

1.6

Solving Linear Inequalities

What you should learn

GOAL 1 Solve simple inequalities.

GOAL 2 Solve compound inequalities, as applied in Example 6.

Why you should learn it

▼ To model **real-life** situations, such as amusement park fees in Ex. 50.

**GOAL 1 SOLVING SIMPLE INEQUALITIES**

Inequalities have properties that are similar to those of equations, but the properties differ in some important ways.

ACTIVITY

Developing Concepts

Investigating Properties of Inequalities

- Write two true inequalities involving integers, one using $<$ and one using $>$.
- Add, subtract, multiply, and divide each side of your inequalities by 2 and -2 . In each case, decide whether the new inequality is true or false.
- Write a general conclusion about the operations you can perform on a true inequality to produce another true inequality.

Inequalities such as $x \leq 1$ and $2n - 3 > 9$ are examples of **linear inequalities** in one variable. A **solution** of an inequality in one variable is a value of the variable that makes the inequality true. For instance, -2 , 0 , 0.872 , and 1 are some of the many solutions of $x \leq 1$.

In the activity you may have discovered some of the following properties of inequalities. You can use these properties to solve an inequality because each transformation produces a new inequality having the same solutions as the original.

TRANSFORMATIONS THAT PRODUCE EQUIVALENT INEQUALITIES

- Add the same number to both sides.
- Subtract the same number from both sides.
- Multiply both sides by the same *positive* number.
- Divide both sides by the same *positive* number.
- Multiply both sides by the same *negative* number and *reverse* the inequality.
- Divide both sides by the same *negative* number and *reverse* the inequality.

The **graph** of an inequality in one variable consists of all points on a real number line that correspond to solutions of the inequality. To graph an inequality in one variable, use an open dot for $<$ or $>$ and a solid dot for \leq or \geq . For example, the graphs of $x < 3$ and $x \geq -2$ are shown below.

Graph of $x < 3$ Graph of $x \geq -2$

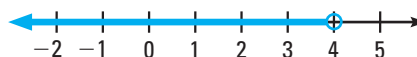
**EXAMPLE 1** Solving an Inequality with a Variable on One SideSolve $5y - 8 < 12$.**SOLUTION**

$$5y - 8 < 12 \quad \text{Write original inequality.}$$

$$5y < 20 \quad \text{Add 8 to each side.}$$

$$y < 4 \quad \text{Divide each side by 5.}$$

▶ The solutions are all real numbers less than 4, as shown in the graph at the right.



✓ **CHECK** As a check, try several numbers that are less than 4 in the original inequality. Also, try checking some numbers that are greater than or equal to 4 to see that they are *not* solutions of the original inequality.

EXAMPLE 2 Solving an Inequality with a Variable on Both SidesSolve $2x + 1 \leq 6x - 1$.**SOLUTION**

$$2x + 1 \leq 6x - 1 \quad \text{Write original inequality.}$$

$$-4x + 1 \leq -1 \quad \text{Subtract } 6x \text{ from each side.}$$

$$-4x \leq -2 \quad \text{Subtract 1 from each side.}$$

$$x \geq \frac{1}{2} \quad \text{Divide each side by } -4 \text{ and reverse the inequality.}$$

▶ The solutions are all real numbers greater than or equal to $\frac{1}{2}$. Check several numbers greater than or equal to $\frac{1}{2}$ in the original inequality.

STUDENT HELP**Study Tip**

Don't forget that when you multiply or divide both sides of an inequality by a negative number, you must *reverse* the inequality to maintain a true statement. For instance, to reverse \leq , replace it with \geq .

**EXAMPLE 3** Using a Simple Inequality

The weight w (in pounds) of an Icelandic saithe is given by

$$w = 10.4t - 2.2$$

where t is the age of the fish in years. Describe the ages of a group of Icelandic saithe that weigh up to 29 pounds. ▶ Source: Marine Research Institute



Icelandic saithe

SOLUTION

$$w \leq 29 \quad \text{Weights are at most 29 pounds.}$$

$$10.4t - 2.2 \leq 29 \quad \text{Substitute for } w.$$

$$10.4t \leq 31.2 \quad \text{Add 2.2 to each side.}$$

$$t \leq 3 \quad \text{Divide each side by 10.4.}$$

▶ The ages are less than or equal to 3 years.

GOAL 2 SOLVING COMPOUND INEQUALITIES

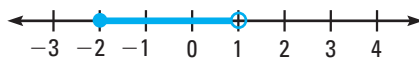
STUDENT HELP

Study Tip

The inequality $a < x < b$ is read as “ x is between a and b .” The inequality $a \leq x \leq b$ is read as “ x is between a and b , inclusive.”

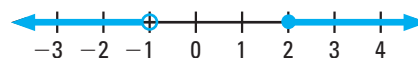
A **compound inequality** is two simple inequalities joined by “and” or “or.” Here are two examples.

$$-2 \leq x < 1$$



All real numbers that are greater than or equal to -2 and less than 1 .

$$x < -1 \text{ or } x \geq 2$$



All real numbers that are less than -1 or greater than or equal to 2 .

EXAMPLE 4 Solving an “And” Compound Inequality

Solve $-2 \leq 3t - 8 \leq 10$.

SOLUTION

To solve, you must isolate the variable between the two inequality signs.

$$-2 \leq 3t - 8 \leq 10 \quad \text{Write original inequality.}$$

$$6 \leq 3t \leq 18 \quad \text{Add 8 to each expression.}$$

$$2 \leq t \leq 6 \quad \text{Divide each expression by 3.}$$

▶ Because t is between 2 and 6, inclusive, the solutions are all real numbers greater than or equal to 2 and less than or equal to 6. Check several of these numbers in the original inequality. The graph is shown below.



EXAMPLE 5 Solving an “Or” Compound Inequality

Solve $2x + 3 < 5$ or $4x - 7 > 9$.

SOLUTION

A solution of this compound inequality is a solution of *either* of its simple parts, so you should solve each part separately.

SOLUTION OF FIRST INEQUALITY

$$2x + 3 < 5 \quad \text{Write first inequality.}$$

$$2x < 2 \quad \text{Subtract 3 from each side.}$$

$$x < 1 \quad \text{Divide each side by 2.}$$

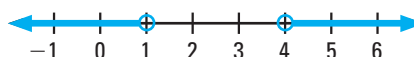
SOLUTION OF SECOND INEQUALITY

$$4x - 7 > 9 \quad \text{Write second inequality.}$$

$$4x > 16 \quad \text{Add 7 to each side.}$$

$$x > 4 \quad \text{Divide each side by 4.}$$

▶ The solutions are all real numbers less than 1 or greater than 4. Check several of these numbers to see that they satisfy one of the simple parts of the original inequality. The graph is shown below.



STUDENT HELP



HOMEWORK HELP

Visit our Web site
www.mcdougallittell.com
for extra examples.



EXAMPLE 6 Using an "And" Compound Inequality

You have added enough antifreeze to your car's cooling system to lower the freezing point to -35°C and raise the boiling point to 125°C . The coolant will remain a liquid as long as the temperature C (in degrees Celsius) satisfies the inequality $-35 < C < 125$. Write the inequality in degrees Fahrenheit.

SOLUTION

Let F represent the temperature in degrees Fahrenheit, and use the formula

$$C = \frac{5}{9}(F - 32).$$

$$-35 < C < 125$$

Write original inequality.

$$-35 < \frac{5}{9}(F - 32) < 125$$

Substitute $\frac{5}{9}(F - 32)$ for C .

$$-63 < F - 32 < 225$$

Multiply each expression by $\frac{9}{5}$, the reciprocal of $\frac{5}{9}$.

$$-31 < F < 257$$

Add 32 to each expression.

- ▶ The coolant will remain a liquid as long as the temperature stays between -31°F and 257°F .

EXAMPLE 7 Using an "Or" Compound Inequality

TRAFFIC ENFORCEMENT You are a state patrol officer who is assigned to work traffic enforcement on a highway. The posted minimum speed on the highway is 45 miles per hour and the posted maximum speed is 65 miles per hour. You need to detect vehicles that are traveling outside the posted speed limits.

- Write these conditions as a compound inequality.
- Rewrite the conditions in kilometers per hour.

SOLUTION

- Let m represent the vehicle speeds in miles per hour. The speeds that you need to detect are given by:

$$m < 45 \text{ or } m > 65$$

- Let k be the vehicle speeds in kilometers per hour. The relationship between miles per hour and kilometers per hour is given by the formula $m \approx 0.621k$. You can rewrite the conditions in kilometers per hour by substituting $0.621k$ for m in each inequality and then solving for k .

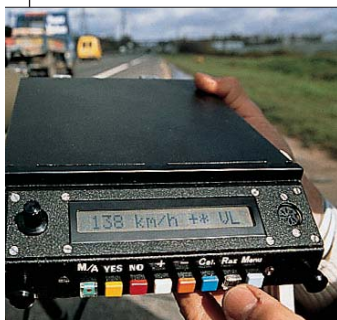
$$m < 45 \quad \text{or} \quad m > 65$$

$$0.621k < 45 \quad \text{or} \quad 0.621k > 65$$

$$k < 72.5 \quad \text{or} \quad k > 105$$

- ▶ You need to detect vehicles whose speeds are less than 72.5 kilometers per hour or greater than 105 kilometers per hour.

FOCUS ON APPLICATIONS



REAL LIFE POLICE RADAR

Police radar guns emit a continuous radio wave of known frequency. The radar gun compares the frequency of the wave reflected from a vehicle to the frequency of the transmitted wave and then displays the vehicle's speed.

GUIDED PRACTICE

Vocabulary Check ✓

1. Explain the difference between a simple linear inequality and a compound linear inequality.

Concept Check ✓

2. Tell whether this statement is *true* or *false*: Multiplying both sides of an inequality by the same number always produces an equivalent inequality. Explain.

3. Explain the difference between solving $2x < 7$ and solving $-2x < 7$.

Skill Check ✓

Solve the inequality. Then graph your solution.

4. $x - 5 < 8$

5. $3x \geq 15$

6. $-x + 4 > 3$

7. $\frac{1}{2}x \leq 6$


8. $x + 8 > -2$

9. $-x - 3 < -5$

Graph the inequality.

10. $-2 \leq x < 5$

11. $x \geq 3$ or $x < -3$

12.  **WINTER DRIVING** You are moving to Montana and need to lower the freezing point of the cooling system in the car from Example 6 to -50°C . This will also raise the boiling point to 140°C . Write a compound inequality that models this situation. Then write the inequality in degrees Fahrenheit.

PRACTICE AND APPLICATIONS

STUDENT HELP

▶ **Extra Practice**
to help you master
skills is on p. 941.

MATCHING INEQUALITIES Match the inequality with its graph.

13. $x \geq 4$

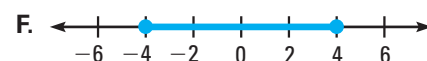
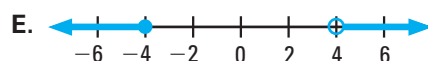
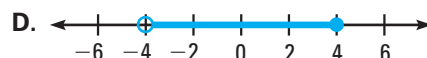
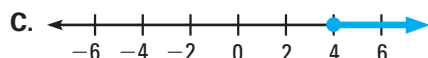
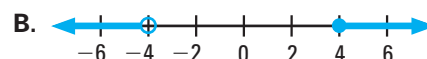
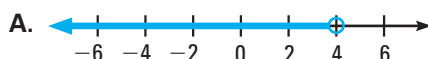
14. $x < 4$

15. $-4 < x \leq 4$

16. $x \geq 4$ or $x < -4$

17. $-4 \leq x \leq 4$

18. $x > 4$ or $x \leq -4$



CHECKING SOLUTIONS Decide whether the given number is a solution of the inequality.

19. $2x + 9 < 16$; 4

20. $10 - x \geq 3$; 7

21. $7x - 12 < 8$; 3

22. $-\frac{1}{3}x - 2 \leq -4$; 9

23. $-3 < 2x \leq 6$; 3

24. $-8 < x - 11 < -6$; 5

STUDENT HELP

▶ **HOMEWORK HELP**
Examples 1, 2: Exs. 13,
14, 19–22, 25–36
Example 3: Exs. 49–51
Examples 4, 5: Exs. 15–18,
23, 24, 37–48
Example 6: Exs. 52–54
Example 7: Exs. 55, 56

SIMPLE INEQUALITIES Solve the inequality. Then graph your solution.

25. $4x + 5 > 25$

26. $7 - n \leq 19$

27. $5 - 2x \geq 27$

28. $\frac{1}{2}x - 4 > -6$

29. $\frac{3}{2}x - 7 < 2$

30. $5 + \frac{1}{3}n \leq 6$

31. $4x - 1 > 14 - x$

32. $-n + 6 < 7n + 4$

33. $4.7 - 2.1x > -7.9$




34. $2(n - 4) \leq 6$

35. $2(4 - x) > 8$


36. $5 - 5x > 4(3 - x)$

COMPOUND INEQUALITIES Solve the inequality. Then graph your solution.

37. $-2 \leq x - 7 \leq 11$ 38. $-16 \leq 3x - 4 \leq 2$ 39. $-5 \leq -n - 6 \leq 0$
40. $-2 < -2n + 1 \leq 7$ 41. $-7 < 6x - 1 < 5$ 42. $-8 < \frac{2}{3}x - 4 < 10$
43. $x + 2 \leq 5$ or $x - 4 \geq 2$ 44. $3x + 2 < -10$ or $2x - 4 > -4$
45. $-5x - 4 < -1.4$ or $-2x + 1 > 11$ 46. $x - 1 \leq 5$ or $x + 3 \geq 10$
47. $-0.1 \leq 3.4x - 1.8 < 6.7$ 48. $0.4x + 0.6 < 2.2$ or $0.6x > 3.6$

49.  **COMMISSION** Your salary is \$1250 per week and you receive a 5% commission on your sales each week. What are the possible amounts (in dollars) that you can sell each week to earn at least \$1500 per week?
50.  **PARK FEES** You have \$50 and are going to an amusement park. You spend \$25 for the entrance fee and \$15 for food. You want to play a game that costs \$.75. Write and solve an inequality to find the possible numbers of times you can play the game. If you play the game the maximum number of times, will you have spent the entire \$50? Explain.
51.  **GRADES** A professor announces that course grades will be computed by taking 40% of a student's project score (0–100 points) and adding 60% of the student's final exam score (0–100 points). If a student gets an 86 on the project, what scores can she get on the final exam to get a course grade of at least 90?

SCIENCE CONNECTION In Exercises 52–54, use the following information.


The international standard for scientific temperature measurement is the Kelvin scale. A Kelvin temperature can be obtained by adding 273.15 to a Celsius temperature. The daytime temperature on Mars ranges from -89.15°C to -31.15°C .  Source: NASA

52. Write the daytime temperature range on Mars as a compound inequality in degrees Celsius.
53. Rewrite the compound inequality in degrees Kelvin.
54. **RESEARCH** Find the high and low temperatures in your area for any particular day. Write three compound inequalities representing the temperature range in degrees Fahrenheit, in degrees Celsius, and in degrees Kelvin.

 **WINTER** In Exercises 55 and 56, use the following information.


The Ontario Winter Severity Index (OWSI) is a weekly calculation used to determine the severity of winter conditions. The OWSI for deer is given by

$$I = \frac{p}{30} + \frac{d}{30} + c$$

where p represents the average Snow Penetration Gauge reading (in centimeters), d represents the average snow depth (in centimeters), and c represents the *chillometer reading*, which is a measure of the cold (in kilowatt-hours) based on temperature and wind chill. An extremely mild winter occurs when $I < 5$ on average, and an extremely severe winter occurs when $I > 6.5$ on average. A deer can tolerate a maximum snow penetration of 50 centimeters. Assume the average snow depth is 60 centimeters.  Source: Snow Network for Ontario Wildlife

55. What weekly chillometer readings will produce extremely severe winter readings?
56. What weekly chillometer readings will produce extremely mild winter readings?

FOCUS ON APPLICATIONS

 **MARS** is the fourth planet from the sun. A Martian year is 687 Earth days long, but a Martian day is only 40 minutes longer than an Earth day. Mars is also much colder than Earth, as discussed in Exs. 52–54.

 **APPLICATION LINK**
www.mcdougallittell.com

Test Preparation

57. **Writing** The first transformation listed in the box on page 41 can be written symbolically as follows: If a , b , and c are real numbers and $a > b$, then $a + c > b + c$. Write similar statements for the other transformations.

58. **MULTI-STEP PROBLEM** You are vacationing at Lake Tahoe, California. You decide to spend a day sightseeing in other places. You want to go from Lake Tahoe to Sacramento, from Sacramento to Sonora, and then from Sonora back to Lake Tahoe. You know that it is about 85 miles from Lake Tahoe to Sacramento and about 75 miles from Sacramento to Sonora.



- The triangle inequality theorem states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side. Write a compound inequality that represents the distance from Sonora to Lake Tahoe.
- CRITICAL THINKING** You are reading a brochure which states that the distance between Sonora and Lake Tahoe is 170 miles. You know that the distance is a misprint. How can you be so sure? Explain.
- You keep a journal of the distances you have traveled. Many of your distances represent triangular circuits. Your friend is reading your journal and states that you must have recorded a wrong distance for one of these circuits. To which one of the following is your friend referring? Explain.

A. 35 miles, 65 miles, 45 miles	B. 15 miles, 50 miles, 64 miles
C. 49 miles, 78 miles, 28 miles	D. 55 miles, 72 miles, 41 miles

★ Challenge

- Write an inequality that has no solution. Show why it has no solution.
- Write an inequality whose solutions are all real numbers. Show why the solutions are all real numbers.

MIXED REVIEW

IDENTIFYING PROPERTIES Identify the property shown. (Review 1.1)

- $(7 \cdot 3) \cdot 11 = 7 \cdot (3 \cdot 11)$
- $34 + (-34) = 0$
- $37 + 29 = 29 + 37$
- $3(9 + 4) = 3(9) + 3(4)$

SOLVING EQUATIONS Solve the equation. Check your solution. (Review 1.3 for 1.7)

- $5x + 4 = -2(x + 3)$
- $2(3 - x) = 16(x + 1)$
- $-(x - 1) + 10 = -3(x - 3)$
- $\frac{1}{8}x + \frac{3}{2} = \frac{3}{4}x - 1$

- CONCERT TRIP** You are going to a concert in another town 48 miles away. You can average 40 miles per hour on the road you plan to take to the concert. What is the minimum number of hours before the concert starts that you should leave to get to the concert on time? (Review 1.5)