

## 1.3

## Solving Linear Equations

*What you should learn*

**GOAL 1** Solve linear equations.

**GOAL 2** Use linear equations to solve **real-life** problems, such as finding how much a broker must sell in **Example 5**.

*Why you should learn it*

▼ To solve **real-life** problems, such as finding the temperature at which dry ice changes to a gas in **Ex. 43**.

**GOAL 1 SOLVING A LINEAR EQUATION**

An **equation** is a statement in which two expressions are equal. A **linear equation** in one variable is an equation that can be written in the form  $ax = b$  where  $a$  and  $b$  are constants and  $a \neq 0$ . A number is a **solution** of an equation if the statement is true when the number is substituted for the variable.

Two equations are **equivalent** if they have the same solutions. For instance, the equations  $x - 4 = 1$  and  $x = 5$  are equivalent because both have the number 5 as their only solution. The following *transformations*, or changes, produce equivalent equations and can be used to solve an equation.

**TRANSFORMATIONS THAT PRODUCE EQUIVALENT EQUATIONS****ADDITION PROPERTY OF EQUALITY**

Add the same number to both sides:  
If  $a = b$ , then  $a + c = b + c$ .

**SUBTRACTION PROPERTY OF EQUALITY**

Subtract the same number from both sides:  
If  $a = b$ , then  $a - c = b - c$ .

**MULTIPLICATION PROPERTY OF EQUALITY**

Multiply both sides by the same nonzero number: If  $a = b$  and  $c \neq 0$ , then  $ac = bc$ .

**DIVISION PROPERTY OF EQUALITY**

Divide both sides by the same nonzero number: If  $a = b$  and  $c \neq 0$ , then  $a \div c = b \div c$ .

**EXAMPLE 1 Solving an Equation with a Variable on One Side**

Solve  $\frac{3}{7}x + 9 = 15$ .

**SOLUTION**

Your goal is to isolate the variable on one side of the equation.

$$\frac{3}{7}x + 9 = 15 \quad \text{Write original equation.}$$

$$\frac{3}{7}x = 6 \quad \text{Subtract 9 from each side.}$$

$$x = \frac{7}{3}(6) \quad \text{Multiply each side by } \frac{7}{3}, \text{ the reciprocal of } \frac{3}{7}.$$

$$x = 14 \quad \text{Simplify.}$$

► The solution is 14.

✓ **CHECK** Check  $x = 14$  in the original equation.

$$\frac{3}{7}(14) + 9 \stackrel{?}{=} 15 \quad \text{Substitute 14 for } x.$$

$$15 = 15 \quad \text{Solution checks.}$$

**EXAMPLE 2** Solving an Equation with a Variable on Both Sides**STUDENT HELP****HOMEWORK HELP**

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for extra examples.

Solve  $5n + 11 = 7n - 9$ .

**SOLUTION**

$$5n + 11 = 7n - 9 \quad \text{Write original equation.}$$

$$11 = 2n - 9 \quad \text{Subtract } 5n \text{ from each side.}$$

$$20 = 2n \quad \text{Add 9 to each side.}$$

$$10 = n \quad \text{Divide each side by 2.}$$

▶ The solution is 10. Check this in the original equation.

**EXAMPLE 3** Using the Distributive Property

Solve  $4(3x - 5) = -2(-x + 8) - 6x$ .

**SOLUTION**

$$4(3x - 5) = -2(-x + 8) - 6x \quad \text{Write original equation.}$$

$$12x - 20 = 2x - 16 - 6x \quad \text{Distributive property}$$

$$12x - 20 = -4x - 16 \quad \text{Combine like terms.}$$

$$16x - 20 = -16 \quad \text{Add } 4x \text{ to each side.}$$

$$16x = 4 \quad \text{Add 20 to each side.}$$

$$x = \frac{1}{4} \quad \text{Divide each side by 16.}$$

▶ The solution is  $\frac{1}{4}$ . Check this in the original equation.

**EXAMPLE 4** Solving an Equation with Fractions

Solve  $\frac{1}{3}x + \frac{1}{4} = x - \frac{1}{6}$ .

**SOLUTION**

$$\frac{1}{3}x + \frac{1}{4} = x - \frac{1}{6} \quad \text{Write original equation.}$$

$$12\left(\frac{1}{3}x + \frac{1}{4}\right) = 12\left(x - \frac{1}{6}\right) \quad \text{Multiply each side by the LCD, 12.}$$

$$4x + 3 = 12x - 2 \quad \text{Distributive property}$$

$$3 = 8x - 2 \quad \text{Subtract } 4x \text{ from each side.}$$

$$5 = 8x \quad \text{Add 2 to each side.}$$

$$\frac{5}{8} = x \quad \text{Divide each side by 8.}$$

▶ The solution is  $\frac{5}{8}$ . Check this in the original equation.

**STUDENT HELP****Skills Review**

For help with finding the LCD, see p. 939.

## FOCUS ON CAREERS


**REAL ESTATE BROKER**

Real estate brokers must have a thorough knowledge not only of the real estate market, but of mathematics as well. Brokers often provide buyers with information about loans, loan rates, and monthly payments.

**CAREER LINK**  
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**GOAL 2** USING LINEAR EQUATIONS IN REAL LIFE**EXAMPLE 5** Writing and Using a Linear Equation

**REAL ESTATE** A real estate broker's base salary is \$18,000. She earns a 4% commission on total sales. How much must she sell to earn \$55,000 total?

**SOLUTION****VERBAL MODEL**

$$\boxed{\text{Total income}} = \boxed{\text{Base salary}} + \boxed{\text{Commission rate}} \cdot \boxed{\text{Total sales}}$$

**LABELS**

$$\text{Total income} = 55,000 \quad (\text{dollars})$$

$$\text{Base salary} = 18,000 \quad (\text{dollars})$$

$$\text{Commission rate} = 0.04 \quad (\text{percent in decimal form})$$

$$\text{Total sales} = x \quad (\text{dollars})$$

**ALGEBRAIC MODEL**

$$55,000 = 18,000 + 0.04x \quad \text{Write linear equation.}$$

$$37,000 = 0.04x \quad \text{Subtract 18,000 from each side.}$$

$$925,000 = x \quad \text{Divide each side by 0.04.}$$

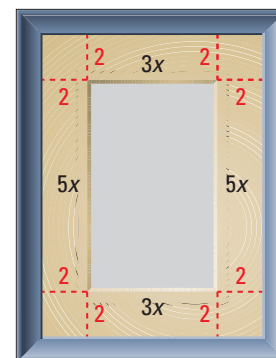
▶ The broker must sell real estate worth a total of \$925,000 to earn \$55,000.

**EXAMPLE 6** Writing and Using a Geometric Formula

You have a 3 inch by 5 inch photo that you want to enlarge, mat, and frame. You want the width of the mat to be 2 inches on all sides. You want the perimeter of the framed photo to be 44 inches. By what percent should you enlarge the photo?

**SOLUTION**

Let  $x$  be the percent (in decimal form) of enlargement relative to the original photo. So, the dimensions of the enlarged photo (in inches) are  $3x$  by  $5x$ . Draw a diagram.

**VERBAL MODEL**

$$\boxed{\text{Perimeter}} = 2 \cdot \boxed{\text{Width}} + 2 \cdot \boxed{\text{Length}}$$

**LABELS**

$$\text{Perimeter} = 44 \quad (\text{inches})$$

$$\text{Width} = 4 + 3x \quad (\text{inches})$$

$$\text{Length} = 4 + 5x \quad (\text{inches})$$

**ALGEBRAIC MODEL**

$$44 = 2(4 + 3x) + 2(4 + 5x) \quad \text{Write linear equation.}$$

$$44 = 16 + 16x \quad \text{Distribute and combine like terms.}$$

$$28 = 16x \quad \text{Subtract 16 from each side.}$$

$$1.75 = x \quad \text{Divide each side by 16.}$$

▶ You should enlarge the photo to 175% of its original size.

**PROBLEM SOLVING STRATEGY**

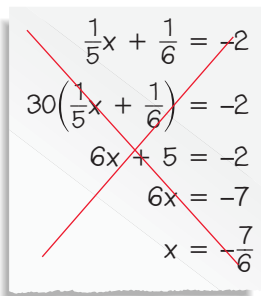
## GUIDED PRACTICE

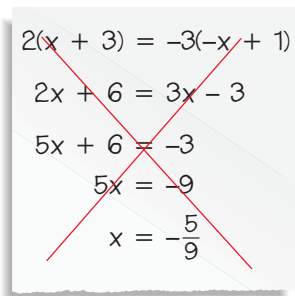
### Vocabulary Check ✓

### Concept Check ✓

1. What is an equation?
2. What does it mean for two equations to be equivalent? Give an example of two equivalent equations.
3. How does an equation such as  $2(x + 3) = 10$  differ from an identity such as  $2(x + 3) = 2x + 6$ ?

### ERROR ANALYSIS Describe the error(s). Then write the correct steps.


4. 

5. 

6. Describe the transformation(s) you would use to solve  $2x - 8 = 14$ .

### Skill Check ✓

#### Solve the equation.

- |                      |  |  |
|----------------------|--|--|
| 7. $x + 4 = 9$       | 8. $4x = 24$                                     | 9. $2x - 3 = 7$                                |
| 10. $0.2x - 8 = 0.6$ | 11. $\frac{1}{3}x + \frac{1}{2} = \frac{11}{12}$ | 12. $\frac{3}{4}x - \frac{2}{3} = \frac{5}{6}$ |
| 13. $1.5x + 9 = 4.5$ | 14. $6x - 4 = 2x + 10$                           | 15. $2(x + 2) = 3(x - 8)$                      |
16.  **REAL ESTATE SALES** The real estate broker's base salary from Example 5 has been raised to \$21,000 and the commission rate has been increased to 5%. How much real estate does the broker have to sell now to earn \$70,000?

## PRACTICE AND APPLICATIONS

### STUDENT HELP

#### Extra Practice

to help you master skills is on p. 940.

### DESCRIBING TRANSFORMATIONS Describe the transformation(s) you would use to solve the equation.

- |                  |                            |                         |
|------------------|----------------------------|-------------------------|
| 17. $x + 5 = -7$ | 18. $\frac{1}{6}x = 3$     | 19. $-\frac{4}{7}x = 6$ |
| 20. $2x - 9 = 0$ | 21. $\frac{x}{3} + 2 = 89$ | 22. $3 = -x - 5$        |

### SOLVING EQUATIONS Solve the equation. Check your solution.

- |                                 |                                     |
|---------------------------------|-------------------------------------|
| 23. $4x + 7 = 27$               | 24. $7s - 29 = -15$                 |
| 25. $3a + 13 = 9a - 8$          | 26. $m - 30 = 6 - 2m$               |
| 27. $15n + 9 = 21$              | 28. $2b + 11 = 15 - 6b$             |
| 29. $2(x + 6) = -2(x - 4)$      | 30. $4(-3x + 1) = -10(x - 4) - 14x$ |
| 31. $-(x + 2) - 2x = -2(x + 1)$ | 32. $-4(3 + x) + 5 = 4(x + 3)$      |

### STUDENT HELP

#### HOMEWORK HELP

Examples 1–4: Exs. 17–40  
Examples 5, 6: Exs. 43–49

**SOLVING EQUATIONS** Solve the equation. Check your solution.

33.  $\frac{7}{2}x - 1 = 2x + 5$

34.  $\frac{1}{2}x - \frac{5}{3} = -\frac{1}{2}x + \frac{19}{4}$

35.  $\frac{3}{4}\left(\frac{4}{5}x - 2\right) = \frac{11}{4}$

36.  $-\frac{2}{3}\left(\frac{6}{5}x - \frac{7}{10}\right) = \frac{17}{20}$

37.  $2.7n + 4.3 = 12.94$

38.  $-4.2n - 6.5 = -14.06$

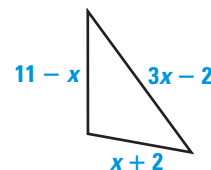
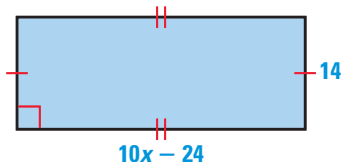
39.  $3.1(x + 2) - 1.5x = 5.2(x - 4)$

40.  $2.5(x - 3) + 1.7x = 10.8(x + 1.5)$

**GEOMETRY CONNECTION** Find the dimensions of the figure.

41. Area = 504

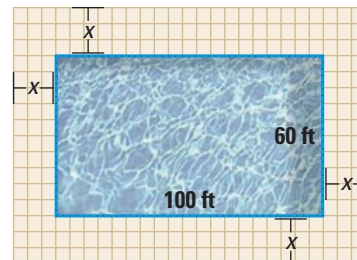
42. Perimeter = 23



In Exercises 43 and 44, use the following formula.

degrees Fahrenheit =  $\frac{9}{5}$ (degrees Celsius) + 32

43. **DRY ICE** Dry ice is solid carbon dioxide. Dry ice does not melt — it changes directly from a solid to a gas. Dry ice changes to a gas at  $-109.3^\circ\text{F}$ . What is this temperature in degrees Celsius?
44. **VETERINARY MEDICINE** The normal body temperature of a dog is  $38.6^\circ\text{C}$ . Your dog's temperature is  $101.1^\circ\text{F}$ . Does your dog have a fever? Explain.
45. **CAR REPAIR** The bill for the repair of your car was \$390. The cost for parts was \$215. The cost for labor was \$35 per hour. How many hours did the repair work take?
46. **SUMMER JOBS** You have two summer jobs. In the first job, you work 28 hours per week and earn \$7.25 per hour. In the second job, you earn \$6.50 per hour and can work as many hours as you want. If you want to earn \$255 per week, how many hours must you work at your second job?
47. **STOCKBROKER** A stockbroker earns a base salary of \$40,000 plus 5% of the total value of the stocks, mutual funds, and other investments that the stockbroker sells. Last year, the stockbroker earned \$71,750. What was the total value of the investments the stockbroker sold?
48. **WORD PROCESSING** You are writing a term paper. You want to include a table that has 5 columns and is 360 points wide. (A point is  $\frac{1}{72}$  of an inch.) You want the first column to be 200 points wide and the remaining columns to be equal in width. How wide should each of the remaining columns be?
49. **WALKWAY CONSTRUCTION** You are building a walkway of uniform width around a 100 foot by 60 foot swimming pool. After completing the walkway, you want to put a fence along the outer edge of the walkway. You have 450 feet of fencing to enclose the walkway. What is the maximum width of the walkway?

**FOCUS ON CAREERS****STOCKBROKER**

Stockbrokers buy and sell stocks, bonds, and other securities for clients as discussed in Ex. 47. Stockbrokers typically study economics in college.

**CAREER LINK**

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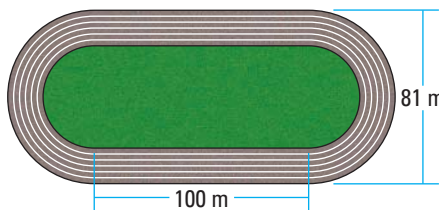




## Test Preparation



50. **MULTI-STEP PROBLEM** You are in charge of constructing a fence around the running track at a high school. The fence is to be built around the track so that there is a uniform gap between the outside edge of the track and the fence.



- What is the maximum width of the gap between the track and the fence if no more than 630 meters of fencing is used? (*Hint:* Use the equation for the circumference of a circle,  $C = 2\pi r$ , to help you.)
- You are charging the school \$10.50 for each meter of fencing. The school has \$5250 in its budget to spend on the fence. How many meters of fencing can you use with this budget?
- CRITICAL THINKING** Explain whether or not it is geometrically reasonable to put up the new fence with the given budget.

## ★ Challenge

**SOLVING EQUATIONS** Solve the equation. If there is no solution, write *no solution*. If the equation is an identity, write *all real numbers*.

51.  $5(x - 4) = 5x + 12$

52.  $3(x + 5) = 3x + 15$

53.  $7x + 14 - 3x = 4x + 14$

54.  $11x - 3 + 2x = 6(x + 4) + 7x$

55.  $-2(4 - 3x) + 7 = -2x + 6 + 8x$

56.  $5(2 - x) = 3 - 2x + 7 - 3x$

### EXTRA CHALLENGE

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## MIXED REVIEW

**GEOMETRY CONNECTION** Find the area of the figure. (**Skills Review, p. 914**)

57. Circle with radius 5 inches

58. Square with side 4 inches

59. Circle with radius 7 inches

60. Square with side 9 inches

**EVALUATING EXPRESSIONS** Evaluate the expression. (**Review 1.2 for 1.4**)

61.  $24 - (9 + 7)$

62.  $-16 + 3(8 - 4)$

63.  $-3 + 6(1 - 3)^2$

64.  $2(3 - 5)^3 + 4(-4 + 7)$

65.  $2x + 3$  when  $x = 4$

66.  $8(x - 2) + 3x$  when  $x = 6$

67.  $5x - 7 + 2x$  when  $x = -3$

68.  $6x - 3(2x + 4)$  when  $x = 5$

**SIMPLIFYING EXPRESSIONS** Simplify the expression. (**Review 1.2**)

69.  $3(7 + x) - 8x$

70.  $2(8 + x) + 2x - x$

71.  $4x - (6 - 3x)$

72.  $2x - 3(4x + 7)$

73.  $3(x + 9) + 2(4 - x)$

74.  $-4(x - 3) - 2(x + 7)$

75.  $2(x^2 + 2) - x + x^2 + 7$

76.  $2(x^2 - 81) - 3x^2$

77.  $x^2 - 5x + 3(x^2 + 7x)$

78.  $4x^2 - 2(x^2 - 3x) + 6x + 8$