

2.1

Functions and Their Graphs

What you should learn

GOAL 1 Represent relations and functions.

GOAL 2 Graph and evaluate linear functions, as applied in Exs. 55 and 56.

Why you should learn it

▼ To model **real-life** quantities, such as the distance a hot air balloon travels in **Example 6**.



GOAL 1 REPRESENTING RELATIONS AND FUNCTIONS

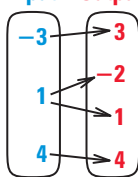
A **relation** is a *mapping*, or pairing, of input values with output values. The set of input values is the **domain**, and the set of output values is the **range**. A relation is a **function** provided there is exactly one output for each input. It is not a function if at least one input has more than one output.

Relations (and functions) between two quantities can be represented in many ways, including mapping diagrams, tables, graphs, equations, and verbal descriptions.

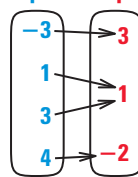
EXAMPLE 1 Identifying Functions

Identify the domain and range. Then tell whether the relation is a function.

a. Input Output



b. Input Output



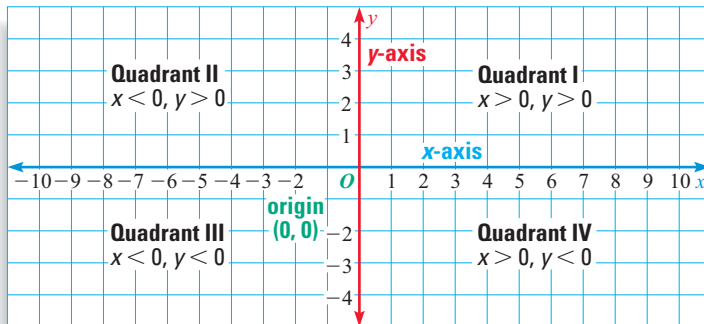
SOLUTION

- a. The domain consists of -3 , 1 , and 4 , and the range consists of -2 , 1 , 3 , and 4 . The relation is not a function because the input 1 is mapped onto both -2 and 1 .
- b. The domain consists of -3 , 1 , 3 , and 4 , and the range consists of -2 , 1 , and 3 . The relation is a function because each input in the domain is mapped onto exactly one output in the range.

A relation can be represented by a set of **ordered pairs** of the form (x, y) . In an ordered pair the first number is the **x-coordinate** and the second number is the **y-coordinate**. To graph a relation, plot each of its ordered pairs in a **coordinate plane**, such as the one shown. A coordinate plane is divided into four **quadrants** by the **x-axis** and the **y-axis**. The axes intersect at a point called the **origin**.

STUDENT HELP

Study Tip
Although the origin O is not usually labeled, it is understood to be the point $(0, 0)$.



EXAMPLE 2 *Graphing Relations*

Graph the relations given in Example 1.

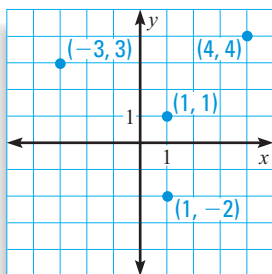
STUDENT HELP

Skills Review

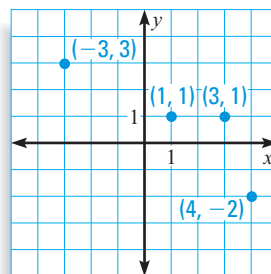
For help with plotting points in a coordinate plane, see p. 933.

SOLUTION

- a. Write the relation as a set of ordered pairs: $(-3, 3)$, $(1, -2)$, $(1, 1)$, $(4, 4)$. Then plot the points in a coordinate plane.



- b. Write the relation as a set of ordered pairs: $(-3, 3)$, $(1, 1)$, $(3, 1)$, $(4, -2)$. Then plot the points in a coordinate plane.



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In Example 2 notice that the graph of the relation that is not a function (the graph on the left) has two points that lie on the same vertical line. You can use this property as a graphical test for functions.

VERTICAL LINE TEST FOR FUNCTIONS

A relation is a function if and only if no vertical line intersects the graph of the relation at more than one point.

FOCUS ON CAREERS



FORESTER

A forester manages, develops, and protects natural resources. To measure the diameter of trees, a forester uses a special tool called diameter tape.



CAREER LINK

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Variables other than x and y are often used when working with relations in real-life situations, as shown in the next example.

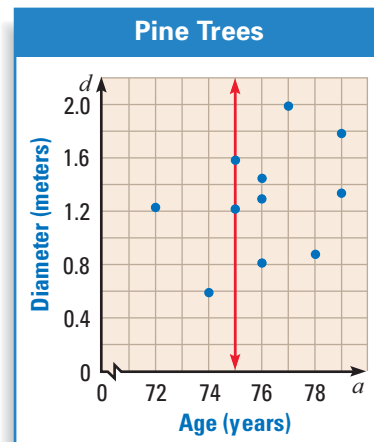
EXAMPLE 3 *Using the Vertical Line Test in Real Life*

FORESTRY The graph shows the ages a and diameters d of several pine trees at Lundbreck Falls in Canada. Are the diameters of the trees a function of their ages? Explain.

► Source: National Geographical Data Center

SOLUTION

The diameters of the trees are not a function of their ages because there is at least one vertical line that intersects the graph at more than one point. For example, a vertical line intersects the graph at the points $(75, 1.22)$ and $(75, 1.58)$. So, at least two trees have the same age but different diameters.



GOAL 2 GRAPHING AND EVALUATING FUNCTIONS

Many functions can be represented by an **equation** in two variables, such as $y = 2x - 7$. An ordered pair (x, y) is a **solution** of such an equation if the equation is true when the values of x and y are substituted into the equation. For instance, $(2, -3)$ is a solution of $y = 2x - 7$ because $-3 = 2(2) - 7$ is a true statement.

In an equation, the input variable is called the **independent variable**. The output variable is called the **dependent variable** and depends on the value of the input variable. For the equation $y = 2x - 7$, the independent variable is x and the dependent variable is y .

The **graph** of an equation in two variables is the collection of all points (x, y) whose coordinates are solutions of the equation.

GRAPHING EQUATIONS IN TWO VARIABLES

To graph an equation in two variables, follow these steps:

- STEP 1** Construct a table of values.
- STEP 2** Graph enough solutions to recognize a pattern.
- STEP 3** Connect the points with a line or a curve.

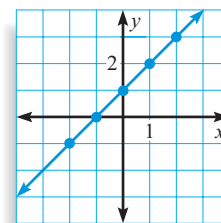
EXAMPLE 4 Graphing a Function

Graph the function $y = x + 1$.

SOLUTION

- 1** Begin by constructing a table of values.

Choose x .	-2	-1	0	1	2
Evaluate y .	-1	0	1	2	3



- 2** Plot the points. Notice the five points lie on a line.
3 Draw a line through the points.

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The function in Example 4 is a **linear function** because it is of the form

$$y = mx + b \quad \text{Linear function}$$

where m and b are constants. The graph of a linear function is a line. By naming a function “ f ” you can write the function using **function notation**.

$$f(x) = mx + b \quad \text{Function notation}$$

The symbol $f(x)$ is read as “the value of f at x ,” or simply as “ f of x .” Note that $f(x)$ is another name for y . The domain of a function consists of the values of x for which the function is defined. The range consists of the values of $f(x)$ where x is in the domain of f . Functions do not have to be represented by the letter f . Other letters such as g or h can also be used.

STUDENT HELP

Study Tip

When you see function notation $f(x)$, remember that it means “the value of f at x .” It does not mean “ f times x .”

EXAMPLE 5 *Evaluating Functions*

Decide whether the function is linear. Then evaluate the function when $x = -2$.

a. $f(x) = -x^2 - 3x + 5$

b. $g(x) = 2x + 6$

SOLUTION

a. $f(x)$ is not a linear function because it has an x^2 -term.

$$f(x) = -x^2 - 3x + 5$$

Write function.

$$\begin{aligned} f(-2) &= -(-2)^2 - 3(-2) + 5 \\ &= 7 \end{aligned}$$

Substitute -2 for x .

Simplify.

b. $g(x)$ is a linear function because it has the form $g(x) = mx + b$.

$$g(x) = 2x + 6$$

Write function.

$$\begin{aligned} g(-2) &= 2(-2) + 6 \\ &= 2 \end{aligned}$$

Substitute -2 for x .

Simplify.

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In Example 5 the domain of each function is all real numbers. In real-life problems the domain is restricted to the numbers that make sense in the real-life context.

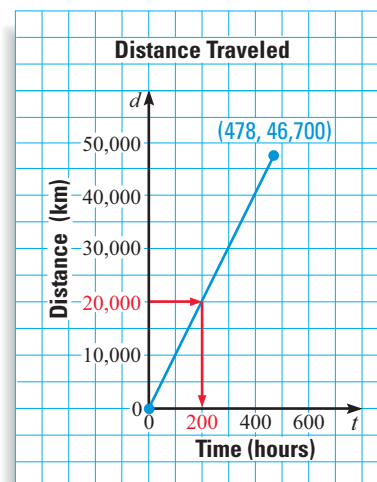
EXAMPLE 6 *Using a Function in Real Life*

BALLOONING In March of 1999, Bertrand Piccard and Brian Jones attempted to become the first people to fly around the world in a balloon. Based on an average speed of 97.8 kilometers per hour, the distance d (in kilometers) that they traveled can be modeled by $d = 97.8t$ where t is the time (in hours). They traveled a total of about 478 hours. The rules governing the record state that the minimum distance covered must be at least 26,700 kilometers. ▶ Source: Breitling

- Identify the domain and range and determine whether Piccard and Jones set the record.
- Graph the function. Then use the graph to approximate how long it took them to travel 20,000 kilometers.

SOLUTION

- Because their trip lasted 478 hours, the domain is $0 \leq t \leq 478$. The distance they traveled was $d = 97.8(478) \approx 46,700$ kilometers, so the range is $0 \leq d \leq 46,700$. Since $46,700 > 26,700$, they did set the record.
- The graph of the function is shown. Note that the graph ends at $(478, 46,700)$. To find how long it took them to travel 20,000 kilometers, start at 20,000 on the d -axis and move right until you reach the graph. Then move down to the t -axis. It took them about 200 hours to travel 20,000 kilometers.

**STUDENT HELP****HOMEWORK HELP**

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for extra examples.

FOCUS ON PEOPLE

REAL LIFE **PICCARD AND JONES** are the first pilots to fly around the world in a balloon. Piccard is a medical doctor in Switzerland specializing in psychiatry, and Jones is a member of the Royal Air Force in the United Kingdom.

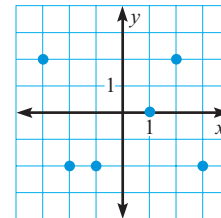
GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

Skill Check ✓

1. What are the domain and range of a relation?
2. Explain why a vertical line, rather than a horizontal line, is used to determine if a graph represents a function.
3. Explain the process for graphing an equation.
4. Identify the domain and range of the relation shown. Then tell whether the relation is a function.



Ex. 4

Graph the function.

5. $y = x - 1$

6. $y = 4x$

7. $y = 2x + 5$

8. $y = x$

9. $y = -2x$

10. $y = -x + 9$

Evaluate the function when $x = 3$.

11. $f(x) = x$

12. $f(x) = 6x$

13. $f(x) = x^2$

14. $g(x) = 2x + 7$

15. $h(x) = -x^2 + 10$

16. $j(x) = x^3 - 7x$



HIGHWAY DRIVING In Exercises 17 and 18, use the following information.

A car has a 16 gallon gas tank. On a long highway trip, gas is used at a rate of about 2 gallons per hour. The gallons of gas g in the car's tank can be modeled by the equation $g = 16 - 2t$ where t is the time (in hours).

17. Identify the domain and range of the function. Then graph the function.
18. At the end of the trip there are 2 gallons of gas left. How long was the trip?

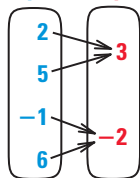
PRACTICE AND APPLICATIONS

STUDENT HELP

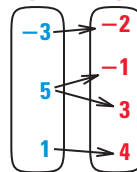
▶ **Extra Practice**
to help you master
skills is on p. 941.

DOMAIN AND RANGE Identify the domain and range.

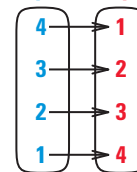
19. **Input** **Output**



20. **Input** **Output**



21. **Input** **Output**



GRAPHS Graph the relation. Then tell whether the relation is a function.

22.	x	0	0	2	2	4	4
	y	-4	4	-3	3	-1	1

23.	x	-5	-4	-3	0	3	4	5
	y	-6	-4	-2	-1	-2	-4	-6

24.	x	-2	-2	0	2	2
	y	1.5	-3.5	0	1.5	-3.5

STUDENT HELP

▶ HOMEWORK HELP

Example 1: Exs. 19–27

Example 2: Exs. 22–27

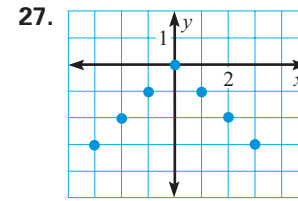
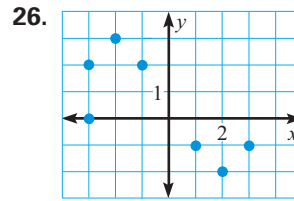
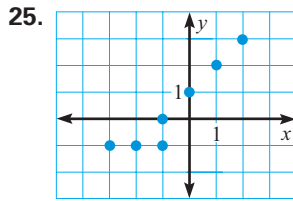
Example 3: Exs. 30–32,
51–54

Example 4: Exs. 34–42

Example 5: Exs. 43–50

Example 6: Exs. 55–58

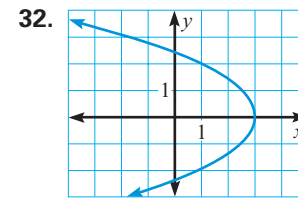
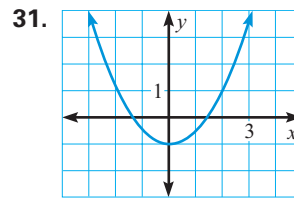
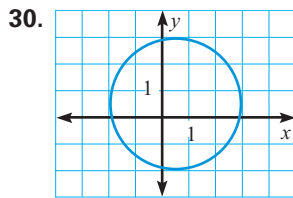
MAPPING DIAGRAMS Use a mapping diagram to represent the relation. Then tell whether the relation is a function.



28. **Writing** Is a function always a relation? Is a relation always a function? Explain your reasoning.

29. **LOGICAL REASONING** Rewrite the vertical line test as two if-then statements.

VERTICAL LINE TEST Use the vertical line test to determine whether the relation is a function.



33. **CRITICAL THINKING** Why does $y = 3$ represent a function, but $x = 3$ does not?

GRAPHING FUNCTIONS Graph the function.

34. $y = x - 3$

35. $y = -x + 6$

36. $y = 2x + 7$

37. $y = -5x + 1$

38. $y = 3x - 4$

39. $y = -2x - 3$

40. $y = 10x$

41. $y = 5$

42. $y = -\frac{2}{3}x + 4$

EVALUATING FUNCTIONS Decide whether the function is linear. Then evaluate the function for the given value of x .

43. $f(x) = x - 11$; $f(4)$

44. $f(x) = 2$; $f(-4)$

45. $f(x) = |x| - 5$; $f(-6)$

46. $f(x) = 9x^3 - x^2 + 2$; $f(2)$

47. $f(x) = -\frac{2}{3}x^2 - x + 5$; $f(6)$

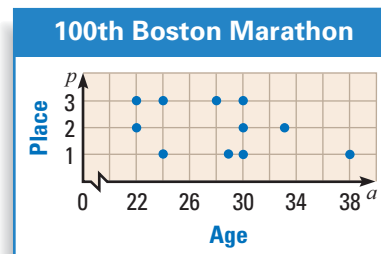
48. $f(x) = -3 + 4x$; $f(-\frac{1}{2})$

49. **GEOMETRY CONNECTION** The volume of a cube with side length s is given by the function $V(s) = s^3$. Find $V(5)$. Explain what $V(5)$ represents.

50. **GEOMETRY CONNECTION** The volume of a sphere with radius r is given by the function $V(r) = \frac{4}{3}\pi r^3$. Find $V(2)$. Explain what $V(2)$ represents.

51. **BOSTON MARATHON** The graph shows the ages and finishing places of the top three competitors in each of the four categories of the 100th Boston Marathon. Is the finishing place of a competitor a function of his or her age? Explain your reasoning.

► Source: Boston Athletic Association



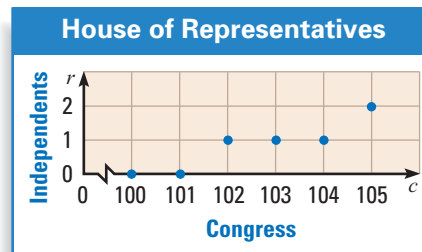
STUDENT HELP

Skills Review
For help with if-then statements, see p. 926.

52. HOUSE OF REPRESENTATIVES

The graph shows the number of Independent representatives for the 100th–105th Congresses. Is the number of Independent representatives a function of the Congress number? Explain your reasoning.

► Source: The Office of the Clerk, United States House of Representatives




STATISTICS CONNECTION In Exercises 53 and 54, use the table which shows the number of shots attempted and the number of shots made by 9 members of the Utah Jazz basketball team in Game 1 of the 1998 NBA Finals. ► Source: NBA

Player	Shots attempted, x	Shots made, y
Bryon Russell	12	6
Karl Malone	25	9
Greg Foster	5	1
Jeff Hornacek	10	2
John Stockton	12	9
Howard Eisley	6	4
Chris Morris	6	3
Greg Ostertag	1	1
Shandon Anderson	5	3

53. Identify the domain and range of the relation. Then graph the relation.

54. Is the relation a function? Explain.

 **WATER PRESSURE** In Exercises 55 and 56, use the information below and in the caption to the photo.

Water pressure can be measured in atmospheres, where 1 atmosphere equals 14.7 pounds per square inch. At sea level the water pressure is 1 atmosphere, and it increases by 1 atmosphere for every 33 feet in depth. Therefore, the water pressure p can be modeled as a function of the depth d by this equation:

$$p = \frac{1}{33}d + 1, \quad 0 \leq d \leq 130$$

55. Identify the domain and range of the function. Then graph the function.

56. What is the water pressure at a depth of 100 feet?

 **CAP SIZES** In Exercises 57 and 58, use the following information.

Your cap size is based on your head circumference (in inches). For head circumferences from $20\frac{7}{8}$ inches to 25 inches, cap size s can be modeled as a function of head circumference c by this equation:

$$s = \frac{c - 1}{3}$$

57. Identify the domain and range of the function. Then graph the function.

58. If you wear a size 7 cap, what is your head circumference?

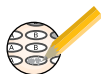
FOCUS ON APPLICATIONS



REAL LIFE WATER PRESSURE

Scuba divers must equalize the pressure on the inside of their bodies with the water pressure on the outside of their bodies. The maximum safe depth for recreational divers is 130 feet.

Test Preparation



QUANTITATIVE COMPARISON In Exercises 59–62, choose the statement that is true about the given quantities.

- (A) The quantity in column A is greater.
- (B) The quantity in column B is greater.
- (C) The two quantities are equal.
- (D) The relationship cannot be determined from the given information.

	Column A	Column B
59.	$f(x) = 3x + 10$ when $x = 0$	$f(x) = 2x - 4$ when $x = 7$
60.	$f(x) = x^2 - 4x - 11$ when $x = 6$	$f(x) = x^2 - 3x + 5$ when $x = 4$
61.	$f(x) = x^3 - 7x + 1$ when $x = -3$	$f(x) = -x^3 - 4$ when $x = 2$
62.	$f(x) = 2x + 8$ when $x = \frac{3}{2}$	$f(x) = -8x + 9$ when $x = -\frac{1}{4}$

★ Challenge

EXTRA CHALLENGE

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63. **TELEPHONE KEYPADS** For the numbers 2 through 9 on a telephone keypad, draw two mapping diagrams: one mapping numbers onto letters, and the other mapping letters onto numbers. Are both relations functions? Explain.



MIXED REVIEW

EVALUATING EXPRESSIONS Evaluate the expression for the given values of x and y . (Review 1.2 for 2.2)

64. $\frac{y-6}{x-9}$ when $x = -3$ and $y = -2$
65. $\frac{y-11}{x-2}$ when $x = -4$ and $y = 5$
66. $\frac{y-(-5)}{x-3}$ when $x = 2$ and $y = 5$
67. $\frac{y-(-1)}{x-(-4)}$ when $x = 6$ and $y = 4$
68. $\frac{4-y}{1-x}$ when $x = 2$ and $y = 3$
69. $\frac{10-y}{14-x}$ when $x = 6$ and $y = 8$

SOLVING EQUATIONS Solve the equation. Check your solution. (Review 1.3)

70. $2x + 13 = 31$
71. $-2.4x + 11.8 = 29.8$
72. $x + 17 = 10 - 3x$
73. $\frac{5}{2} - 7x = 40 + x$
74. $-\frac{1}{3}(x - 15) = -48$
75. $6x + 5 = 0.5(x + 6) - 4$

CHECKING SOLUTIONS Decide whether the given number is a solution of the inequality. (Review 1.6)

76. $3x - 4 < 10$; 5
77. $\frac{1}{2}x - 8 \leq 0$; 16
78. $10 - x \geq 6$; 2
79. $3 + 2x > -5$; -2
80. $-5 \leq x + 8 < 15$; $\frac{3}{2}$
81. $x - 2.7 < -1$ or $3x > 6.9$; 2.5