3.2 Solving Linear Systems Algebraically

What you should learn

GOAL 1 Use algebraic methods to solve linear systems.

GOAL 2 Use linear systems to model **real-life** situations, such as catering an event in **Example 5**.

Why you should learn it

▼ To solve **real-life** problems, such as how to plan a 40 minute workout in **Ex. 57**.

GOAL 1 USING ALGEBRAIC METHODS TO SOLVE SYSTEMS

In this lesson you will study two algebraic methods for solving linear systems. The first method is called *substitution*.

THE SUBSTITUTION METHOD

STEP 1 Solve one of the equations for one of its variables.

STEP 2 Substitute the expression from Step 1 into the other equation and solve for the other variable.

STEP 3 Substitute the value from Step 2 into the revised equation from Step 1 and solve.

EXAMPLE 1 *The Substitution Method*

Solve the linear system using the substitution method.

$3x + 4y = -4$ **Equation 1**

$x + 2y = 2$ **Equation 2**

SOLUTION

1 Solve Equation 2 for x.

$x + 2y = 2$ **Write Equation 2.**

$x = -2y + 2$ **Revised Equation 2**

2 Substitute the expression for x into Equation 1 and solve for y.

$3x + 4y = -4$ **Write Equation 1.**

$3(-2y + 2) + 4y = -4$ **Substitute $-2y + 2$ for x.**

$y = 5$ **Solve for y.**

3 Substitute the value of y into revised Equation 2 and solve for x.

$x = -2y + 2$ **Write revised Equation 2.**

$x = -2(5) + 2$ **Substitute 5 for y.**

$x = -8$ **Simplify.**

▶ The solution is $(-8, 5)$.

✓ **CHECK** Check the solution by substituting back into the original equations.

$3x + 4y = -4$ **Write original equations.** $x + 2y = 2$

$3(-8) + 4(5) \stackrel{?}{=} -4$ **Substitute for x and y.** $-8 + 2(5) \stackrel{?}{=} 2$

$-4 = -4$ ✓ **Solution checks.** $2 = 2$ ✓

CHOOSING A METHOD In Step 1 of Example 1, you could have solved for either x or y in either Equation 1 or Equation 2. It was easiest to solve for x in Equation 2 because the x-coefficient is 1. In general you should solve for a variable whose coefficient is 1 or -1.

$x - 5y = 11$ ← Solve for x.
$2x + 7y = -3$

$4x - 2y = -1$
$3x - y = 8$ ← Solve for y.

If neither variable has a coefficient of 1 or -1, you can still use substitution. In such cases, however, the *linear combination* method may be better. The goal of this method is to add the equations to obtain an equation in one variable.

THE LINEAR COMBINATION METHOD

STEP 1 Multiply one or both of the equations by a constant to obtain coefficients that differ only in sign for one of the variables.

STEP 2 Add the revised equations from Step 1. Combining like terms will eliminate one of the variables. Solve for the remaining variable.

STEP 3 Substitute the value obtained in Step 2 into either of the original equations and solve for the other variable.

EXAMPLE 2 The Linear Combination Method: Multiplying One Equation

Solve the linear system using the linear combination method.

$2x - 4y = 13$ Equation 1
$4x - 5y = 8$ Equation 2

SOLUTION

1 Multiply the first equation by -2 so that the x-coefficients differ only in sign.

$2x - 4y = 13$ $\times -2$ $\rightarrow$ $-4x + 8y = -26$
$4x - 5y = 8$ $\rightarrow$ $4x - 5y = 8$

2 Add the revised equations and solve for y.

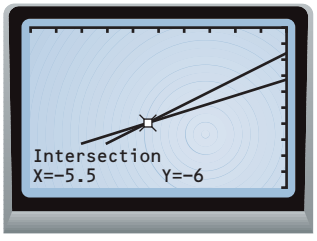
$3y = -18$
$y = -6$

3 Substitute the value of y into one of the original equations. Solve for x.

$2x - 4y = 13$ Write Equation 1.
$2x - 4(-6) = 13$ Substitute -6 for y.
$2x + 24 = 13$ Simplify.
$x = -\frac{11}{2}$ Solve for x.

▶ The solution is $\left(-\frac{11}{2}, -6\right)$.

✓**CHECK** You can check the solution algebraically using the method shown in Example 1. You can also use a graphing calculator to check the solution.



STUDENT HELP

Study Tip
In Example 2, one x-coefficient is a multiple of the other. In this case, it is easier to eliminate the x-terms because you need to multiply only one equation by a constant.

EXAMPLE 3 The Linear Combination Method: Multiplying Both Equations

Solve the linear system using the linear combination method.

$7x - 12y = -22$ **Equation 1**
$-5x + 8y = 14$ **Equation 2**

SOLUTION

Multiply the first equation by **2** and the second equation by **3** so that the coefficients of y differ only in sign.

$$7x - 12y = -22 \quad \xrightarrow{\times 2} \quad 14x - 24y = -44$$
$$-5x + 8y = 14 \quad \xrightarrow{\times 3} \quad -15x + 24y = 42$$

Add the revised equations and solve for x.

$$-x = -2$$
$$x = 2$$

Substitute the value of x into one of the original equations. Solve for y.

$-5x + 8y = 14$ **Write Equation 2.**
$-5(2) + 8y = 14$ **Substitute 2 for x.**
$y = 3$ **Solve for y.**

▶ The solution is (2, 3). Check the solution algebraically or graphically.

EXAMPLE 4 Linear Systems with Many or No Solutions

Solve the linear system.

a. $x - 2y = 3$
$2x - 4y = 7$

b. $6x - 10y = 12$
$-15x + 25y = -30$

SOLUTION

a. Since the coefficient of x in the first equation is 1, use substitution.

Solve the first equation for x.

$x - 2y = 3$
$x = 2y + 3$

Substitute the expression for x into the second equation.

$2x - 4y = 7$ **Write second equation.**
$2(2y + 3) - 4y = 7$ **Substitute $2y + 3$ for x.**
$6 = 7$ **Simplify.**

▶ Because the statement $6 = 7$ is never true, there is *no solution.*

b. Since no coefficient is 1 or -1, use the linear combination method.

Multiply the first equation by **5** and the second equation by **2**.

$$6x - 10y = 12 \quad \xrightarrow{\times 5} \quad 30x - 50y = 60$$
$$-15x + 25y = -30 \quad \xrightarrow{\times 2} \quad -30x + 50y = -60$$

Add the revised equations.

$$0 = 0$$

▶ Because the equation $0 = 0$ is always true, there are *infinitely many solutions.*

GOAL 2 USING LINEAR SYSTEMS IN REAL LIFE

EXAMPLE 5 *Using a Linear System as a Model*

CATERING A caterer is planning a party for 64 people. The customer has \$150 to spend. A \$39 pan of pasta feeds 14 people and a \$12 sandwich tray feeds 6 people. How many pans of pasta and how many sandwich trays should the caterer make?

SOLUTION

PROBLEM SOLVING STRATEGY

VERBAL MODEL

(People per pan) · (Pans of pasta) + (People per tray) · (Sandwich trays) = (People at the party)

(Price per pan) · (Pans of pasta) + (Price per tray) · (Sandwich trays) = (Money to spend on food)

LABELS

Equation 1

People per pan of pasta = **14** (people)

Pans of pasta = P (pans)

People per sandwich tray = **6** (people)

Sandwich trays = S (trays)

People at the party = **64** (people)

Equation 2

Price per pan of pasta = **39** (dollars)

Pans of pasta = P (pans)

Price per sandwich tray = **12** (dollars)

Sandwich trays = S (trays)

Money to spend on food = **150** (dollars)

ALGEBRAIC MODEL

Equation 1: $14P + 6S = 64$ — People at the party

Equation 2: $39P + 12S = 150$ — Money to spend on food

Use the linear combination method to solve the system.

Multiply Equation 1 by -2 so that the coefficients of S differ only in sign.

$$14P + 6S = 64 \quad \xrightarrow{\times\ -2} \quad -28P - 12S = -128$$

$$39P + 12S = 150 \quad \longrightarrow \quad 39P + 12S = 150$$

Add the revised equations and solve for P.

$$11P = 22$$
$$P = 2$$

Substitute the value of P into one of the original equations and solve for S.

$14P + 6S = 64$ — Write Equation 1.

$14(2) + 6S = 64$ — Substitute 2 for P.

$28 + 6S = 64$ — Multiply.

$S = 6$ — Solve for S.

▶ The caterer should make 2 pans of pasta and 6 sandwich trays for the party.

FOCUS ON CAREERS

CATERER A caterer prepares food for special events. When planning a meal, a caterer needs to consider both the cost of the food and the number of guests.

CAREER LINK
www.mcdougallittell.com

GUIDED PRACTICE

Vocabulary Check ✓

1. Complete this statement: To solve a linear system where one of the coefficients is 1 or -1, it is usually easiest to use the ? method.

Concept Check ✓

2. Read Step 3 in the box on page 148. Why do you think it recommends substituting into the revised equation from Step 1 instead of one of the original equations?

3. When solving a linear system algebraically, how do you know when there is no solution? How do you know when there are infinitely many solutions?

Skill Check ✓

Solve the system using the substitution method.

4. $x + 3y = -2$
$-4x - 5y = 8$

5. $3x + 2y = 10$
$2x - y = 9$

6. $-3x + y = -7$
$5x - 2y = 12$

Solve the system using the linear combination method.

7. $-3x + 2y = -6$
$5x - 2y = 18$

8. $5x - 2y = 12$
$-9x - 8y = 19$

9. $4x - 3y = 0$
$-10x + 7y = -2$

10. **BUSINESS** Selling frozen yogurt at a fair, you make \$565 and use 250 cones. A single-scoop cone costs \$2 and a double-scoop cone costs \$2.50. How many of each type of cone did you sell?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 943.

SUBSTITUTION METHOD **Solve the system using the substitution method.**

11. $2x + 3y = 5$
$x - 5y = 9$

12. $-2x + y = 6$
$4x - 2y = 5$

13. $-x + 2y = 3$
$4x - 5y = -3$

14. $5x + 3y = 4$
$5x + y = 16$

15. $4x + 6y = 15$
$-x + 2y = 5$

16. $3x - y = 4$
$5x + 3y = 9$

17. $\frac{1}{2}x + y = 9$
$7x + 4y = 24$

18. $-3x + y = 2$
$8x - 15y = 7$

19. $5x + 6y = -45$
$x - \frac{1}{2}y = 8$

20. $-x - 4y = -3$
$2x + y = 15$

21. $x + 2y = 2$
$7x - 3y = -20$

22. $3x - y = 4$
$-9x + 3y = -12$

LINEAR COMBINATION METHOD **Solve the system using the linear combination method.**

23. $3x + 5y = -16$
$3x - 2y = -9$

24. $3x + 2y = 6$
$-6x - 3y = -6$

25. $-6x + 5y = 4$
$7x - 10y = -8$

26. $7x - 4y = -3$
$2x + 5y = -7$

27. $-9x + 6y = 0$
$-12x + 8y = 0$

28. $5x + 6y = -16$
$2x + 10y = 5$

29. $21x - 8y = -1$
$9x + 5y = 8$

30. $-15x - 2y = -31$
$4x + 6y = 11$

31. $\frac{1}{4}x + 5y = 37$
$-4x + 2y = 13$

32. $7x + 2y = -3$
$-14x - 4y = 6$

33. $6x - y = -2$
$-18x + 3y = 4$

34. $-5x + 2y = -10$
$3x - 6y = -18$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 11–22, 35–49
Examples 2, 3: Exs. 23–49
Example 4: Exs. 11–49
Example 5: Exs. 54–62

CHOOSING A METHOD **Solve the system using any algebraic method.**

35. $-5x + 7y = 11$
$-5x + 3y = 19$

36. $x - y = 3$
$-2x + 2y = -6$

37. $2x - 5y = 10$
$-3x + 4y = -15$

38. $-3x + y = 11$
$5x - 2y = -16$

39. $-4x - 6y = 11$
$6x + 9y = -3$

40. $x - 4y = -2$
$-3x + 8y = -1$

41. $2x + 5y = 17$
$-5x - 7y = -10$

42. $-3x + 7y = 6$
$5x - y = 10$

43. $-2x + 3y = 20$
$4x + 4y = -15$

44. $3x - 7y = 20$
$-11x + 10y = 5$

45. $x - y = 17$
$\frac{1}{2}x - 3y = 1$

46. $4x + 9y = -10$
$-8x - 12y = 8$

47. $12x + 3y = 16$
$-36x - 9y = 32$

48. $-x + 5y = 17$
$2x - 10y = -34$

49. $\frac{1}{3}x + y = 9$
$-2x + 2y = -6$

50. *Writing* Explain how you can tell whether the system has infinitely many solutions or no solution without trying to solve the system.

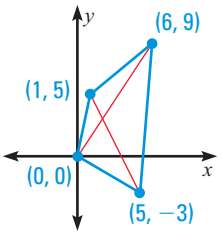
a. $5x - 2y = 6$
$-10x + 4y = -12$

b. $-2x + y = 8$
$-6x + 3y = 12$

GEOMETRY CONNECTION **Find the coordinates of the point where the diagonals of the quadrilateral intersect.**

STUDENT HELP
INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 51–53.

51.



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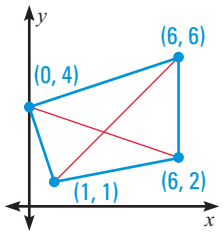

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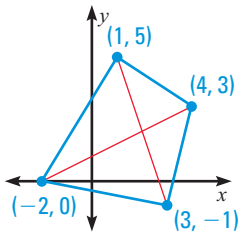
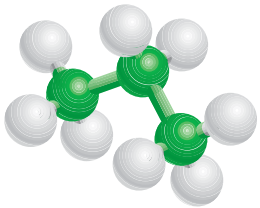


54. **BREAKING EVEN** You are starting a business selling boxes of hand-painted greeting cards. To get started, you spend $36 on paint and paintbrushes that you need. You buy boxes of plain cards for $3.50 per box, paint the cards, and then sell them for $5 per box. How many boxes must you sell for your earnings to equal your expenses? What will your earnings and expenses equal when you break even?

55. **HOME ELECTRONICS** To connect a VCR to a television set, you need a cable with special connectors at both ends. Suppose you buy a 6 foot cable for $15.50 and a 3 foot cable for $10.25. Assuming that the cost of a cable is the sum of the cost of the two connectors and the cost of the cable itself, what would you expect to pay for a 4 foot cable? Explain how you got your answer.

56. **SCIENCE CONNECTION** Weights of atoms and molecules are measured in *atomic mass units* (u). A molecule of C_2H_6 (ethane) is made up of 2 carbon atoms and 6 hydrogen atoms and weighs 30.07 u. A molecule of C_3H_8 (propane) is made up of 3 carbon atoms and 8 hydrogen atoms and weighs 44.097 u. Find the weights of a carbon atom and a hydrogen atom.


Ethane molecule


Propane molecule

FOCUS ON APPLICATIONS

SWIMMING One way swimmers improve their racing times is by training at high altitudes. Many elite swimmers train at the Olympic Training Center in Colorado Springs, Colorado, at an altitude of 6035 feet above sea level.

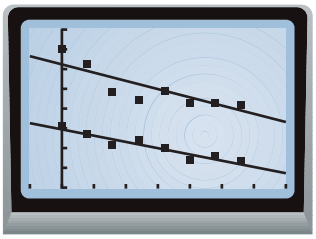
57. CROSS-TRAINING You want to burn 380 Calories during 40 minutes of exercise. You burn about 8 Calories per minute inline skating and 12 Calories per minute swimming. How long should you spend doing each activity?

58. RENTING AN APARTMENT Two friends rent an apartment for $975 per month. Since one bedroom is 60 square feet larger than the other bedroom, each person's rent contribution is based on bedroom size. Each person agrees to pay $3.25 per square foot of bedroom area. Let x be the area (in square feet) of the larger bedroom, and let y be the area (in square feet) of the smaller bedroom. Write and solve a system of linear equations to find the area of each bedroom.

SWIMMING In Exercises 59–62, use the table below of winning times in the Olympic 100 meter freestyle swimming event for the period 1968–1996.

Years since 1968, x	0	4	8	12	16	20	24	28
Men's time (sec), m	52.2	51.2	50.0	50.4	49.8	48.6	49.0	48.7
Women's time (sec), w	60.0	58.6	55.7	54.8	55.9	54.9	54.6	54.5

INTERNET **DATA UPDATE** of USA Swimming data at www.mcdougallittell.com



59. Use a graphing calculator to make scatter plots of the data pairs (x, m) and (x, w).

60. For each scatter plot, find an equation of the line of best fit. Graph the equations, as shown.

61. Find the coordinates of the intersection point of the lines. Describe what this point represents.

62. CRITICAL THINKING Why might a linear model not be appropriate for projecting winning times far into the future?

Test Preparation

QUANTITATIVE COMPARISON In Exercises 63 and 64, choose the statement that is true about the given quantities.

(A) The quantity in column A is greater.

(B) The quantity in column B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined from the given information.

	Column A	Column B
63.	The x-coordinate of the solution of: $7x - y = 19$, $10x + 2y = 34$	3
64.	-5	The y-coordinate of the solution of: $-2x + 6y = -26$, $x + 3y = 11$

Challenge

65. CRITICAL THINKING Find values of r, s, and t that produce the solution(s).

$$-3x - 5y = 9$$
$$rx + sy = t$$

a. no solution **b.** infinitely many solutions **c.** a solution of $(2, -3)$

MIXED REVIEW

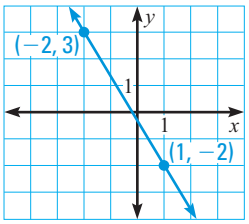
ABSOLUTE VALUE EQUATIONS **Solve the equation.** (Review 1.7)

66. $|6x| = 12$ **67.** $|x + 5| = 3$ **68.** $|2x - 1| = 7$

69. $|4x + 1| = 5$ **70.** $|3x - 2| = 8$ **71.** $|-x + 10| = 14$

WRITING EQUATIONS **Write an equation of the line.** (Review 2.4)

72.



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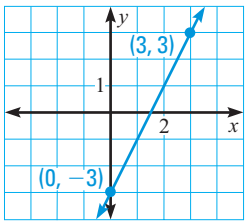

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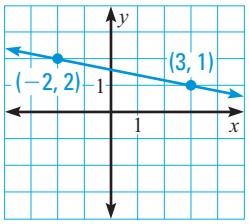

GRAPHING INEQUALITIES **Graph the inequality in a coordinate plane.** (Review 2.6 for 3.3)

75. $y < 4$ **76.** $x \geq -2$ **77.** $3x - y \geq 0$

78. $y < -x + 4$ **79.** $4x - y < 5$ **80.** $y \geq -2x - 1$

81. **CONSUMER ECONOMICS** You plan to buy a pair of jeans for $25 and some T-shirts for $12 each. You have only $60 to spend. Write and solve an inequality for the number of T-shirts you can buy. (Review 1.6 for 3.3)

QUIZ 1

Self-Test for Lessons 3.1 and 3.2

Use a graph to solve the system. (Lesson 3.1)

1. $y = 2x + 5$
$y = -2x - 3$

2. $y = -4x + 1$
$y = x - 4$

3. $-3x + 2y = 4$
$6x - 4y = 14$

4. $-2x - y = -2$
$3x - 3y = 15$

5. $y = -x + 5$
$3x - y = -1$

6. $4x + 5y = -9$
$x + 3y = -4$

Tell how many solutions the linear system has. (Lessons 3.1 and 3.2)

7. $6x + 6y = 3$
$4x + 4y = 2$

8. $-2x + y = 13$
$x - 4y = -31$

9. $-5x + 7y = 10$
$15x - 21y = 22$

10. $3x - 3y = 3$
$-4x + y = -21$

11. $x - 6y = 6$
$-3x + 2y = -2$

12. $-4x + 8y = 24$
$-x + 2y = 6$

Solve the system using any algebraic method. (Lesson 3.2)

13. $-2x + 2y = -5$
$x + y = -5$

14. $-3x + 2y = -6$
$5x - 2y = 18$

15. $-4x - y = -1$
$12x + 3y = 3$

16. $-3x - 4y = -2$
$x + 2y = 3$

17. $3x - 8y = 11$
$-6x + 16y = -5$

18. $3x - 8y = -7$
$-5x - 6y = 3$

19. **THEATER** Tickets for your school's play are $3 for students and $5 for non-students. On opening night 937 tickets are sold and $3943 is collected. How many tickets were sold to students? to non-students? (Lesson 3.2)