

1.2

Algebraic Expressions and Models

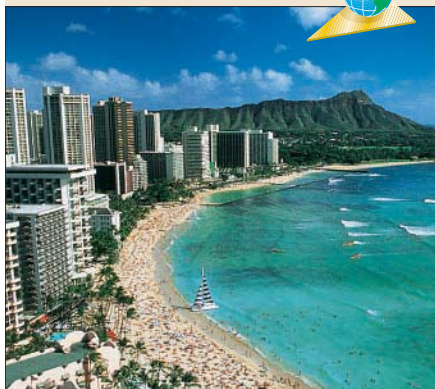
What you should learn

GOAL 1 Evaluate algebraic expressions.

GOAL 2 Simplify algebraic expressions by combining like terms, as applied in Example 6.

Why you should learn it

▼ To solve **real-life** problems, such as finding the population of Hawaii in Ex. 57.

**GOAL 1** EVALUATING ALGEBRAIC EXPRESSIONS

A **numerical expression** consists of numbers, operations, and grouping symbols. In Lesson 1.1 you worked with addition, subtraction, multiplication, and division. In this lesson you will work with *exponentiation*, or raising to a power.

Exponents are used to represent repeated factors in multiplication. For instance, the expression 2^5 represents the number that you obtain when 2 is used as a factor 5 times.

$$2^5 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{5 \text{ factors of } 2} \quad \text{2 to the fifth power}$$

The number 2 is the **base**, the number 5 is the **exponent**, and the expression 2^5 is a **power**. The exponent in a power represents the number of times the base is used as a factor. For a number raised to the first power, you do not usually write the exponent 1. For instance, you usually write 2^1 simply as 2.

EXAMPLE 1 Evaluating Powers

a. $(-3)^4 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) = 81$

b. $-3^4 = -(3 \cdot 3 \cdot 3 \cdot 3) = -81$

.....

In Example 1, notice how parentheses are used in part (a) to indicate that the base is -3 . In the expression -3^4 , however, the base is 3, not -3 . An **order of operations** helps avoid confusion when evaluating expressions.

ORDER OF OPERATIONS

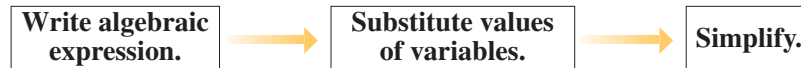
1. First, do operations that occur within grouping symbols.
2. Next, evaluate powers.
3. Then, do multiplications and divisions from left to right.
4. Finally, do additions and subtractions from left to right.

EXAMPLE 2 Using Order of Operations

$$\begin{aligned} -4 + 2(-2 + 5)^2 &= -4 + 2(3)^2 && \text{Add within parentheses.} \\ &= -4 + 2(9) && \text{Evaluate power.} \\ &= -4 + 18 && \text{Multiply.} \\ &= 14 && \text{Add.} \end{aligned}$$

A **variable** is a letter that is used to represent one or more numbers. Any number used to replace a variable is a **value of the variable**. An expression involving variables is called an **algebraic expression**.

When the variables in an algebraic expression are replaced by numbers, you are *evaluating* the expression, and the result is called the **value of the expression**. To evaluate an algebraic expression, use the following flow chart.



EXAMPLE 3 *Evaluating an Algebraic Expression*

Evaluate $-3x^2 - 5x + 7$ when $x = -2$.

$$\begin{aligned}
 -3x^2 - 5x + 7 &= -3(-2)^2 - 5(-2) + 7 && \text{Substitute } -2 \text{ for } x. \\
 &= -3(4) - 5(-2) + 7 && \text{Evaluate power.} \\
 &= -12 + 10 + 7 && \text{Multiply.} \\
 &= 5 && \text{Add.}
 \end{aligned}$$

.....

An expression that represents a real-life situation is a **mathematical model**. When you create the expression, you are *modeling* the real-life situation.



EXAMPLE 4 *Writing and Evaluating a Real-Life Model*

You have \$50 and are buying some movies on videocassettes that cost \$15 each. Write an expression that shows how much money you have left after buying n movies. Evaluate the expression when $n = 2$ and $n = 3$.

SOLUTION



VERBAL MODEL	<div style="border: 1px solid black; padding: 2px; display: inline-block;">Original amount</div> - <div style="border: 1px solid black; padding: 2px; display: inline-block;">Price per movie</div> · <div style="border: 1px solid black; padding: 2px; display: inline-block;">Number of movies bought</div>
LABELS	Original amount = 50 (dollars) Price per movie = 15 (dollars per movie) Number of movies bought = n (movies)
ALGEBRAIC MODEL	$50 - 15n$

When you buy **2** movies, you have $50 - 15(2) = \$20$ left.

When you buy **3** movies, you have $50 - 15(3) = \$5$ left.

UNIT ANALYSIS You can use unit analysis to check your verbal model.

$$\text{dollars} - \left(\frac{\text{dollars}}{\text{movie}} \right) (\text{movies}) = \text{dollars} - \text{dollars} = \text{dollars}$$

GOAL 2 SIMPLIFYING ALGEBRAIC EXPRESSIONS

STUDENT HELP

Study Tip

For an expression like $2x - 3$, think of the expression as $2x + (-3)$, so the terms are $2x$ and -3 .

STUDENT HELP

Skills Review

For help with opposites, see p. 936.

For an expression such as $2x + 3$, the parts that are added together, $2x$ and 3 , are called **terms**. When a term is the product of a number and a power of a variable, such as $2x$ or $4x^3$, the number is the **coefficient** of the power.

Terms such as $3x^2$ and $-5x^2$ are **like terms** because they have the same variable part. **Constant terms** such as -4 and 2 are also like terms. The distributive property lets you *combine like terms* that have variables by adding the coefficients.

EXAMPLE 5 Simplifying by Combining Like Terms

- a. $7x + 4x = (7 + 4)x$ **Distributive property**
 $= 11x$ **Add coefficients.**
- b. $3n^2 + n - n^2 = (3n^2 - n^2) + n$ **Group like terms.**
 $= 2n^2 + n$ **Combine like terms.**
- c. $2(x + 1) - 3(x - 4) = 2x + 2 - 3x + 12$ **Distributive property**
 $= (2x - 3x) + (2 + 12)$ **Group like terms.**
 $= -x + 14$ **Combine like terms.**
-

Two algebraic expressions are **equivalent** if they have the same value for all values of their variable(s). For instance, the expressions $7x + 4x$ and $11x$ are equivalent, as are the expressions $5x - (6x + y)$ and $-x - y$. A statement such as $7x + 4x = 11x$ that equates two equivalent expressions is called an **identity**.

EXAMPLE 6 Using a Real-Life Model

MUSIC You want to buy either a CD or a cassette as a gift for each of 10 people. CDs cost \$13 each and cassettes cost \$8 each. Write an expression for the total amount you must spend. Then evaluate the expression when 4 of the people get CDs.

SOLUTION

VERBAL MODEL	<div style="border: 1px solid black; padding: 2px; display: inline-block;">Price per CD</div> · <div style="border: 1px solid black; padding: 2px; display: inline-block;">Number of CDs</div> + <div style="border: 1px solid black; padding: 2px; display: inline-block;">Price per cassette</div> · <div style="border: 1px solid black; padding: 2px; display: inline-block;">Number of cassettes</div>
↓	
LABELS	CD price = 13 (dollars per CD) Number of CDs = n (CDs) Cassette price = 8 (dollars per cassette) Number of cassettes = $10 - n$ (cassettes)
↓	
ALGEBRAIC MODEL	$13n + 8(10 - n) = 13n + 80 - 8n$ $= 5n + 80$

▶ When $n = 4$, the total cost is $5(4) + 80 = 20 + 80 = \$100$.

FOCUS ON PEOPLE



HEITARO NAKAJIMA could be called the inventor of the compact disc (CD). He was head of the research division of the company that developed the first CDs in 1982. A CD usually has a diameter of 12 centimeters, just the right size to hold Beethoven's Ninth Symphony.

GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

- Copy 8^4 and label the base and the exponent. What does each number represent?
- Identify the terms of $6x^3 - 17x + 5$.
- Explain how the order of operations is used to evaluate $3 - 8^2 \div 4 + 1$.

ERROR ANALYSIS Find the error. Then write the correct steps.

4. ~~$$\begin{aligned} 5 + 2(16 \div 2)^2 &= 5 + 2(16 \div 4) \\ &= 5 + 2(4) \\ &= 5 + 8 \\ &= 13 \end{aligned}$$~~

5. ~~$$\begin{aligned} 4x - (3y + 7x) &= 4x - 3y + 7x \\ &= (4x + 7x) - 3y \\ &= (4 + 7)x - 3y \\ &= 11x - 3y \end{aligned}$$~~

Skill Check ✓

Evaluate the expression for the given value of x .

- $x - 8$ when $x = 2$
- $3x + 14$ when $x = -3$
- $x(x + 4)$ when $x = 5$
- $x^2 - 9$ when $x = 6$

Simplify the expression.

- $9y - 14y$
- $11x + 6y - 2x + 3y$
- $3(x + 4) - (6 + 2x)$
- $3x^2 - 5x + 5x^2 - 3x$
- RETAIL BUYING** When you arrive at the music store to buy the CDs and cassettes for the 10 people mentioned in Example 6, you find that the store is having a sale. CDs now cost \$11 each and cassettes now cost \$7 each. Write an expression for the new total amount you will spend. Then evaluate the expression when 6 of the people get CDs.

PRACTICE AND APPLICATIONS

STUDENT HELP

► **Extra Practice** to help you master skills is on p. 940.

WRITING WITH EXPONENTS Write the expression using exponents.

- eight to the third power
- x to the fifth power
- 5 to the n th power
- $x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$

EVALUATING POWERS Evaluate the power.

- 4^4
- $(-4)^4$
- -2^5
- $(-2)^5$
- 5^3
- 3^5
- 2^8
- 8^2

USING ORDER OF OPERATIONS Evaluate the expression.

- $13 + 20 - 9$
- $14 \cdot 3 - 2$
- $6 \cdot 2 + 35 \div 5$
- $-6 + 3(-3 + 7)^2$
- $24 - 8 \cdot 12 \div 4$
- $16 \div (2 + 6) \cdot 10$

EVALUATING EXPRESSIONS Evaluate the expression for the given value of x .

- $x - 12$ when $x = 7$
- $6x + 9$ when $x = 4$
- $25x(x - 4)$ when $x = -1$
- $x^2 + 5 - x$ when $x = 5$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 15–26
Example 2: Exs. 27–32
Example 3: Exs. 33–46
Example 4: Exs. 56–61
Example 5: Exs. 47–52
Example 6: Exs. 56–61

EVALUATING EXPRESSIONS Evaluate the expression for the given values of x and y .

- 37. $x^4 + 3y$ when $x = 2$ and $y = -8$
- 38. $(3x)^2 - 7y^2$ when $x = 3$ and $y = 2$
- 39. $9x + 8y$ when $x = 4$ and $y = 5$
- 40. $5\left(\frac{x}{y}\right) - x$ when $x = 6$ and $y = \frac{2}{3}$
- 41. $\frac{x^2}{2y + 1}$ when $x = -3$ and $y = 2$
- 42. $\frac{(x + 3)^2}{3y - 2}$ when $x = 2$ and $y = 4$
- 43. $\frac{x + y}{x - y}$ when $x = -4$ and $y = 9$
- 44. $\frac{2x + y}{3y + x}$ when $x = 10$ and $y = 6$
- 45. $\frac{4(x - 2y)}{x + y}$ when $x = 4$ and $y = -2$
- 46. $\frac{4y - x}{3(2x + y)}$ when $x = -3$ and $y = 3$

SIMPLIFYING EXPRESSIONS Simplify the expression.

- 47. $7x^2 + 12x - x^2 - 40x$
- 48. $4x^2 + x - 3x - 6x^2$
- 49. $12(n - 3) + 4(n - 13)$
- 50. $5(n^2 + n) - 3(n^2 - 2n)$
- 51. $4x - 2y + y - 9x$
- 52. $8(y - x) - 2(x - y)$

STUDENT HELP

Skills Review

For help with area, see p. 914.



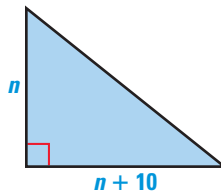
HOMEWORK HELP

Visit our Web site www.mcdougallittell.com for help with problem solving in Ex. 56.

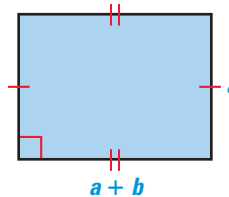
GEOMETRY CONNECTION

Write an expression for the area of the figure. Then evaluate the expression for the given value(s) of the variable(s).

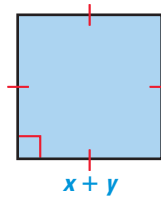
53. $n = 40$



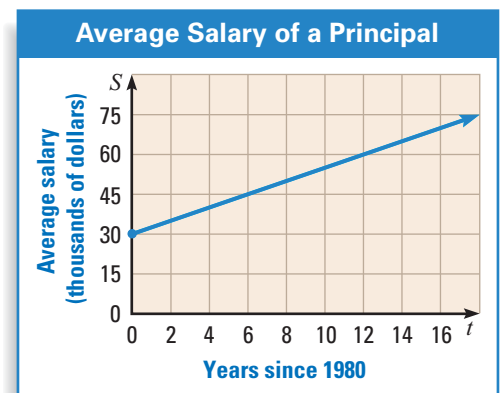
54. $a = 8, b = 3$



55. $x = 12, y = 5$



56. **AVERAGE SALARIES** In 1980, a public high school principal's salary was approximately \$30,000. From 1980 through 1996, the average salary of principals at public high schools increased by an average of \$2500 per year. Use the verbal model and labels below to write an algebraic model that gives a public high school principal's average salary t years after 1980. Evaluate the expression when $t = 5, 10,$ and 15 .



Source: Educational Research Service

VERBAL MODEL

$$\boxed{\text{Salary in 1980}} + \boxed{\text{Average increase per year}} \cdot \boxed{\text{Years since 1980}}$$

LABELS

Salary in 1980 = 30 (thousands of dollars)

Average increase per year = 2.5 (thousands of dollars per year)

Years since 1980 = t (years)

FOCUS ON CAREERS




PHYSICAL THERAPIST

Physical therapists help restore function, improve mobility, and relieve pain in patients with injuries or disease.


CAREER LINK

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Test Preparation

57. **REAL LIFE SOCIAL STUDIES CONNECTION** For 1980 through 1998, the population (in thousands) of Hawaii can be modeled by $13.2t + 965$ where t is the number of years since 1980. What was the population of Hawaii in 1998? What was the population increase from 1980 to 1998? ▶ Source: U.S. Bureau of the Census
58. **INTERNET PHYSICAL THERAPY** In 1996 there were approximately 115,000 physical therapy jobs in the United States. The number of jobs is expected to increase by 8100 each year. Write an expression that gives the total number of physical therapy jobs each year since 1996. Evaluate the expression for the year 2010.
 **DATA UPDATE** of U.S. Bureau of Labor Statistics data at www.mcdougallittell.com
59. **INTERNET MOVIE RENTALS** You buy a VCR for \$149 and plan to rent movies each month. Each rental costs \$3.85. Write an expression that gives the total amount you spend during the first twelve months that you own the VCR, including the price of the VCR. Evaluate the expression if you rent 6 movies each month.
60. **INTERNET USED CARS** You buy a used car with 37,148 miles on the odometer. Based on your regular driving habits, you plan to drive the car 15,000 miles each year that you own it. Write an expression for the number of miles that appears on the odometer at the end of each year. Evaluate the expression to find the number of miles that will appear on the odometer after you have owned the car for 4 years.
61. **INTERNET WALK-A-THON** You are taking part in a charity walk-a-thon where you can either walk or run. You walk at 4 kilometers per hour and run at 8 kilometers per hour. The walk-a-thon lasts 3 hours. Money is raised based on the total distance you travel in the 3 hours. Your sponsors donate \$15 for each kilometer you travel. Write an expression that gives the total amount of money you raise. Evaluate the expression if you walk for 2 hours and run for 1 hour.

QUANTITATIVE COMPARISON In Exercises 62–67, choose the statement that is true about the given quantities.

- (A) The quantity in column A is greater.
 (B) The quantity in column B is greater.
 (C) The two quantities are equal.
 (D) The relationship cannot be determined from the given information.

	Column A	Column B
62.	2^6	$(-2)^6$
63.	-4^4	$(-4)^4$
64.	x^4	x^5
65.	$3(x - 2)$ when $x = 4$	$3x - 6$ when $x = 4$
66.	$x + 10(x^2 - 3)$ when $x = 3$	x^6 when $x = 2$
67.	$2(x^2 - 1)$	$2x^2 - 1$


Challenge

68. **MATH CLUB SHIRTS** The math club is ordering shirts for its 8 members. The club members have a choice of either a \$15 T-shirt or a \$25 sweatshirt. Make a table showing the total amount of money needed for each possible combination of T-shirts and sweatshirts that the math club can order. Describe any patterns you see. Write an expression that gives the total cost of the shirts. Explain what each term in the expression represents.


EXTRA CHALLENGE

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MIXED REVIEW

LEAST COMMON DENOMINATOR Find the least common denominator.

(Skills Review, p. 908)

69. $\frac{1}{2}, \frac{3}{4}, \frac{4}{5}$

70. $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}$

71. $\frac{1}{3}, \frac{2}{5}, -\frac{14}{15}$

72. $\frac{1}{4}, -\frac{3}{8}, \frac{7}{12}$

73. $\frac{1}{3}, \frac{4}{5}, \frac{6}{7}$

74. $\frac{1}{2}, \frac{3}{4}, -\frac{1}{16}$

USING A NUMBER LINE Graph the numbers on a number line. Then decide which number is greater and use the symbol $<$ or $>$ to show the relationship.

(Review 1.1)

75. $-\sqrt{3}, -3$

76. $-\frac{1}{2}, -\frac{11}{2}$

77. $2.75, \frac{7}{2}$

IDENTIFYING PROPERTIES Identify the property shown. (Review 1.1)

78. $(7 \cdot 9)8 = 7(9 \cdot 8)$

79. $-13 + 13 = 0$

80. $27 + 6 = 6 + 27$

81. $19 \cdot 1 = 19$

FINDING RECIPROCAL Give the reciprocal of the number. (Review 1.1 for 1.3)

82. -22

83. $\frac{7}{8}$

84. 12

85. $-\frac{5}{4}$

86. $\frac{11}{16}$

87. $-\frac{1}{9}$

88. 37

89. -14

QUIZ 1

Self-Test for Lessons 1.1 and 1.2

Graph the numbers on a number line. Then write the numbers in increasing order. (Lesson 1.1)

1. $\frac{9}{2}, -2.5, 0, -\frac{3}{4}, 1$

2. $\frac{10}{3}, 0.8, \frac{15}{8}, -1.5, -0.25$

Identify the property shown. (Lesson 1.1)

3. $5(3 - 7) = 5 \cdot 3 - 5 \cdot 7$

4. $(8 + 6) + 4 = 8 + (6 + 4)$

Evaluate the expression for the given value(s) of the variable(s). (Lesson 1.2)

5. $12x - 21$ when $x = 3$

6. $7x - (9x + 5)$ when $x = \frac{1}{3}$

7. $x^2 + 5x - 8$ when $x = -3$

8. $x^3 + 4(x - 1)$ when $x = 4$

9. $x^2 - 11x + 40y - 14$ when $x = 5$ and $y = -2$


Simplify the expression. (Lesson 1.2)

10. $3x - 2y - 9y + 4 + 5x$

11. $3(x - 2) - (4 + x)$

12. $5x^2 - 3x + 8x - 6 - 7x^2$

13. $4(x + 2x) - 2(x^2 - x)$

14.  **COMPUTER DISKS** You are buying a total of 15 regular floppy disks and high capacity storage disks for your computer. Regular floppy disks cost \$.35 each and high capacity disks cost \$13.95 each. Write an expression for the total amount you spend on computer disks. (Lesson 1.2)