# 6.3

# Adding, Subtracting, and Multiplying Polynomials

*What you should learn*

GOAL 1 Add, subtract, and multiply polynomials.

GOAL 2 Use polynomial operations in **real-life** problems, such as finding net farm income in **Example 7**.

*Why you should learn it*

▼ To combine **real-life** polynomial models into a new model, such as the model for the power needed to keep a bicycle moving at a certain speed in **Ex. 66**.


## GOAL 1 ADDING, SUBTRACTING, AND MULTIPLYING

To add or subtract polynomials, add or subtract the coefficients of like terms. You can use a vertical or horizontal format.

### EXAMPLE 1 *Adding Polynomials Vertically and Horizontally*

Add the polynomials.

**a.**
$$\begin{array}{r} 3x^3 + 2x^2 - x - 7 \\ +\ x^3 - 10x^2 + 8 \\ \hline 4x^3 - 8x^2 - x + 1 \end{array}$$

**b.** $(9x^3 - 2x + 1) + (5x^2 + 12x - 4) = 9x^3 + 5x^2 - 2x + 12x + 1 - 4$

$= 9x^3 + 5x^2 + 10x - 3$

### EXAMPLE 2 *Subtracting Polynomials Vertically and Horizontally*

Subtract the polynomials.

**a.**
$$\begin{array}{r} 8x^3 - 3x^2 - 2x + 9 \\ -\ (2x^3 + 6x^2 - x + 1) \end{array} \quad \rightarrow \quad \begin{array}{r} 8x^3 - 3x^2 - 2x + 9 \\ -2x^3 - 6x^2 + x - 1 \\ \hline 6x^3 - 9x^2 - x + 8 \end{array}$$

Add the opposite.

**b.** $(2x^2 + 3x) - (3x^2 + x - 4) = 2x^2 + 3x - 3x^2 - x + 4$ Add the opposite.

$= -x^2 + 2x + 4$

To multiply two polynomials, each term of the first polynomial must be multiplied by each term of the second polynomial.

### EXAMPLE 3 *Multiplying Polynomials Vertically*

Multiply the polynomials.

$$\begin{array}{r} -x^2 + 2x + 4 \\ \times \quad x - 3 \\ \hline 3x^2 - 6x - 12 \\ -x^3 + 2x^2 + 4x \\ \hline -x^3 + 5x^2 - 2x - 12 \end{array}$$

Multiply $-x^2 + 2x + 4$ by $-3$.

Multiply $-x^2 + 2x + 4$ by $x$.

Combine like terms.

**STUDENT HELP**

**Look Back**
For help with simplifying expressions, see p. 251.

## EXAMPLE 4 Multiplying Polynomials Horizontally

Multiply the polynomials.

$$\begin{aligned}(x-3)(3x^2-2x-4) &= (x-3)3x^2-(x-3)2x-(x-3)4\\ &= 3x^3-9x^2-2x^2+6x-4x+12\\ &= 3x^3-11x^2+2x+12\end{aligned}$$

## EXAMPLE 5 Multiplying Three Binomials

**STUDENT HELP**

**Look Back**
For help with multiplying binomials, see p. 251.

Multiply the polynomials.

$$\begin{aligned}(x-1)(x+4)(x+3) &= (x^2+3x-4)(x+3)\\ &= (x^2+3x-4)x+(x^2+3x-4)3\\ &= x^3+3x^2-4x+3x^2+9x-12\\ &= x^3+6x^2+5x-12\end{aligned}$$

. . . . . .

Some binomial products occur so frequently that it is worth memorizing their *special product patterns*. You can verify these products by multiplying.

### SPECIAL PRODUCT PATTERNS

| | |
|---|---|
| **SUM AND DIFFERENCE** | **Example** |
| $(a+b)(a-b)=a^2-b^2$ | $(x+3)(x-3)=x^2-9$ |
| **SQUARE OF A BINOMIAL** | |
| $(a+b)^2=a^2+2ab+b^2$ | $(y+4)^2=y^2+8y+16$ |
| $(a-b)^2=a^2-2ab+b^2$ | $(3t^2-2)^2=9t^4-12t^2+4$ |
| **CUBE OF A BINOMIAL** | |
| $(a+b)^3=a^3+3a^2b+3ab^2+b^3$ | $(x+1)^3=x^3+3x^2+3x+1$ |
| $(a-b)^3=a^3-3a^2b+3ab^2-b^3$ | $(p-2)^3=p^3-6p^2+12p-8$ |

## EXAMPLE 6 Using Special Product Patterns

Multiply the polynomials.

**a.** $(4n-5)(4n+5)=(4n)^2-5^2$ — Sum and difference

$$=16n^2-25$$

**b.** $(9y-x^2)^2=(9y)^2-2(9y)(x^2)+(x^2)^2$ — Square of a binomial

$$=81y^2-18x^2y+x^4$$

**c.** $(ab+2)^3=(ab)^3+3(ab)^2(2)+3(ab)(2)^2+2^3$ — Cube of a binomial

$$=a^3b^3+6a^2b^2+12ab+8$$

FOCUS ON APPLICATIONS

**FARMING** The number of farms in the United States has been decreasing steadily since the 1930s. However, the average size of farms has been increasing.

## GOAL 2 USING POLYNOMIAL OPERATIONS IN REAL LIFE

### EXAMPLE 7 *Subtracting Polynomial Models*

**FARMING** From 1985 through 1995, the gross farm income $G$ and farm expenses $E$ (in billions of dollars) in the United States can be modeled by

$$G = -0.246t^2 + 7.88t + 159 \quad \text{and} \quad E = 0.174t^2 + 2.54t + 131$$

where $t$ is the number of years since 1985. Write a model for the *net* farm income $N$ for these years. ▶ Source: U.S. Department of Agriculture

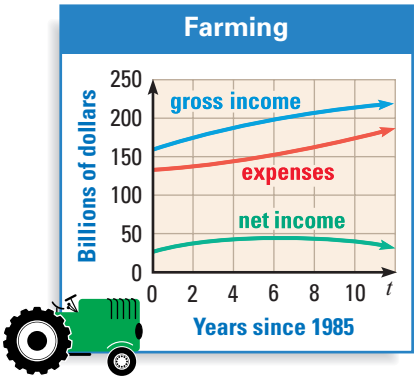
**SOLUTION**

To find a model for the net farm income, subtract the expenses model from the gross income model.

$$\begin{array}{r} -0.246t^2 + 7.88t + 159 \\ -\ (0.174t^2 + 2.54t + 131) \\ \hline -0.420t^2 + 5.34t + 28 \end{array}$$

▶ The net farm income can be modeled by $N = -0.42t^2 + 5.34t + 28$.

The graphs of the models are shown. Although $G$ and $E$ both increase, the net income $N$ eventually decreases because $E$ increases faster than $G$.




### EXAMPLE 8 *Multiplying Polynomial Models*

**Publishing**

From 1982 through 1995, the number of softbound books $N$ (in millions) sold in the United States and the average price per book $P$ (in dollars) can be modeled by

$$N = 1.36t^2 + 2.53t + 1076 \quad \text{and} \quad P = 0.314t + 3.42$$

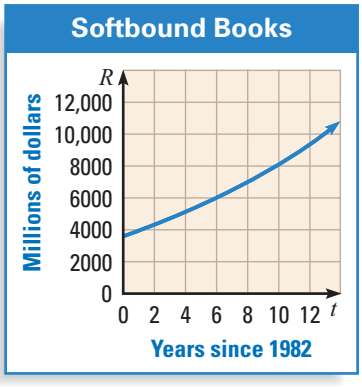
where $t$ is the number of years since 1982. Write a model for the total revenue $R$ received from the sales of softbound books. What was the total revenue from softbound books in 1990? ▶ Source: Book Industry Study Group, Inc.

**SOLUTION**

To find a model for $R$, multiply the models for $N$ and $P$.

$$\begin{array}{r} 1.36t^2 + 2.53t + 1076 \\ \times \qquad 0.314t + 3.42 \\ \hline 4.6512t^2 + 8.6526t + 3679.92 \\ 0.42704t^3 + 0.79442t^2 + 337.864t \\ \hline 0.42704t^3 + 5.44562t^2 + 346.5166t + 3679.92 \end{array}$$

▶ The total revenue can be modeled by $R = 0.427t^3 + 5.45t^2 + 347t + 3680$. The graph of the revenue model is shown at the right. By substituting $t = 8$ into the model for $R$, you can calculate that the revenue was about \$7020 million, or \$7.02 billion, in 1990.



# GUIDED PRACTICE

**Vocabulary Check** ✓

1. When you add or subtract polynomials, you add or subtract the coefficients of ?.

**Concept Check** ✓

2. **ERROR ANALYSIS** Describe the error in the subtraction shown below.

$$(x^2 - 3x + 4) - (x^2 + 7x - 2) = x^2 - 3x + 4 - x^2 + 7x - 2$$
$$= 4x + 2$$

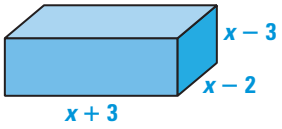
3. When you multiply a polynomial of degree 2 by a polynomial of degree 4, what is the degree of the product?

**Skill Check** ✓

**Perform the indicated operation.**

4. $(4x^2 + 3) + (3x^2 + 8)$
5. $(2x^3 - 4x^2 + 5) + (-x^2 - 3x + 1)$
6. $(x^2 + 7x - 5) - (3x^2 + 1)$
7. $(x^2 + 1) - (3x^2 - 4x + 3)$
8. $(x + 2)(2x^2 + 3)$
9. $(x^2 + 3x + 10)(4x^2 - 2x - 7)$
10. $(x - 1)(2x + 1)(x + 5)$
11. $(-3x + 1)^3$

12. **GEOMETRY CONNECTION** Write a polynomial model in standard form for the volume of the rectangular prism shown at the right.




# PRACTICE AND APPLICATIONS

**STUDENT HELP**

**Extra Practice** to help you master skills is on p. 948.

**ADDING AND SUBTRACTING POLYNOMIALS** **Find the sum or difference.**

13. $(8x^2 + 1) + (3x^2 - 2)$
14. $(3x^3 + 10x + 5) - (x^3 - 4x + 6)$
15. $(x^2 - 6x + 5) - (x^2 + x - 2)$
16. $(16 - 13x) + (10x - 11)$
17. $(7x^3 - 1) - (15x^3 + 4x^2 - x + 3)$
18. $8x + (14x + 3 - 41x^2 + x^3)$
19. $(4x^2 - 11x + 10) + (5x - 31)$
20. $(9x^3 - 4 + x^2 + 8x) - (7x^3 - 3x + 7)$
21. $(-3x^3 + x - 11) - (4x^3 + x^2 - x)$
22. $(6x^2 - 19x + 5) - (19x^2 - 4x + 9)$
23. $(10x^3 - 4x^2 + 3x) - (x^3 - x^2 + 1)$
24. $(50x - 3) + (8x^3 + 7x^2 + x + 4)$
25. $(10x - 3 + 7x^2) + (x^3 - 2x + 17)$
26. $(3x^3 - 5x^4 - 10x + 1) + (17x^4 - x^3)$

**MULTIPLYING POLYNOMIALS** **Find the product of the polynomials.**

27. $x(x^2 + 6x - 7)$
28. $10x^2(x - 5)$
29. $-4x(x^2 - 8x + 3)$
30. $5x(3x^2 - x + 3)$
31. $(x - 4)(x - 7)$
32. $(x + 9)(x - 2)$
33. $(x + 3)(x^2 - 4x + 9)$
34. $(x + 8)(x^2 - 7x - 3)$
35. $(2x + 5)(3x^3 - x^2 + x)$
36. $(6x + 2)(2x^2 - 6x + 1)$
37. $(x + 11)(x^2 - 5x + 9)$
38. $(4x^2 - 1)(x^2 - 6x + 9)$
39. $(x - 1)(x^3 + 2x^2 + 2)$
40. $(x + 1)(5x^3 - x^2 + x - 4)$
41. $(3x^2 - 2)(x^2 + 4x + 3)$
42. $(-x^3 - 2)(x^2 + 3x - 3)$
43. $(x^2 + x + 4)(2x^2 - x + 1)$
44. $(x^2 - x - 3)(x^2 + 4x + 2)$

**STUDENT HELP**

**HOMEWORK HELP**

**Examples 1, 2:** Exs. 13–26
**Examples 3, 4:** Exs. 27–44
**Example 5:** Exs. 45–52
**Example 6:** Exs. 53–61
**Example 7:** Exs. 64, 65, 69
**Example 8:** Exs. 66–68

**MULTIPLYING THREE BINOMIALS** **Find the product of the binomials.**

**45.** $(x + 9)(x - 2)(x - 7)$

**46.** $(x + 3)(x - 4)(x - 5)$

**47.** $(x + 5)(x + 7)(-x + 1)$

**48.** $(2x - 3)(x + 7)(x + 6)$

**49.** $(x - 9)(x - 2)(3x + 2)$

**50.** $(x - 1)(-2x - 5)(x - 8)$

**51.** $(2x + 1)(3x + 1)(x + 4)$

**52.** $(4x - 1)(2x - 1)(3x - 2)$

**SPECIAL PRODUCTS** **Find the product.**

**53.** $(x + 7)(x - 7)$

**54.** $(x + 4)^2$

**55.** $(4x - 3)^3$

**56.** $(10x + 3)(10x - 3)$

**57.** $(6 - x^2)^2$

**58.** $(2y + 5x)^2$

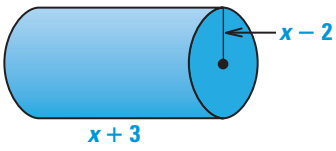
**59.** $(3x + 7)^3$

**60.** $(7y - x)^2$

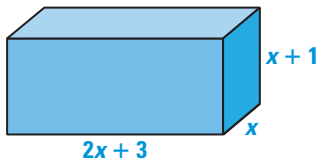
**61.** $(2x + 3y)^3$

**GEOMETRY CONNECTION** **Write the volume of the figure as a polynomial in standard form.**

**62.** $V = \pi r^2 h$




**63.** $V = lwh$




**64.** **MOTOR VEHICLE SALES** For 1983 through 1996, the number of cars $C$ (in thousands) and the number of trucks and buses $T$ (in thousands) sold that were manufactured in the United States can be modeled by

$$C = -1.63t^4 + 49.5t^3 - 476t^2 + 1370t + 6705$$

$$T = -1.052t^4 + 31.6t^3 - 296t^2 + 1097t + 2290$$

where $t$ is the number of years since 1983. Find a model that represents the total number of vehicles sold that were manufactured in the United States. How many vehicles were sold in 1990?

**65.** **SOCIAL STUDIES CONNECTION** For 1980 through 1996, the population $P$ (in thousands) of the United States and the number of people $S$ (in thousands) age 85 and over can be modeled by

$$P = -0.804t^4 + 26.9t^3 - 262t^2 + 3010t + 227{,}000$$

$$S = 0.0206t^4 - 0.670t^3 + 6.42t^2 + 213t + 7740$$

where $t$ is the number of years since 1980. Find a model that represents the number of people in the United States under the age of 85. How many people were under the age of 85 in 1995?

INTERNET **DATA UPDATE** of U.S. Bureau of the Census data at www.mcdougallittell.com

**66.** **BICYCLING** The equation $P = 0.00267sF$ gives the power $P$ (in horsepower) needed to keep a certain bicycle moving at speed $s$ (in miles per hour), where $F$ is the force of road and air resistance (in pounds). On level ground this force is given by $F = 0.0116s^2 + 0.789$. Write a polynomial function (in terms of $s$ only) for the power needed to keep the bicycle moving at speed $s$ on level ground. How much power does a cyclist need to exert to keep the bicycle moving at 10 miles per hour?

**FOCUS ON CAREERS**

REAL LIFE **GERONTOLOGIST** A gerontologist studies the biological, psychological, and sociological phenomena associated with old age. As people's life expectancies have increased, demand for gerontologists has grown.

INTERNET **CAREER LINK** www.mcdougallittell.com

**67.** **EDUCATION** For 1980 through 1995, the number of degrees $D$ (in thousands) earned by people in the United States and the percent of degrees $P$ earned by women can be modeled by

$$D = -0.096t^4 + 3t^3 - 27t^2 + 91t + 1700$$
$$P = 0.43t + 49$$

where $t$ is the number of years since 1980. Find a model that represents the number of degrees $W$ (in thousands) earned by women from 1980 to 1995. How many degrees were earned by women in 1991? ▶ Source: U.S. Bureau of the Census

**68.** **PUBLISHING** From 1985 through 1993, the number of hardback books $N$ (in millions) sold in the United States and the average price per book $P$ (in dollars) can be modeled by

$$N = -0.27t^3 + 3.9t^2 + 7.9t + 650$$
$$P = 0.67t + 9.4$$

where $t$ is the number of years since 1985. Write a model that represents the total revenue $R$ (in millions of dollars) received from the sales of hardback books. What was the revenue in 1991?

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with problem solving in Ex. 69.

**69.** **PERSONAL FINANCE** Suppose two brothers each make three deposits in accounts earning the same annual interest rate $r$ (expressed as a decimal).

EagleBank

Porter, Mark J. #05-8922-4310

| Date | Transaction | Amount |
|---|---|---|
| 1/1/97 | Deposit | $6000.00 |
| 1/1/98 | Deposit | $8000.00 |
| 1/1/99 | Deposit | $9000.00 |

WorldBank

Porter, Tom R. #12-4600-2541

| Date | Transaction | Amount |
|---|---|---|
| 1/1/97 | Deposit | $4000.00 |
| 1/1/98 | Deposit | $5000.00 |
| 1/1/99 | Deposit | $7000.00 |

Mark's account is worth $6000(1 + r)^3 + 8000(1 + r)^2 + 9000(1 + r)$ on January 1, 2000. Find the value of Tom's account on January 1, 2000. Then find the total value of the two accounts on January 1, 2000. Write the total value as a polynomial in standard form.

## Test Preparation

**70.** **MULTIPLE CHOICE** What is the sum of $2x^4 + 5x^3 - 8x^2 - x + 10$ and $8x^4 - 4x^3 + x^2 - x + 2$?

(A) $10x^4 + x^3 - 9x^2 + 12$ (B) $10x^4 + x^3 - 9x^2 - 2x + 12$

(C) $10x^4 + x^3 - 7x^2 - 2x + 12$ (D) $10x^4 + 9x^3 - 7x^2 - 2x + 12$

**71.** **MULTIPLE CHOICE** $(3x - 8)^3 = \underline{?}$

(A) $27x^3 - 216x^2 + 576x - 512$ (B) $27x^3 - 216x^2 + 576x + 512$

(C) $27x^3 - 72x^2 + 576x - 512$ (D) $27x^3 - 216x^2 + 72x - 512$

## Challenge

**72.** **FINDING A PATTERN** Look at the following polynomials and their factorizations.

$$x^2 - 1 = (x - 1)(x + 1)$$
$$x^3 - 1 = (x - 1)(x^2 + x + 1)$$
$$x^4 - 1 = (x - 1)(x^3 + x^2 + x + 1)$$

**a.** Factor $x^5 - 1$ and $x^6 - 1$. Check your answers by multiplying.

EXTRA CHALLENGE
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**b.** In general, how can $x^n - 1$ be factored? Show that this factorization works by multiplying the factors.

# MIXED REVIEW

**SOLVING QUADRATIC EQUATIONS** Solve the equation. (Review 5.2 for 6.4)

**73.** $4x^2 - 36 = 0$ **74.** $x^2 + 3x - 40 = 0$ **75.** $x^2 + 16x + 64 = 0$

**76.** $x^2 - x - 56 = 0$ **77.** $2x^2 - 7x - 15 = 0$ **78.** $6x^2 + 10x - 4 = 0$

**WRITING QUADRATIC FUNCTIONS** Write a quadratic function in standard form whose graph passes through the given points. (Review 5.8)

**79.** $(-4, 0), (2, 0), (1, 6)$ **80.** $(10, 0), (1, 0), (4, 3)$

**81.** $(-6, 0), (6, 0), (-3, -9)$ **82.** $(-3, 0), (5, 0), (-2, 7)$

**SIMPLIFYING ALGEBRAIC EXPRESSIONS** Simplify the expression. Tell which properties of exponents you used. (Review 6.1)

**83.** $x^5 \cdot \frac{1}{x^2}$ **84.** $\frac{x^4y^5}{xy^3}$ **85.** $-5^{-2}y^0$

**86.** $(4x^{-3})^4 \cdot \left(\frac{x^6}{2}\right)^2$ **87.** $\frac{3x^5y^8}{6xy^{-3}}$ **88.** $\frac{6x^4y^2}{30x^2y^{-1}}$

# QUIZ 1

*Self-Test for Lessons 6.1–6.3*

**Evaluate the expression.** (Lesson 6.1)

**1.** $7^0 \cdot 5^{-3}$ **2.** $\left(\frac{4}{9}\right)^{-2}$ **3.** $\left(\frac{5}{3^2}\right)^2$

**4.** $3^2 \cdot (3^2 \cdot 2^4)^{-1}$ **5.** $(8^2 \cdot 8^{-3})^2 \cdot 8^2$ **6.** $\frac{(2^5 \cdot 3^2)^{-1}}{2^{-2} \cdot 3^2}$

**Simplify the expression.** (Lesson 6.1)

**7.** $(-5)^{-2}y^0$ **8.** $(3x^3y^6)^{-2}$ **9.** $(x^3y^{-5})(x^2y)^2$

**10.** $(x^2y^{-3})(xy^2)$ **11.** $\left(\frac{2x}{y^2}\right)^{-3}$ **12.** $\frac{x^6y^{-2}}{x^{-1}y^5}$

**Graph the polynomial function.** (Lesson 6.2)

**13.** $f(x) = x^4 - 2$ **14.** $f(x) = -2x^5 + 3$ **15.** $f(x) = 3x^3 + 5x - 2$

**16.** $f(x) = -x^3 + x^2 - 2$ **17.** $f(x) = x^3 - 2x$ **18.** $f(x) = -x^4 - 3x + 6$

**Perform the indicated operation.** (Lesson 6.3)

**19.** $(7x^3 + 8x - 11) + (3x^2 - x + 8)$ **20.** $(-2x^2 + 4x) + (5x^2 - x - 11)$

**21.** $(-5x^2 + 12x - 9) - (-7x^2 - 6x - 7)$ **22.** $(3x^2 + 4x - 1) - (-x^3 + 2x + 5)$

**23.** $(x + 5)(4x^2 - x - 1)$ **24.** $(x - 3)(x + 2)(2x + 5)$

**25.** $(x - 6)^3$ **26.** $(2x^2 + 3)^2$

**27.** **ASTRONOMY** Suppose NASA launches a spacecraft that can travel at a speed of 25,000 miles per hour in space. How long would it take the spacecraft to reach Jupiter if Jupiter is about 495,000,000 miles away? Use scientific notation to get your answer. (Lesson 6.1)