2.6 Linear Inequalities in Two Variables

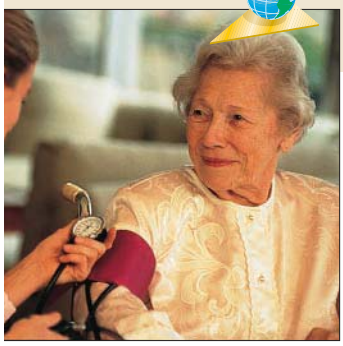
What you should learn

GOAL 1 Graph linear inequalities in two variables.

GOAL 2 Use linear inequalities to solve **real-life** problems, such as finding the number of minutes you can call relatives using a calling card in **Example 4**.

Why you should learn it

▼ To model **real-life** data, such as blood pressures in your arm and ankle in **Ex. 45**.



GOAL 1 GRAPHING LINEAR INEQUALITIES

A **linear inequality** in two variables is an inequality that can be written in one of the following forms:

$$Ax + By < C, \quad Ax + By \leq C, \quad Ax + By > C, \quad Ax + By \geq C$$

An ordered pair (x, y) is a **solution** of a linear inequality if the inequality is true when the values of x and y are substituted into the inequality. For instance, $(-6, 2)$ is a solution of $y \geq 3x - 9$ because $2 \geq 3(-6) - 9$ is a true statement.

EXAMPLE 1 Checking Solutions of Inequalities

Check whether the given ordered pair is a solution of $2x + 3y \geq 5$.

a. $(0, 1)$ **b.** $(4, -1)$ **c.** $(2, 1)$

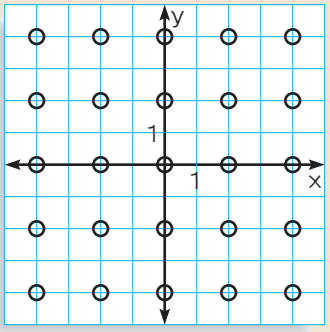
SOLUTION

ORDERED PAIR	SUBSTITUTE	CONCLUSION
a. $(0, 1)$	$2(0) + 3(1) = 3 \not\geq 5$	$(0, 1)$ is not a solution.
b. $(4, -1)$	$2(4) + 3(-1) = 5 \geq 5$	$(4, -1)$ is a solution.
c. $(2, 1)$	$2(2) + 3(1) = 7 \geq 5$	$(2, 1)$ is a solution.

ACTIVITY
Developing Concepts

Investigating the Graph of an Inequality

1. Copy the scatter plot.
2. Test each circled point to see whether it is a solution of $x + y \geq 1$. If it is a solution, color it blue. If it is not a solution, color it red.
3. Graph the line $x + y = 1$. What relationship do you see between the colored points and the line?
4. Describe a general strategy for graphing an inequality in two variables.



The **graph** of a linear inequality in two variables is the graph of all solutions of the inequality. The boundary line of the inequality divides the coordinate plane into two **half-planes**: a shaded region which contains the points that are solutions of the inequality, and an unshaded region which contains the points that are not.

GRAPHING A LINEAR INEQUALITY

The graph of a linear inequality in two variables is a half-plane. To graph a linear inequality, follow these steps:

STEP 1 Graph the boundary line of the inequality. Use a dashed line for $<$ or $>$ and a solid line for $\leq$ or $\geq$.

STEP 2 To decide which side of the boundary line to shade, test a point *not* on the boundary line to see whether it is a solution of the inequality. Then shade the appropriate half-plane.

EXAMPLE 2 Graphing Linear Inequalities in One Variable

STUDENT HELP

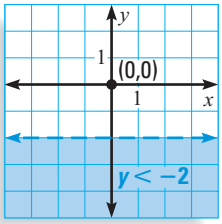
Look Back
For help with inequalities in one variable, see p. 42.

Graph (**a**) $y < -2$ and (**b**) $x \leq 1$ in a coordinate plane.

SOLUTION

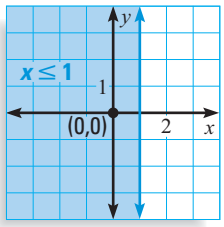
a. ***Graph*** the boundary line $y = -2$. Use a dashed line because $y < -2$.

Test the point (0, 0). Because (0, 0) is *not* a solution of the inequality, shade the half-plane below the line.



b. ***Graph*** the boundary line $x = 1$. Use a solid line because $x \leq 1$.

Test the point (0, 0). Because (0, 0) *is* a solution of the inequality, shade the half-plane to the left of the line.



EXAMPLE 3 Graphing Linear Inequalities in Two Variables

STUDENT HELP

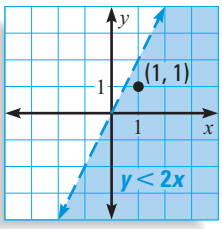
Study Tip
Because your test point must *not* be on the boundary line, you may not always be able to use (0, 0) as a convenient test point. In such cases test a different point, such as (1, 1) or (1, 0).

Graph (**a**) $y < 2x$ and (**b**) $2x - 5y \geq 10$.

SOLUTION

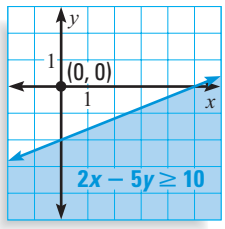
a. ***Graph*** the boundary line $y = 2x$. Use a dashed line because $y < 2x$.

Test the point (1, 1). Because (1, 1) *is* a solution of the inequality, shade the half-plane below the line.



b. ***Graph*** the boundary line $2x - 5y = 10$. Use a solid line because $2x - 5y \geq 10$.

Test the point (0, 0). Because (0, 0) is *not* a solution of the inequality, shade the half-plane below the line.



GOAL 2 USING LINEAR INEQUALITIES IN REAL LIFE

REAL LIFE
Communication

EXAMPLE 4 *Writing and Using a Linear Inequality*

You have relatives living in both the United States and Mexico. You are given a prepaid phone card worth \$50. Calls within the continental United States cost \$.16 per minute and calls to Mexico cost \$.44 per minute.

a. Write a linear inequality in two variables to represent the number of minutes you can use for calls within the United States and for calls to Mexico.

b. Graph the inequality and discuss three possible solutions in the context of the real-life situation.

SOLUTION

PROBLEM SOLVING STRATEGY

a. **VERBAL MODEL**

United States rate	·	United States time	+	Mexico rate	·	Mexico time	≤	Value of card

LABELS

United States rate = **0.16**	(dollars per minute)
United States time = x	(minutes)
Mexico rate = **0.44**	(dollars per minute)
Mexico time = y	(minutes)
Value of card = **50**	(dollars)

ALGEBRAIC MODEL

$$0.16x + 0.44y \leq 50$$

STUDENT HELP

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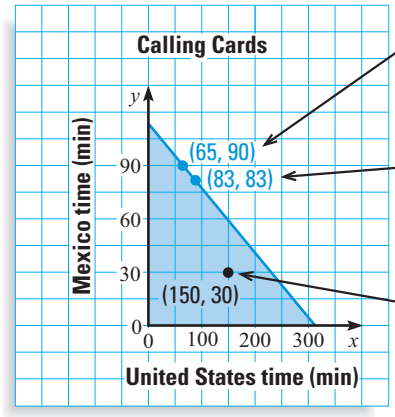
b. ***Graph*** the boundary line $0.16x + 0.44y = 50$. Use a solid line because $0.16x + 0.44y \leq 50$.

Test the point (0, 0). Because (0, 0) *is* a solution of the inequality, shade the half-plane below the line. Finally, because x and y cannot be negative, restrict the graph to points in the first quadrant.

Possible solutions are points within the shaded region shown.



One solution is to spend 65 minutes on calls within the United States and 90 minutes on calls to Mexico. The total cost will be \$50.

To split the time evenly, you could spend 83 minutes on calls within the United States and 83 minutes on calls to Mexico. The total cost will be \$49.80.

You could instead spend 150 minutes on calls within the United States and only 30 minutes on calls to Mexico. The total cost will be \$37.20.

GUIDED PRACTICE

Vocabulary Check ✓

1. Compare the graph of a linear inequality with the graph of a linear equation.

Concept Check ✓

2. Would you use a dashed line or a solid line for the graph of $Ax + By < C$? for the graph of $Ax + By \leq C$? Explain.

Tell whether the statement is *true* or *false*. Explain.

3. The point $\left(\frac{4}{3}, 0\right)$ is a solution of $3x - y > 4$.

4. The graph of $y < 3x + 5$ is the half-plane below the line $y = 3x + 5$.

Skill Check ✓

GRAPHING INEQUALITIES **Graph the inequality in a coordinate plane.**

5. $x > 5$
6. $y < -4$
7. $3x \leq 1$
8. $-y \geq \frac{4}{3}$
9. $y \geq -x + 7$
10. $y > \frac{2}{3}x - 1$
11. $2x - 3y < 6$
12. $x + 5y \leq -10$

13. **CALLING CARDS** Look back at Example 4. Suppose you have relatives living in China instead of Mexico. Calls to China cost $.75 per minute. Write and graph a linear inequality showing the number of minutes you can use for calls within the United States and for calls to China. Then discuss three possible solutions in the context of the real-life situation.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 942.

CHECKING SOLUTIONS **Check whether the given ordered pairs are solutions of the inequality.**

14. $x \leq -5$; $(0, 2)$, $(-5, 1)$
15. $2y \geq 7$; $(1, -6)$, $(0, 4)$
16. $y < -9x + 7$; $(-2, 2)$, $(3, -8)$
17. $19x + y \geq -0.5$; $(2, 3)$, $(-1, 0)$

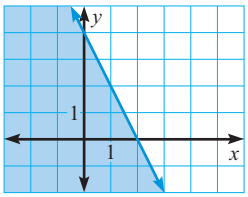
INEQUALITIES IN ONE VARIABLE **Graph the inequality in a coordinate plane.**

18. $x \leq 6$
19. $-x \geq 20$
20. $10x \geq \frac{10}{3}$
21. $-3y < 21$
22. $8y > -4$
23. $y < 0.75$

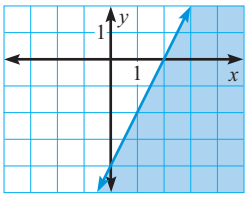
MATCHING GRAPHS **Match the inequality with its graph.**

24. $2x - y \geq 4$
25. $-2x - y < 4$
26. $2x + y \leq 4$

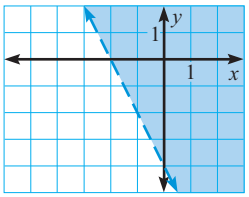
A.



B.



C.



STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 14–17
Example 2: Exs. 18–23, 33–44
Example 3: Exs. 24–44
Example 4: Exs. 45–51

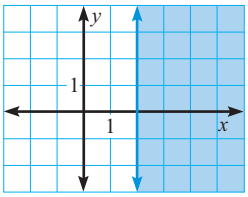
INEQUALITIES IN TWO VARIABLES **Graph the inequality.**

27. $y \leq 3x + 11$
28. $y > -4 - x$
29. $y < 0.75x - 5$
30. $3x + 12y > 4$
31. $9x - 9y > -36$
32. $\frac{3}{2}x + \frac{2}{3}y > 1$

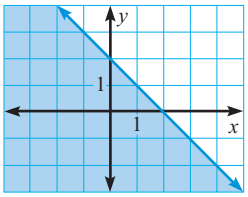
MATCHING GRAPHS **Match the inequality with its graph.**

33. $x + y > 2$ **34.** $x \geq 2$ **35.** $y \leq -x + 2$

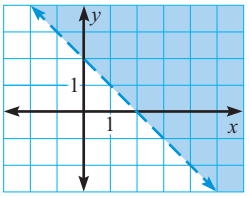
A.



B.



C.



GRAPHING INEQUALITIES **Graph the inequality in a coordinate plane.**

36. $9x - 2y \leq -18$ **37.** $y < 3x - \frac{3}{4}$ **38.** $5x > -20$

39. $y \geq \frac{1}{5}x + 10$ **40.** $4y \leq -6$ **41.** $2x + 3y > 4$

42. $6x \geq -\frac{1}{3}y$ **43.** $0.25x + 3y > 19$ **44.** $x + y < 0$

45. **HEALTH RISKS** By comparing the blood pressure in your ankle with the blood pressure in your arm, a physician can determine whether your arteries are becoming clogged with plaque. If the blood pressure in your ankle is less than 90% of the blood pressure in your arm, you may be at risk for heart disease. Write and graph an inequality that relates the unacceptable blood pressure in your ankle to the blood pressure in your arm.

NUTRITION **In Exercises 46 and 47, use the following information.**
Teenagers should consume at least 1200 milligrams of calcium per day. Suppose you get calcium from two different sources, skim milk and cheddar cheese. One cup of skim milk supplies 296 milligrams of calcium, and one slice of cheddar cheese supplies 338 milligrams of calcium. ▶ Source: *Nutrition in Exercise and Sport*

46. Write and graph an inequality that represents the amounts of skim milk and cheddar cheese you need to consume to meet your daily requirement of calcium.

47. Determine how many cups of skim milk you should drink if you have eaten two slices of cheddar cheese.

MOVIES **In Exercises 48 and 49, use the following information.**
You receive a gift certificate for $25 to your local movie theater. Matinees are $4.50 each and evening shows are $7.50 each.

48. Write and graph an inequality that represents the numbers of matinees and evening shows you can attend.

49. Give three possible combinations of the numbers of matinees and evening shows you can attend.

FOOTBALL **In Exercises 50 and 51, use the following information.**
In one of its first five games of a season, a football team scored a school record of 63 points. In all of the first five games, points came from touchdowns worth 7 points and field goals worth 3 points.

50. Write and graph an inequality that represents the numbers of touchdowns and field goals the team could have scored in any of the first five games.

51. Give five possible numbers of points scored, including the number of touchdowns and the number of field goals, for the first five games.

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Test Preparation

52. MULTI-STEP PROBLEM You want to open your own truck rental company. You do some research and find that the majority of truck rental companies in your area charge a flat fee of $29.99, plus $.29 for every mile driven. You want to charge less so that you can advertise your lower rate and get more business.

a. Write and graph an equation for the cost of renting a truck from other truck rental companies.

b. Shade the region of the coordinate plane where the amount you will charge must fall.

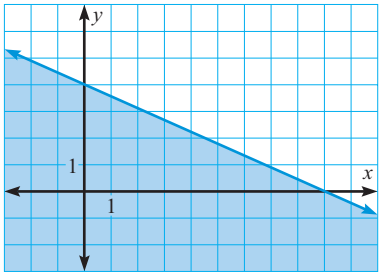
c. To charge less than your competitors, will you offer a lower flat fee, a lower rate per mile, or both? Explain your choice.

d. Write and graph an equation for the cost of renting a truck from your company in the same coordinate plane used in part (a).

e. CRITICAL THINKING Why can't you offer a lower rate per mile but a higher flat fee and still always charge less?

★ Challenge

VISUAL THINKING In Exercises 53–55, use the graph shown.



53. Write the inequality whose graph is shown.

54. Explain how you came up with the inequality.

55. What real-life situation could the first–quadrant portion of the graph represent?

EXTRA CHALLENGE
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MIXED REVIEW

SCIENTIFIC NOTATION Write the number in scientific notation. (Skills Review, p. 913)

56. 10,000,000 **57.** 1,650,000,000 **58.** 203,000

59. 0.00067 **60.** 0.0000009 **61.** 0.0808

GRAPHING EQUATIONS Graph the equation. (Review 2.3 for 2.7)

62. $y = \frac{5}{2}x - 5$ **63.** $y = -5x - 1$ **64.** $y = -\frac{1}{2}x + 6$

65. $x - y = 4$ **66.** $2x + y = 6$ **67.** $-4x + y = 4$

WRITING EQUATIONS Write an equation of the line that passes through the given points. (Review 2.4 for 2.7)

68. (2, 2), (5, 5) **69.** (0, 7), (5, 1) **70.** (−1, 6), (8, −2)

71. (3, 2), (3, −4) **72.** (1, 9), (−10, −6) **73.** (4, −8), (−7, −8)

74. GARDENING The horizontal middle of the United States is at about 40°N latitude. As a rule of thumb, plants will bloom earlier south of 40°N latitude and later north of 40°N latitude. The function $w = \frac{3}{5}(l - 40)$ gives the number of weeks w (earlier or later) that plants at latitude l°N will bloom compared with those at 40°N. The equation is valid from 35°N to 45°N latitude. Identify the domain and range of the function and then graph the function. (Review 2.1)