9.5 Addition, Subtraction, and Complex Fractions

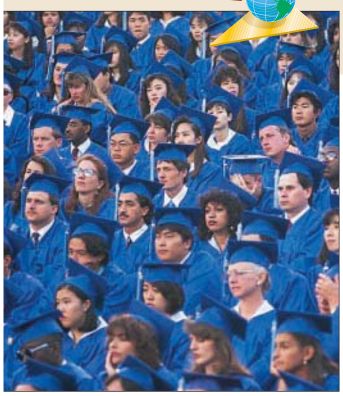
What *you should learn*

GOAL 1 Add and subtract rational expressions, as applied in **Example 4**.

GOAL 2 Simplify complex fractions, as applied in **Example 6**.

Why *you should learn it*

▼ To solve **real-life** problems, such as modeling the total number of male college graduates in **Ex. 47**.



GOAL 1 WORKING WITH RATIONAL EXPRESSIONS

As with numerical fractions, the procedure used to add (or subtract) two rational expressions depends upon whether the expressions have *like* or *unlike* denominators.

To add (or subtract) two rational expressions with *like* denominators, simply add (or subtract) their numerators and place the result over the common denominator.

EXAMPLE 1 *Adding and Subtracting with Like Denominators*

Perform the indicated operation.

a. $\frac{4}{3x} + \frac{5}{3x}$ **b.** $\frac{2x}{x+3} - \frac{4}{x+3}$

SOLUTION

a. $\frac{4}{3x} + \frac{5}{3x} = \frac{4+5}{3x} = \frac{9}{3x} = \frac{3}{x}$ **Add numerators and simplify expression.**

b. $\frac{2x}{x+3} - \frac{4}{x+3} = \frac{2x-4}{x+3}$ **Subtract numerators.**

To add (or subtract) rational expressions with *unlike* denominators, first find the least common denominator (LCD) of the rational expressions. Then, rewrite each expression as an equivalent rational expression using the LCD and proceed as with rational expressions with like denominators.

EXAMPLE 2 *Adding with Unlike Denominators*

Add: $\frac{5}{6x^2} + \frac{x}{4x^2 - 12x}$

SOLUTION

First find the least common denominator of $\frac{5}{6x^2}$ and $\frac{x}{4x^2 - 12x}$.

It helps to factor each denominator: $6x^2 = 6 \cdot x \cdot x$ and $4x^2 - 12x = 4 \cdot x \cdot (x - 3)$.

The LCD is $12x^2(x - 3)$. Use this to rewrite each expression.

$$\frac{5}{6x^2} + \frac{x}{4x^2 - 12x} = \frac{5}{6x^2} + \frac{x}{4x(x-3)} = \frac{5[2(x-3)]}{6x^2[2(x-3)]} + \frac{x(3x)}{4x(x-3)(3x)}$$

$$= \frac{10x - 30}{12x^2(x-3)} + \frac{3x^2}{12x^2(x-3)}$$

$$= \frac{3x^2 + 10x - 30}{12x^2(x-3)}$$

STUDENT HELP

Skills Review
For help with LCDs, see p. 908.

EXAMPLE 3 *Subtracting With Unlike Denominators*

Subtract: $\dfrac{x+1}{x^2+4x+4} - \dfrac{2}{x^2-4}$

SOLUTION

$$\frac{x+1}{x^2+4x+4} - \frac{2}{x^2-4} = \frac{x+1}{(x+2)^2} - \frac{2}{(x-2)(x+2)}$$

$$= \frac{(x+1)(x-2)}{(x+2)^2(x-2)} - \frac{2(x+2)}{(x-2)(x+2)(x+2)}$$

$$= \frac{x^2-x-2-(2x+4)}{(x+2)^2(x-2)}$$

$$= \frac{x^2-3x-6}{(x+2)^2(x-2)}$$

STUDENT HELP

Look Back
For help with multiplying polynomials, see p. 338.

REAL LIFE *Statistics*

EXAMPLE 4 *Adding Rational Models*

The distribution of heights for American men and women aged 20–29 can be modeled by

$$y_1 = \frac{0.143}{1 + 0.008(x-70)^4} \quad \textbf{American men's heights}$$

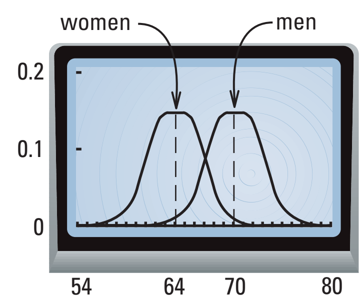
$$y_2 = \frac{0.143}{1 + 0.008(x-64)^4} \quad \textbf{American women's heights}$$

where x is the height (in inches) and y is the percent (in decimal form) of adults aged 20–29 whose height is $x \pm 0.5$ inches. ▶ Source: *Statistical Abstract of the United States*

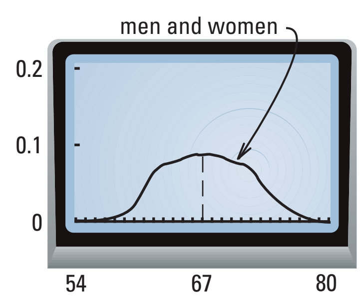
a. Graph each model. What is the most common height for men aged 20–29? What is the most common height for women aged 20–29?

b. Write a model that shows the distribution of the heights of *all* adults aged 20–29. Graph the model and find the most common height.

SOLUTION



a. From the graphing calculator screen shown at the top right, you can see that the most common height for men is 70 inches (14.3%). The second most common heights are 69 inches and 71 inches (14.2% each). For women, the curve has the same shape, but is shifted to the left so that the most common height is 64 inches. The second most common heights are 63 inches and 65 inches.



b. To find a model for the distribution of all adults aged 20–29, add the two models and divide by 2.

$$y = \frac{1}{2}\left[\frac{0.143}{1+0.008(x-70)^4} + \frac{0.143}{1+0.008(x-64)^4}\right]$$

From the graph shown at the bottom right, you can see that the most common height is 67 inches.

GOAL 2 SIMPLIFYING COMPLEX FRACTIONS

A **complex fraction** is a fraction that contains a fraction in its numerator or denominator. To simplify a complex fraction, write its numerator and its denominator as single fractions. Then divide by multiplying by the reciprocal of the denominator.

EXAMPLE 5 *Simplifying a Complex Fraction*

Simplify: $\dfrac{\frac{2}{x+2}}{\frac{1}{x+2}+\frac{2}{x}}$

SOLUTION

$$\dfrac{\frac{2}{x+2}}{\frac{1}{x+2}+\frac{2}{x}} = \dfrac{\frac{2}{x+2}}{\frac{3x+4}{x(x+2)}}$$ **Add fractions in denominator.**

$$= \frac{2}{x+2} \cdot \frac{x(x+2)}{3x+4}$$ **Multiply by reciprocal.**

$$= \frac{2x\cancel{(x+2)}}{\cancel{(x+2)}(3x+4)}$$ **Divide out common factor.**

$$= \frac{2x}{3x+4}$$ **Write in simplified form.**

Another way to simplify a complex fraction is to multiply the numerator and denominator by the least common denominator of *every* fraction in the numerator and denominator.

EXAMPLE 6 *Simplifying a Complex Fraction*

PHOTOGRAPHY The focal length f of a thin camera lens is given by

$$f = \dfrac{1}{\frac{1}{p}+\frac{1}{q}}$$

where p is the distance between an object being photographed and the lens and q is the distance between the lens and the film. Simplify the complex fraction.

SOLUTION

$$f = \dfrac{1}{\frac{1}{p}+\frac{1}{q}}$$ **Write equation.**

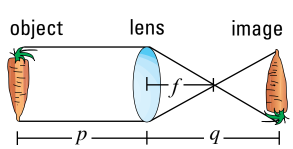
$$= \frac{pq}{pq} \cdot \dfrac{1}{\frac{1}{p}+\frac{1}{q}}$$ **Multiply numerator and denominator by *pq*.**

$$= \frac{pq}{q+p}$$ **Simplify.**

STUDENT HELP

HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

FOCUS ON APPLICATIONS



PHOTOGRAPHY The focal length of a camera lens is the distance between the lens and the point where light rays converge after passing through the lens.

GUIDED PRACTICE

Vocabulary Check ✓

1. Give two examples of a complex fraction.

Concept Check ✓

2. How is adding (or subtracting) rational expressions similar to adding (or subtracting) numerical fractions?
3. Describe two ways to simplify a complex fraction.
4. Why isn't $(x+1)^3$ the LCD of $\frac{1}{x+1}$ and $\frac{1}{(x+1)^2}$? What is the LCD?

Skill Check ✓

Perform the indicated operation and simplify.

5. $\frac{2x}{x+5} + \frac{7}{x+5}$
6. $\frac{7}{5x} + \frac{8}{3x}$
7. $\frac{x}{x-4} - \frac{6}{x+3}$

Simplify the complex fraction.

8. $\dfrac{\frac{x}{5} + 4}{8 + \frac{1}{x}}$
9. $\dfrac{\frac{x+2}{5} - 5}{8 + \frac{4}{x}}$
10. $\dfrac{\frac{15}{2x+2}}{\frac{6}{x} - \frac{1}{2}}$

11. **FINANCE** For a loan paid back over t years, the monthly payment is given by $M = \dfrac{Pi}{1 - \left(\frac{1}{1+i}\right)^{12t}}$ where P is the principal and i is the annual interest rate. Show that this formula is equivalent to $M = \dfrac{Pi(1+i)^{12t}}{(1+i)^{12t} - 1}$.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 953.

OPERATIONS WITH LIKE DENOMINATORS **Perform the indicated operation and simplify.**

12. $\frac{7}{6x} + \frac{11}{6x}$
13. $\frac{23}{10x^2} - \frac{x}{10x^2}$
14. $\frac{4x}{x+1} - \frac{3}{x+1}$
15. $\frac{5x^2}{x+8} + \frac{5x}{x+8}$
16. $\frac{6x^2}{x-2} - \frac{12x}{x-2}$
17. $\frac{x}{x^2-5x} - \frac{5}{x^2-5x}$

FINDING LCDS **Find the least common denominator.**

18. $\frac{14}{4(x+1)}, \frac{7}{4x}$
19. $\frac{4}{21x^2}, \frac{x}{3x^2-15x}$
20. $\frac{5x+2}{4x^2-1}, \frac{3}{x}, \frac{9x}{2x+1}$
21. $\frac{1}{x(x-6)}, \frac{12}{x^2-3x-18}$
22. $\frac{3x+1}{x(x-7)}, \frac{3}{x^2-6x-7}$
23. $\frac{1}{x^2-3x-28}, \frac{x}{x^2+6x+8}$

LOGICAL REASONING **Tell whether the statement is *always true, sometimes true,* or *never true.* Explain your reasoning.**

24. The LCD of two rational expressions is the product of the denominators.
25. The LCD of two rational expressions will have a degree greater than or equal to that of the denominator with the higher degree.

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 12–17
Examples 2, 3: Exs. 18–23, 26–37
Example 4: Exs. 47–51
Example 5: Exs. 38–46
Example 6: Exs. 52, 53

STUDENT HELP

Look Back
For help with the negative exponents in Exs. 41 and 42, see p. 323.

OPERATIONS WITH UNLIKE DENOMINATORS **Perform the indicated operation(s) and simplify.**

26. $\frac{6}{4x^2} + \frac{2}{5x}$

27. $-\frac{4}{7x} - \frac{5}{3x}$

28. $\frac{7}{6(x-2)} - \frac{x+3}{6x}$

29. $\frac{6x+1}{x^2-9} + \frac{4}{x-3}$

30. $\frac{10}{x^2-5x-14} + \frac{2}{x-7}$

31. $\frac{5x-1}{x^2+2x-8} - \frac{6}{x+4}$

32. $\frac{4x^2}{3x+5} - \frac{10}{x+8}$

33. $\frac{2-5x}{x-10} + \frac{1}{3x+2}$

34. $\frac{x^2+x-3}{x^2-12x+32} + \frac{3x}{x-8}$

35. $\frac{2x+1}{x^2+8x+16} - \frac{3}{x^2-16}$

36. $\frac{4x}{x+1} + \frac{5}{2x-3} - \frac{4}{x}$

37. $\frac{10x}{3x^2-3} + \frac{4}{x-1} + \frac{5}{6x}$

SIMPLIFYING COMPLEX FRACTIONS **Simplify the complex fraction.**

38. $\frac{\frac{x}{2}-5}{6+\frac{3}{x}}$

39. $\frac{\frac{20}{x+1}}{\frac{1}{4}-\frac{7}{x+1}}$

40. $\frac{\frac{1}{2x^2-2}}{\frac{2}{x+1}+\frac{x}{x^2-2x-3}}$

41. $\frac{\frac{1}{x}-\frac{x}{x^{-1}+1}}{\frac{3}{x}}$

42. $\frac{\frac{1-x}{x^4}}{x^{-2}-\frac{2}{x^3+x^2}}$

43. $\frac{\frac{1}{4x+3}-\frac{5}{3(4x+3)}}{\frac{x}{4x+3}}$

44. $\frac{\frac{4}{x^2-9}+\frac{2}{x-3}}{\frac{1}{x+3}+\frac{1}{x-3}}$

45. $\frac{\frac{1}{x^3+64}}{\frac{5}{x^2-16}-\frac{2}{3x^2+12x}}$

46. $\frac{\frac{3}{2x^2+6x+18}+\frac{x}{x^3-27}}{\frac{5x}{3x-9}-\frac{3}{x-3}}$

47. **COLLEGE GRADUATES** From the 1984–85 school year through the 1993–94 school year, the number of female college graduates F and the total number of college graduates G in the United States can be modeled by

$$F = \frac{-19{,}600t + 493{,}000}{-0.0580t + 1} \quad \text{and} \quad G = \frac{7560t^2 + 978{,}000}{0.00418t^2 + 1}$$

where t is the number of school years since the 1984–85 school year. Write a model for the number of male college graduates. ▶ Source: U.S. Department of Education

FOCUS ON CAREERS

PHARMACIST
In addition to mixing and dispensing prescription drugs, pharmacists advise patients and physicians on the use of medications. This includes warning of possible side effects and recommending drug dosages, as discussed in Exs. 48–51.

CAREER LINK
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DRUG ABSORPTION **In Exercises 48–51, use the following information.**
The amount A (in milligrams) of an oral drug, such as aspirin, in a person's bloodstream can be modeled by

$$A = \frac{391t^2 + 0.112}{0.218t^4 + 0.991t^2 + 1}$$

where t is the time (in hours) after one dose is taken. ▶ Source: *Drug Disposition in Humans*

48. Graph the equation using a graphing calculator.

49. A second dose of the drug is taken 1 hour after the first dose. Write an equation to model the amount of the second dose in the bloodstream.

50. Write and graph a model for the total amount of the drug in the bloodstream after the second dose is taken.

51. About how long after the second dose has been taken is the greatest amount of the drug in the bloodstream?

ELECTRONICS **In Exercises 52 and 53, use the following information.**
If three resistors in a parallel circuit have resistances R_1, R_2, and R_3 (all in ohms), then the total resistance R_t (in ohms) is given by this formula:

$$R_t = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

52. Simplify the complex fraction.

53. You have three resistors in a parallel circuit with resistances 6 ohms, 12 ohms, and 24 ohms. What is the total resistance of the circuit?

Test Preparation

54. **MULTI-STEP PROBLEM** From 1988 through 1997, the total dollar value V (in millions of dollars) of the United States sound-recording industry can be modeled by

$$V = \frac{5783 + 1134t}{1 + 0.025t}$$

where t represents the number of years since 1988.

▶ Source: Recording Industry Association of America

a. Calculate the percent change in dollar value from 1988 to 1989.

b. Develop a general formula for the percent change in dollar value from year t to year $t + 1$.

c. Enter the formula into a graphing calculator or spreadsheet. Observe the changes from year to year for 1988 through 1997. Describe what you observe from the data.

★ Challenge

CRITICAL THINKING **In Exercises 55 and 56, use the following expressions.**

$$2 + \frac{1}{1 + \frac{1}{2}},\ 2 + \frac{1}{1 + \frac{1}{2 + \frac{2}{3}}},\ 2 + \frac{1}{1 + \frac{1}{2 + \frac{2}{3 + \frac{3}{4}}}}$$

55. The expressions form a pattern. Continue the pattern two more times. Then simplify all five expressions.

56. The expressions are getting closer and closer to some value. What is it?

EXTRA CHALLENGE
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MIXED REVIEW

SOLVING LINEAR EQUATIONS **Solve the equation.** (Review 1.3 for 9.6)

57. $\frac{1}{2}x - 7 = 5$

58. $6 - \frac{1}{10}x = -1$

59. $\frac{3}{4}x + \frac{1}{2} = x - \frac{5}{6}$

60. $\frac{3}{8}x + 4 = -8$

61. $-\frac{1}{12}x - 3 = \frac{5}{2}$

62. $2 = -\frac{4}{3}x + 10$

63. $-5x - \frac{3}{4}x = \frac{51}{2}$

64. $2x + \frac{7}{8}x = -23$

65. $x = 12 + \frac{5}{6}x$

SOLVING QUADRATIC EQUATIONS **Solve the equation.** (Review 5.2, 5.3 for 9.6)

66. $x^2 - 5x - 24 = 0$

67. $5x^2 - 8 = 4(x^2 + 3)$

68. $6x^2 + 13x - 5 = 0$

69. $3(x - 5)^2 = 27$

70. $2(x + 7)^2 - 1 = 49$

71. $2x(x + 6) = 7 - x$