2.4 Writing Equations of Lines

What *you should learn*

GOAL 1 Write linear equations.

GOAL 2 Write direct variation equations, as applied in **Example 7**.

Why *you should learn it*

▼ To model **real-life** quantities, such as the number of calories you burn while dancing in **Ex. 64**.

REAL LIFE

GOAL 1 WRITING LINEAR EQUATIONS

In Lesson 2.3 you learned to find the slope and y-intercept of a line whose equation is given. In this lesson you will study the reverse process. That is, you will learn to write an equation of a line using one of the following: the slope and y-intercept of the line, the slope and a point on the line, or two points on the line.

CONCEPT SUMMARY — **WRITING AN EQUATION OF A LINE**

SLOPE-INTERCEPT FORM Given the slope m and the y-intercept b, use this equation:

$$y = mx + b$$

POINT-SLOPE FORM Given the slope m and a point (x_1, y_1), use this equation:

$$y - y_1 = m(x - x_1)$$

TWO POINTS Given two points (x_1, y_1) and (x_2, y_2), use the formula

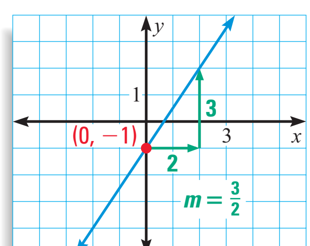
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

to find the slope m. Then use the point-slope form with this slope and either of the given points to write an equation of the line.

Every nonvertical line has only one slope and one y-intercept, so the slope-intercept form is unique. The point-slope form, however, depends on the point that is used. Therefore, in this book equations of lines will be simplified to slope-intercept form so a unique solution may be given.

EXAMPLE 1 *Writing an Equation Given the Slope and the y-intercept*

Write an equation of the line shown.



SOLUTION

From the graph you can see that the slope is $m = \frac{3}{2}$. You can also see that the line intersects the y-axis at the point $(0, -1)$, so the y-intercept is $b = -1$.

Because you know the slope and the y-intercept, you should use the slope-intercept form to write an equation of the line.

$y = mx + b$ **Use slope-intercept form.**

$y = \frac{3}{2}x - 1$ **Substitute $\frac{3}{2}$ for m and -1 for b.**

▶ An equation of the line is $y = \frac{3}{2}x - 1$.

EXAMPLE 2 Writing an Equation Given the Slope and a Point

Write an equation of the line that passes through (2, 3) and has a slope of $-\frac{1}{2}$.

SOLUTION

Because you know the slope and a point on the line, you should use the point-slope form to write an equation of the line. Let $(x_1, y_1) = (2, 3)$ and $m = -\frac{1}{2}$.

$y - y_1 = m(x - x_1)$ **Use point-slope form.**

$y - 3 = -\frac{1}{2}(x - 2)$ **Substitute for m, x_1, and y_1.**

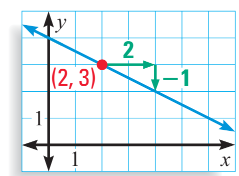
Once you have used the point-slope form to find an equation, you can simplify the result to the slope-intercept form.

$y - 3 = -\frac{1}{2}(x - 2)$ **Write point-slope form.**

$y - 3 = -\frac{1}{2}x + 1$ **Distributive property**

$y = -\frac{1}{2}x + 4$ **Write in slope-intercept form.**

✓**CHECK** You can check the result graphically. Draw the line that passes through the point (2, 3) with a slope of $-\frac{1}{2}$. Notice that the line has a y-intercept of 4, which agrees with the slope-intercept form found above.



EXAMPLE 3 Writing Equations of Perpendicular and Parallel Lines

STUDENT HELP

HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

Write an equation of the line that passes through (3, 2) and is (**a**) perpendicular and (**b**) parallel to the line $y = -3x + 2$.

SOLUTION

a. The given line has a slope of $m_1 = -3$. So, a line that is perpendicular to this line must have a slope of $m_2 = -\frac{1}{m_1} = \frac{1}{3}$. Because you know the slope and a point on the line, use the point-slope form with $(x_1, y_1) = (3, 2)$ to find an equation of the line.

$y - y_1 = m_2(x - x_1)$ **Use point-slope form.**

$y - 2 = \frac{1}{3}(x - 3)$ **Substitute for m_2, x_1, and y_1.**

$y - 2 = \frac{1}{3}x - 1$ **Distributive property**

$y = \frac{1}{3}x + 1$ **Write in slope-intercept form.**

b. For a parallel line use $m_2 = m_1 = -3$ and $(x_1, y_1) = (3, 2)$.

$y - y_1 = m_2(x - x_1)$ **Use point-slope form.**

$y - 2 = -3(x - 3)$ **Substitute for m_2, x_1, and y_1.**

$y - 2 = -3x + 9$ **Distributive property**

$y = -3x + 11$ **Write in slope-intercept form.**

FOCUS ON PEOPLE

BARBARA JORDAN was the first African-American woman elected to Congress from a southern state. She was a member of the House of Representatives from 1973 to 1979.

EXAMPLE 4 *Writing an Equation Given Two Points*

Write an equation of the line that passes through $(-2, -1)$ and $(3, 4)$.

SOLUTION

The line passes through $(x_1, y_1) = (-2, -1)$ and $(x_2, y_2) = (3, 4)$, so its slope is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-1)}{3 - (-2)} = \frac{5}{5} = 1$$

Because you know the slope and a point on the line, use the point-slope form to find an equation of the line.

$y - y_1 = m(x - x_1)$ — Use point-slope form.

$y - (-1) = 1[x - (-2)]$ — Substitute for m, x_1, and y_1.

$y + 1 = x + 2$ — Simplify.

$y = x + 1$ — Write in slope-intercept form.

EXAMPLE 5 *Writing and Using a Linear Model*

POLITICS In 1970 there were 160 African-American women in elected public office in the United States. By 1993 the number had increased to 2332. Write a linear model for the number of African-American women who held elected public office at any given time between 1970 and 1993. Then use the model to predict the number of African-American women who will hold elected public office in 2010.

INTERNET DATA UPDATE of Joint Center for Political and Economic Studies data at www.mcdougallittell.com

SOLUTION

The average rate of change in officeholders is $m = \frac{2332 - 160}{1993 - 1970} \approx 94.4$.

You can use the average rate of change as the slope in your linear model.

PROBLEM SOLVING STRATEGY

VERBAL MODEL: Number of officeholders = Number in 1970 + Average rate of change · Years since 1970

LABELS:

Number of officeholders = y (people)

Number in 1970 = **160** (people)

Average rate of change = **94.4** (people per year)

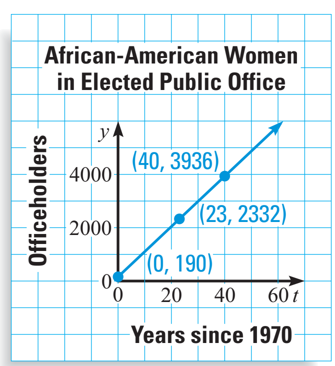
Years since 1970 = t (years)

ALGEBRAIC MODEL: $y = 160 + 94.4\,t$

In 2010, which is 40 years since 1970, you can predict that there will be

$$y = 160 + 94.4(40) \approx 3936$$

African-American women in elected public office. You can graph the model to check your prediction visually.



GOAL 2 WRITING DIRECT VARIATION EQUATIONS

Two variables x and y show **direct variation** provided $y = kx$ and $k \neq 0$. The nonzero constant k is called the **constant of variation**, and y is said to *vary directly* with x. The graph of $y = kx$ is a line through the origin.

EXAMPLE 6 *Writing and Using a Direct Variation Equation*

The variables x and y vary directly, and $y = 12$ when $x = 4$.

a. Write and graph an equation relating x and y. **b.** Find y when $x = 5$.

SOLUTION

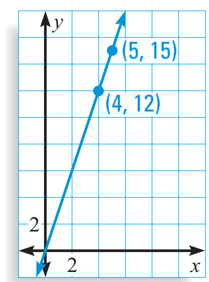
a. Use the given values of x and y to find the constant of variation.

$y = kx$ — Write direct variation equation.

$12 = k(4)$ — Substitute 12 for y and 4 for x.

$3 = k$ — Solve for k.

The direct variation equation is $y = 3x$. The graph of $y = 3x$ is shown.



b. When $x = 5$, the value of y is $y = 3(5) = 15$.

. . . .

The equation for direct variation can be rewritten as $\frac{y}{x} = k$. This tells you that a set of data pairs (x, y) shows direct variation if the quotient of y and x is constant.

REAL LIFE Jewelry

EXAMPLE 7 *Identifying Direct Variation*

Tell whether the data show direct variation. If so, write an equation relating x and y.

a.

14-karat Gold Chains (1 gram per inch)					
Length, *x* (inches)	16	18	20	24	30
Price, *y* (dollars)	288	324	360	432	540

b.

Loose Diamonds (round, colorless, very small flaws)					
Weight, *x* (carats)	0.5	0.7	1.0	1.5	2.0
Price, *y* (dollars)	2250	3430	6400	11,000	20,400

SOLUTION For each data set, check whether the quotient of y and x is constant.

a. For the 14-karat gold chains, $\frac{288}{16} = \frac{324}{18} = \frac{360}{20} = \frac{432}{24} = \frac{540}{30} = 18$. The data do show direct variation, and the direct variation equation is $y = 18x$.

b. For the loose diamonds, $\frac{2250}{0.5} = 4500$, but $\frac{3430}{0.7} = 4900$. The data do not show direct variation.

GUIDED PRACTICE

Vocabulary Check ✓

1. Define the constant of variation for two variables x and y that vary directly.

Concept Check ✓

2. How can you find an equation of a line given the slope and the y-intercept of the line? given the slope and a point on the line? given two points on the line?

3. Give a real-life example of two quantities that vary directly.

Skill Check ✓

Write an equation of the line that has the given properties.

4. slope: $\frac{2}{5}$, y-intercept: 2
5. slope: 2, passes through $(0, -4)$
6. slope: -3, passes through $(5, 2)$
7. slope: $-\frac{3}{4}$, passes through $(-7, 0)$
8. passes through $(4, 8)$ and $(1, 2)$
9. passes through $(0, 2)$ and $(-5, 0)$
10. Write an equation of the line that passes through $(1, -6)$ and is perpendicular to the line $y = 3x + 7$.
11. Write an equation of the line that passes through $(3, 9)$ and is parallel to the line $y = 5x - 15$.
12. **LAW OF SUPPLY** The *law of supply* states that the quantity supplied of an item varies directly with the price of that item. Suppose that for \$4 per tape 5 million cassette tapes will be supplied. Write an equation that relates the number c (in millions) of cassette tapes supplied to the price p (in dollars) of the tapes. Then determine how many cassette tapes will be supplied for \$5 per tape.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 942.

SLOPE-INTERCEPT FORM **Write an equation of the line that has the given slope and *y*-intercept.**

13. $m = 5, b = -3$
14. $m = -3, b = -4$
15. $m = -4, b = 0$
16. $m = 0, b = 4$
17. $m = \frac{3}{5}, b = 6$
18. $m = -\frac{3}{4}, b = \frac{7}{3}$

POINT-SLOPE FORM **Write an equation of the line that passes through the given point and has the given slope.**

19. $(0, 4), m = 2$
20. $(1, 0), m = 3$
21. $(-6, 5), m = 0$
22. $(9, 3), m = -\frac{2}{3}$
23. $(3, -2), m = -\frac{4}{3}$
24. $(7, -4), m = \frac{2}{5}$
25. Write an equation of the line that passes through $(1, -1)$ and is perpendicular to the line $y = -\frac{1}{2}x + 6$.
26. Write an equation of the line that passes through $(6, -10)$ and is perpendicular to the line that passes through $(4, -6)$ and $(3, -4)$.
27. Write an equation of the line that passes through $(2, -7)$ and is parallel to the line $x = 5$.
28. Write an equation of the line that passes through $(4, 6)$ and is parallel to the line that passes through $(6, -6)$ and $(10, -4)$.

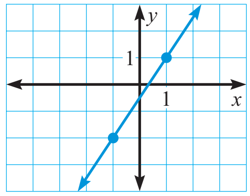
STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 13–18
Example 2: Exs. 19–24
Example 3: Exs. 25–28
Example 4: Exs. 29–40
Example 5: Exs. 59–62
Example 6: Exs. 43–54
Example 7: Exs. 55–58, 63–68

VISUAL THINKING **Write an equation of the line.**

29.



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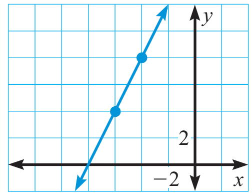

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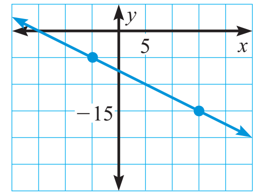

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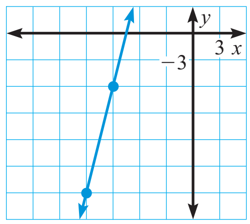

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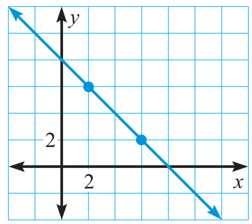

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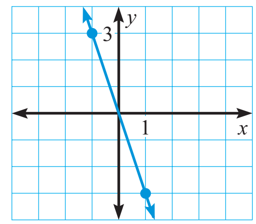


WRITING EQUATIONS **Write an equation of the line that passes through the given points.**

35. $(8, 5), (11, 14)$ **36.** $(-5, 9), (-4, 7)$ **37.** $(-8, 8), (0, 1)$

38. $(2, 0), (4, -6)$ **39.** $(-20, -10), (5, 15)$ **40.** $(-2, 0), (0, 6)$

41. LOGICAL REASONING Redo Example 2 by substituting the given point and slope into $y = mx + b$. Then solve for b to write an equation of the line. Explain why using this method does not change the equation of the line.

42. LOGICAL REASONING Redo Example 4 by substituting $(3, 4)$ for (x_1, y_1) into $y - y_1 = m(x - x_1)$. Then rewrite the equation in slope-intercept form. Explain why using the point $(3, 4)$ does not change the equation of the line.

RELATING VARIABLES **The variables *x* and *y* vary directly. Write an equation that relates the variables. Then find *y* when *x* = 8.**

43. $x = 2, y = 7$ **44.** $x = -6, y = 15$ **45.** $x = -3, y = 9$

46. $x = 24, y = 4$ **47.** $x = 1, y = \frac{1}{2}$ **48.** $x = 0.8, y = 1.6$

RELATING VARIABLES **The variables *x* and *y* vary directly. Write an equation that relates the variables. Then find *x* when *y* = −5.**

49. $x = 6, y = 3$ **50.** $x = 9, y = 15$ **51.** $x = -5, y = -1$

52. $x = 100, y = 2$ **53.** $x = \frac{5}{2}, y = \frac{5}{4}$ **54.** $x = -0.3, y = 2.2$

IDENTIFYING DIRECT VARIATION **Tell whether the data show direct variation. If so, write an equation relating *x* and *y*.**

55.

x	2	4	6	8	10
y	1	2	3	4	5

56.

x	1	2	3	4	5
y	5	4	3	2	1

57.

x	3	6	9	12	15
y	−3	−6	−9	−12	−15

58.

x	−5	−4	−3	−2	−1
y	10	8	6	4	2

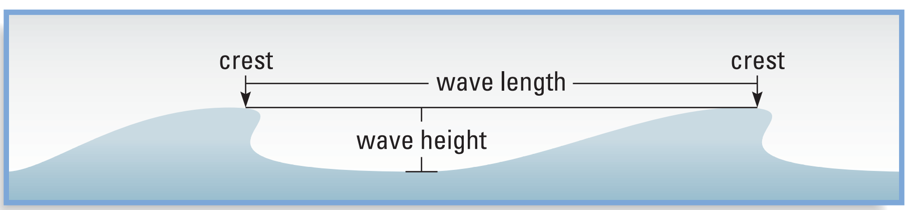
59. **POPULATION OF OREGON** From 1990 to 1996 the population of Oregon increased by about 60,300 people per year. In 1996 the population was about 3,204,000. Write a linear model for the population P of Oregon from 1990 to 1996. Let t represent the number of years since 1990. Then estimate the population of Oregon in 2014. ▶ Source: *Statistical Abstract of the United States*

60. **AIRFARE** In 1998 an airline offered a special airfare of $201 to fly from Cincinnati to Washington, D.C., a distance of 386 miles. Special airfares offered for longer flights increased by about $.138 per mile. Write a linear model for the special airfares a based on the total number of miles t of the flight. Estimate the airfare offered for a flight from Boston to Sacramento, a distance of 2629 miles.

61. **BOOKSTORE SALES** In 1990 retail sales at bookstores were about $7.4 billion. In 1997 retail sales at bookstores were about $11.8 billion. Write a linear model for retail sales s (in billions of dollars) at bookstores from 1990 through 1997. Let t represent the number of years since 1990. Then estimate the retail sales at bookstores in 2012. ▶ Source: American Booksellers Association

62. **SCIENCE CONNECTION** The velocity of sound in dry air increases as the temperature increases. At 40°C sound travels at a rate of about 355 meters per second. At 49°C it travels at a rate of about 360 meters per second. Write a linear model for the velocity v (in meters per second) of sound based on the temperature T (in degrees Celsius). Then estimate the velocity of sound at 60°C.
▶ Source: *CRC Handbook of Chemistry and Physics*

63. **BREAKING WAVES** The height h (in feet) at which a wave breaks varies directly with the wave length l (in feet), which is the distance from the crest of one wave to the crest of the next. A wave that breaks at a height of 4 feet has a wave length of 28 feet. Write a linear model that gives h as a function of l. Then estimate the wave length of a wave that breaks at a height of 5.5 feet.
▶ Source: Rhode Island Sea Grant



FOCUS ON APPLICATIONS

HAILSTONES The largest hailstone ever recorded fell at Coffeyville, Kansas. It weighed 1.67 pounds and had a radius of about 2.75 inches.

64. **DANCING** The number C of calories a person burns performing an activity varies directly with the time t (in minutes) the person spends performing the activity. A 160 pound person can burn 73 Calories by dancing for 20 minutes. Write a linear model that gives C as a function of t. Then estimate how long a 160 pound person should dance to burn 438 Calories. ▶ Source: *Health Journal*

65. **HAILSTONES** Hailstones are formed when frozen raindrops are caught in updrafts and carried into high clouds containing water droplets. As a rule of thumb, the radius r (in inches) of a hailstone varies directly with the time t (in seconds) that the hailstone is in a high cloud. After a hailstone has been in a high cloud for 60 seconds, its radius is 0.25 inch. Write a linear model that gives r as a function of t. Then estimate how long a hailstone was in a high cloud if its radius measures 2.75 inches. ▶ Source: National Oceanic and Atmospheric Administration

66. **GEOMETRY CONNECTION** When the length of a rectangle is fixed, the area A (in square inches) of the rectangle varies directly with its width w (in inches). When the width of a particular rectangle is 12 inches, its area is 36 square inches. Write an equation that gives A as a function of w. Then find A when w is 7.5 inches.

STATISTICS CONNECTION **Tell whether the data show direct variation. If so, write an equation relating *x* and *y*.**

67.

Applesauce					
Ounces, *x*	8	16	24	36	48
Price, *y*	\$.89	\$1.25	\$1.39	\$2.09	\$2.49

68.

Fresh Apples					
Pounds, *x*	1	1.5	2	2.5	3
Price, *y*	\$.89	\$1.34	\$1.78	\$2.23	\$2.49

Test Preparation

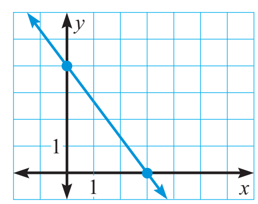
69. MULTI-STEP PROBLEM Besides slope-intercept and point-slope forms, another form that can be used to write equations of lines is *intercept form*: $\frac{x}{a} + \frac{y}{b} = 1$

a. Graph $\frac{x}{5} + \frac{y}{3} = 1$.

b. Graph $\frac{x}{-2} + \frac{y}{9} = 1$.

c. *Writing* Geometrically, what do a and b represent in the intercept form of a linear equation?

d. Write an equation of the line shown using intercept form.



e. Write an equation of the line with x-intercept -5 and y-intercept -8 using intercept form.

f. Write an equation of the line that passes through $(0, -3)$ and $(2, 0)$ using intercept form.

★ **Challenge**

70. SLOPE-INTERCEPT FORM Derive the slope-intercept form of a linear equation from the slope formula using $(0, b)$ as the coordinates of the point where the line crosses the y-axis and an arbitrary point (x, y).

MIXED REVIEW

SOLVING EQUATIONS **Solve the equation.** (Review 1.7)

71. $|x - 10| = 17$ **72.** $|7 - 2x| = 5$ **73.** $|-x - 9| = 1$

74. $|4x + 1| = 0.5$ **75.** $|22x + 6| = 9.2$ **76.** $|5.2x + 7| = 3.8$

FINDING SLOPE **Find the slope of the line passing through the given points.** (Review 2.2 for 2.5)

77. $(1, -7), (2, 7)$ **78.** $(-1, -1), (-5, -4)$ **79.** $(2, 4), (5, 10)$

80. $(5, -2), (-3, -1)$ **81.** $(-2, 4), (2, 4)$ **82.** $(-4, -1), (5, -4)$

83. $(0, -8), (-9, 10)$ **84.** $(6, 11), (6, -5)$ **85.** $(-11, 4), (-4, 11)$

GRAPHING EQUATIONS **Graph the equation.** (Review 2.3 for 2.5)

86. $y = \frac{3}{4}x - 5$ **87.** $y = -\frac{1}{5}x + 2$ **88.** $y = -\frac{3}{7}x + 2$

89. $3x + 7y = 42$ **90.** $2x - 8y = -15$ **91.** $-5x + 3y = 10$

92. $x = 0$ **93.** $y = -3$ **94.** $y = x$