ferris wheel rotates one revolution every 50 seconds. What is its angular velocity in radians per second?
26. A clothes dryer is rotating at 500 revolutions per minute. Determine its angular velocity in radians per second.

27. Change 85 radians per second to revolutions per minute (rpm).

Determine the linear velocity of a point rotating at the given angular velocity at a distance *r* from the center of the rotating object. Round to the nearest tenth.

28. $\omega = 16.6$ radians per second, $r = 8$ centimeters
29. $\omega = 27.4$ radians per second, $r = 4$ feet
30. $\omega = 6.1\pi$ radians per minute, $r = 1.8$ meters
31. $\omega = 75.3\pi$ radians per second, $r = 17$ inches
32. $\omega = 805.6$ radians per minute, $r = 39$ inches
33. $\omega = 64.5\pi$ radians per minute, $r = 88.9$ millimeters

34. A pulley is turned $120°$ per second.
    a. Find the number of revolutions per minute (rpm).
    b. If the radius of the pulley is 5 inches, find the linear velocity in inches per second.

35. Consider the tip of each hand of a clock. Find the linear velocity in millimeters per second for each hand.
    a. second hand which is 30 millimeters
    b. minute hand which is 27 millimeters long
    c. hour hand which is 18 millimeters long

Applications and Problem Solving

Real World Application

36. **Entertainment** The diameter of a Ferris wheel is 80 feet.
    a. If the Ferris wheel makes one revolution every 45 seconds, find the linear velocity of a person riding in the Ferris wheel.
    b. Suppose the linear velocity of a person riding in the Ferris wheel is 8 feet per second. What is the time for one revolution of the Ferris wheel?

37. **Entertainment** The Kit Carson County Carousel makes 3 revolutions per minute.
    a. Find the linear velocity in feet per second of someone riding a horse that is $22\frac{1}{2}$ feet from the center.
    b. The linear velocity of the person on the inside of the carousel is 3.1 feet per second. How far is the person from the center of the carousel?
    c. How much faster is the rider on the outside going than the rider on the inside?

38. **Critical Thinking** Two children are playing on the seesaw. The lighter child is 9 feet from the fulcrum, and the heavier child is 6 feet from the fulcrum. As the lighter child goes from the ground to the highest point, she travels through an angle of $35°$ in $\frac{1}{2}$ second.
    a. Find the angular velocity of each child.
    b. What is the linear velocity of each child?

39. **Bicycling** A bicycle wheel is 30 inches in diameter.
    a. To the nearest revolution, how many times will the wheel turn if the bicycle is ridden for 3 miles?
    b. Suppose the wheel turns at a constant rate of 2.75 revolutions per second. What is the linear speed in miles per hour of a point on the tire?

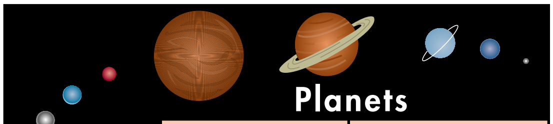
interNET CONNECTION

Research
For information about the other planets, visit **www.amc.glencoe.com**

40. **Space** The radii and times needed to complete one rotation for the four planets closest to the sun are given at the right.

a. Find the linear velocity of a point on each planet's equator.

b. Compare the linear velocity of a point on the equator of Mars with a point on the equator of Earth.

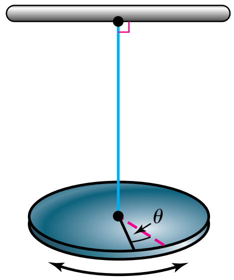


Planets

	Radius (kilometers)	Time for One Rotation (hours)
Mercury	2440	1407.6
Venus	6052	5832.5
Earth	6356	23.935
Mars	3375	24.623

Source: NASA

41. **Physics** A torsion pendulum is an object suspended by a wire or rod so that its plane of rotation is horizontal and it rotates back and forth around the wire without losing energy. Suppose that the pendulum is rotated θ_m radians and released. Then the angular displacement θ at time t is $\theta = \theta_m \cos \omega t$, where ω is the angular frequency in radians per second. Suppose the angular frequency of a certain torsion pendulum is π radians per second and its initial rotation is $\frac{\pi}{4}$ radians.



a. Write the equation for the angular displacement of the pendulum.

b. What are the first two values of t for which the angular displacement of the pendulum is 0?

42. **Space** Low Earth orbiting (LEO) satellites orbit between 200 and 500 miles above Earth. In order to keep the satellites at a constant distance from Earth, they must maintain a speed of 17,000 miles per hour. Assume Earth's radius is 3960 miles.

a. Find the angular velocity needed to maintain a LEO satellite at 200 miles above Earth.

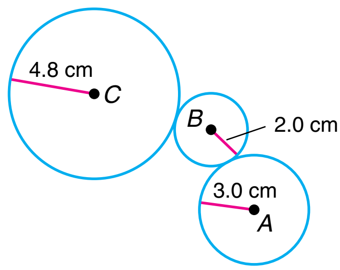
b. How far above Earth is a LEO with an angular velocity of 4 radians per hour?

c. Describe the angular velocity of any LEO satellite.

43. **Critical Thinking** The figure at the right is a side view of three rollers that are tangent to one another.

a. If roller A turns counterclockwise, in which directions do rollers B and C turn?

b. If roller A turns at 120 revolutions per minute, how many revolutions per minute do rollers B and C turn?



Mixed Review

44. Find the area of a sector if the central angle measures 105° and the radius of the circle is 7.2 centimeters. *(Lesson 6-1)*

45. **Geometry** Find the area of a regular pentagon that is inscribed in a circle with a diameter of 7.3 centimeters. *(Lesson 5-4)*

Extra Practice See p. A36.

46. Write 35° 20′ 55″ as a decimal to the nearest thousandth. *(Lesson 5-1)*

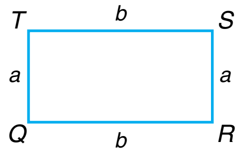
47. Solve $10 + \sqrt{k - 5} = 8$. *(Lesson 4-7)*

48. Write a polynomial equation of least degree with roots -4, $3\boldsymbol{i}$, and $-3\boldsymbol{i}$. *(Lesson 4-1)*

49. Graph $y > x^3 + 1$. *(Lesson 3-3)*

50. Write the slope-intercept form of the equation of the line through points at (8, 5) and (−6, 0). *(Lesson 1-4)*

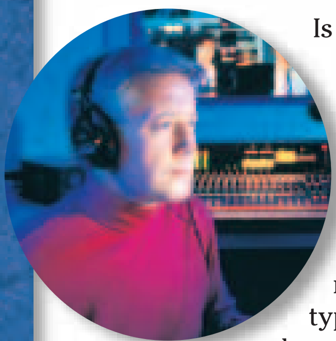
51. **SAT/ACT Practice** The perimeter of rectangle $QRST$ is p, and $a = \frac{3}{4}b$. Find the value of b in terms of p.



A $\frac{p}{7}$ **B** $\frac{4p}{7}$ **C** $\frac{7}{p}$ **D** $\frac{2p}{7}$ **E** $\frac{7p}{4}$

CAREER CHOICES

Audio Recording Engineer



Is music your forte? Do you enjoy being creative and solving problems? If you answered yes to these questions, you may want to consider a career as an audio recording engineer. This type of engineer is in charge of all the technical aspects of recording music, speech, sound effects, and dialogue.

Some aspects of the career include controlling the recording equipment, tackling technical problems that arise during recording, and communicating with musicians and music producers. You would need to keep up-to-date on the latest recording equipment and technology. The music producer may direct the sounds you produce through use of the equipment, or you may have the opportunity to design and perfect your own sounds for use in production.

CAREER OVERVIEW

Degree Preferred:
two- or four-year degree in audio engineering

Related Courses:
mathematics, music, computer science, electronics

Outlook:
number of jobs expected to increase at a slower pace than the average through the year 2006

Sound	Decibels
Threshold of Hearing	0
Average Whisper (4 feet)	20
Broadcast Studio (no program in progress)	30
Soft Recorded Music	36
Normal Conversation (4 feet)	60
Moderate Discotheque	90
Personal Stereo	up to 120
Percussion Instruments at a Symphony Concert	up to 130
Rock Concert	up to 140

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