CHAPTER 5

Chapter Summary

WHAT did you learn?

WHY did you learn it?

Graph quadratic functions. (5.1)	Model the suspension cables on the Golden Gate Bridge. (p. 252)
Write quadratic functions in standard, intercept, and vertex forms. (5.1, 5.2, 5.5)	Find the amount of fertilizer that maximizes the sugar yield from sugarbeets. (p. 285)
Find zeros of quadratic functions. (5.2)	Determine what subscription price to charge for a Web site in order to maximize revenue. (p. 259)
Solve quadratic equations.	
• by factoring (5.2)	Calculate dimensions for a mural. (p. 262)
• by finding square roots (5.3)	Find a falling rock's time in the air. (p. 268)
• by completing the square (5.5)	Tell how a firefighter should position a hose. (p. 288)
• by using the quadratic formula (5.6)	Find the speed and duration of a thrill ride. (p. 297)
Perform operations with complex numbers. (5.4)	Determine whether a complex number belongs to the Mandelbrot set. (p. 276)
Find the discriminant of a quadratic equation. (5.6)	Identify the number and type of solutions of a quadratic equation. (p. 293)
Graph quadratic inequalities in two variables. (5.7)	Calculate the weight that a rope can support. (p. 304)
Solve quadratic inequalities in one variable. (5.7)	Relate a driver's age and reaction time. (p. 302)
Find quadratic models for data. (5.8)	Determine the effect of wind on a runner's performance. (p. 311)

How does Chapter 5 fit into the BIGGER PICTURE of algebra?

In Chapter 5 you saw the relationship between the *solutions* of the quadratic equation $ax^2 + bx + c = 0$, the *zeros* of the quadratic function $y = ax^2 + bx + c$, and the *x-intercepts* of this function's graph. You'll continue to see this relationship with other types of functions. Also, the graph of a quadratic function—a parabola—is one of the four conic sections. You'll study all the conic sections in Chapter 10.

STUDY STRATEGY

How did you troubleshoot?

Here is an example of a trouble spot identified and eliminated, following the **Study Strategy** on page 248.

Troubleshoot

Trouble spot: Changing a quadratic function from standard form to vertex form by completing the square.

How to eliminate: Remember to add the same constant to *both* sides of the equation for the function.

Example:

$$y = x^2 + 10x - 3$$
$$y + 25 = (x^2 + 10x + 25) - 3$$
$$y + 25 = (x + 5)^2 - 3$$
$$y = (x + 5)^2 - 28$$

CHAPTER 5

Chapter Review

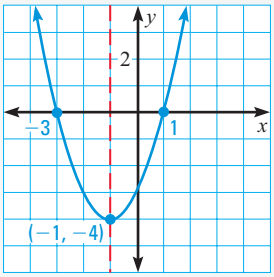
VOCABULARY

- quadratic function, p. 249
- parabola, p. 249
- vertex of a parabola, p. 249
- axis of symmetry, p. 249
- standard form of a quadratic function, p. 250
- vertex form of a quadratic function, p. 250
- intercept form of a quadratic function, p. 250
- binomial, p. 256
- trinomial, p. 256
- factoring, p. 256
- monomial, p. 257
- quadratic equation, p. 257
- standard form of a quadratic equation, p. 257
- zero product property, p. 257
- zero of a function, p. 259
- square root, p. 264
- radical sign, p. 264
- radicand, p. 264
- radical, p. 264
- rationalizing the denominator, p. 265
- imaginary unit i, p. 272
- complex number, p. 272
- standard form of a complex number, p. 272
- imaginary number, p. 272
- pure imaginary number, p. 272
- complex plane, p. 273
- complex conjugates, p. 274
- absolute value of a complex number, p. 275
- completing the square, p. 282
- quadratic formula, p. 291
- discriminant, p. 293
- quadratic inequality, pp. 299, 301
- best-fitting quadratic model, p. 308

5.1 GRAPHING QUADRATIC FUNCTIONS

Examples on pp. 249–252

EXAMPLE You can graph a quadratic function given in standard form, vertex form, or intercept form. For instance, the same function is given below in each of these forms, and its graph is shown.



Standard form: $y = x^2 + 2x - 3$;

axis of symmetry: $x = -\frac{b}{2a} = -\frac{2}{2(1)} = -1$

Vertex form: $y = (x + 1)^2 - 4$; vertex: $(-1, -4)$

Intercept form: $y = (x + 3)(x - 1)$; x-intercepts: -3, 1

Graph the quadratic function.

1. $y = x^2 + 4x + 7$ **2.** $y = -3(x - 2)^2 + 5$ **3.** $y = \frac{1}{2}(x + 1)(x - 5)$

5.2–5.3 SOLVING BY FACTORING AND BY FINDING SQUARE ROOTS

Examples on pp. 256–259, 264–266

EXAMPLES You can use factoring or square roots to solve quadratic equations.

Solving by factoring:

$x^2 - 4x - 21 = 0$

$(x + 3)(x - 7) = 0$

$x + 3 = 0$ or $x - 7 = 0$

$x = -3$ or $x = 7$

Solving by finding square roots:

$4x^2 - 7 = 65$

$4x^2 = 72$

$x^2 = 18$

$x = \pm\sqrt{18} = \pm 3\sqrt{2}$

Solve the quadratic equation.

4. $x^2 + 11x + 24 = 0$ **5.** $x^2 - 8x + 16 = 0$ **6.** $2x^2 + 3x + 1 = 0$

7. $3u^2 = -4u + 15$ **8.** $25v^2 - 30v = -9$ **9.** $2x^2 = 200$

10. $5x^2 - 2 = 13$ **11.** $4(t + 6)^2 = 160$ **12.** $-(k - 1)^2 + 7 = -43$

5.4 COMPLEX NUMBERS

Examples on pp. 272–276

EXAMPLES You can add, subtract, multiply, and divide complex numbers. You can also find the absolute value of a complex number.

Addition: $(1 + 8i) + (2 - 3i) = (1 + 2) + (8 - 3)i = 3 + 5i$

Subtraction: $(1 + 8i) - (2 - 3i) = (1 - 2) + (8 + 3)i = -1 + 11i$

Multiplication: $(1 + 8i)(2 - 3i) = 2 - 3i + 16i - 24i^2 = 2 + 13i - 24(-1) = 26 + 13i$

Division: $\frac{1 + 8i}{2 - 3i} = \frac{1 + 8i}{2 - 3i} \cdot \frac{2 + 3i}{2 + 3i} = \frac{-22 + 19i}{13} = -\frac{22}{13} + \frac{19}{13}i$

Absolute value: $|1 + 8i| = \sqrt{1^2 + 8^2} = \sqrt{65}$

In Exercises 13–16, write the expression as a complex number in standard form.

13. $(7 - 4i) + (-2 + 5i)$ **14.** $(2 + 11i) - (6 - i)$

15. $(3 + 10i)(4 - 9i)$ **16.** $\frac{8 + i}{1 - 2i}$

17. Find the absolute value of $6 + 9i$.

5.5 COMPLETING THE SQUARE

Examples on pp. 282–285

EXAMPLES You can use completing the square to solve quadratic equations and change quadratic functions from standard form to vertex form.

Solving an equation:

$x^2 + 6x + 13 = 0$

$x^2 + 6x = -13$

$x^2 + 6x + 9 = -13 + 9$

$(x + 3)^2 = -4$

$x + 3 = \pm\sqrt{-4}$

$x = -3 \pm 2i$

Writing a function in vertex form:

$y = x^2 + 6x + 13$

$y + \underline{?} = (x^2 + 6x + \underline{?}) + 13$

$y + 9 = (x^2 + 6x + 9) + 13$

$y + 9 = (x + 3)^2 + 13$

$y = (x + 3)^2 + 4$

Note that the vertex is $(-3, 4)$.

Solve the quadratic equation by completing the square.

18. $x^2 + 4x = 3$ **19.** $x^2 - 10x + 26 = 0$ **20.** $2w^2 + w - 7 = 0$ $-\frac{1}{4} + \frac{\sqrt{57}}{4}, -\frac{1}{4} - \frac{\sqrt{57}}{4}$

Write the quadratic function in vertex form and identify the vertex.

21. $y = x^2 - 8x + 17$ **22.** $y = -x^2 - 2x - 6$ **23.** $y = 4x^2 + 16x + 23$

5.6 THE QUADRATIC FORMULA AND THE DISCRIMINANT

Examples on pp. 291–294

EXAMPLE You can use the quadratic formula to solve any quadratic equation.

$$3x^2 - 5x = -1$$

$$3x^2 - 5x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(1)}}{2(3)} = \frac{5 \pm \sqrt{13}}{6}$$

Use the quadratic formula to solve the equation.

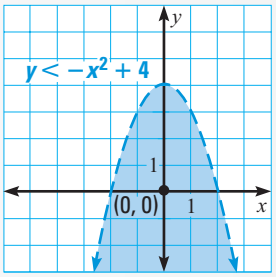
24. $x^2 - 8x + 5 = 0$ **25.** $9x^2 = 1 - 7x$ **26.** $5v^2 + 6v + 7 = v^2 - 4v$

5.7 GRAPHING AND SOLVING QUADRATIC INEQUALITIES

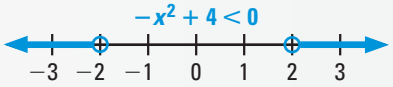
Examples on pp. 299–302

EXAMPLES You can graph a quadratic inequality in two variables and solve a quadratic inequality in one variable.

Graphing an inequality in two variables: To graph $y < -x^2 + 4$, draw the dashed parabola $y = -x^2 + 4$. Test a point inside the parabola, such as (0, 0). Since (0, 0) is a solution of the inequality, shade the region inside the parabola.



Solving an inequality in one variable: To solve $-x^2 + 4 < 0$, graph $y = -x^2 + 4$ and identify the x-values where the graph lies below the x-axis. Or, solve $-x^2 + 4 = 0$ to find the critical x-values -2 and 2, then test an x-value in each interval determined by -2 and 2 to find the solution. The solution is $x < -2$ or $x > 2$.



Graph the quadratic inequality.

27. $y \geq x^2 - 4x + 4$ **28.** $y < x^2 + 6x + 5$ **29.** $y > -2x^2 + 3$

Solve the quadratic inequality.

30. $x^2 - 3x - 4 \leq 0$ **31.** $2x^2 + 7x + 2 \geq 0$ **32.** $9x^2 > 49$

5.8 MODELING WITH QUADRATIC FUNCTIONS

Examples on pp. 306–308

EXAMPLE You can write a quadratic function given characteristics of its graph.

To find a function for the parabola with vertex $(1, -3)$ and passing through $(0, -1)$, use the vertex form $y = a(x - h)^2 + k$ with $(h, k) = (1, -3)$ to write $y = a(x - 1)^2 - 3$. Use the point $(0, -1)$ to find a: $-1 = a(0 - 1)^2 - 3$, so $-1 = a - 3$, and therefore $a = 2$. The function is $y = 2(x - 1)^2 - 3$.

Write a quadratic function whose graph has the given characteristics.

33. vertex: (6, 1)
point on graph: (4, 5)

34. x-intercepts: -4, 3
point on graph: (1, 20)

35. points on graph:
$(-5, 1)$, $(-4, -2)$, $(3, 5)$

CHAPTER 5

Chapter Test

Graph the quadratic function.

1. $y = -2x^2 + 8x - 5$ **2.** $y = (x + 3)^2 + 1$ **3.** $y = -\frac{1}{3}(x + 1)(x - 5)$

4. Write $y = 4(x - 3)^2 - 7$ in standard form.

Factor the expression.

5. $x^2 - x - 20$ **6.** $9x^2 + 6x + 1$ **7.** $3u^2 - 108$

8. Write $y = x^2 - 10x + 16$ in intercept form and give the function's zeros.

9. Simplify the radical expressions $\sqrt{500}$ and $\sqrt{\frac{8}{3}}$.

10. Plot these numbers in the same complex plane: $4 + 2i$, $-5 + i$, and $-3i$.

Write the expression as a complex number in standard form.

11. $(3 + i) + (1 - 5i)$ **12.** $(-4 + 2i) - (7 - 3i)$ **13.** $(8 + i)(6 + 2i)$ **14.** $\frac{9 + 2i}{1 - 4i}$

15. Is $c = -0.5i$ in the Mandelbrot set? Use absolute value to justify your answer.

Find the value of *c* that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

16. $x^2 - 4x + c$ **17.** $x^2 + 11x + c$ **18.** $x^2 - 0.6x + c$

19. Write $y = x^2 + 18x - 4$ in vertex form and identify the vertex.

Solve the quadratic equation using any appropriate method.

20. $7x^2 - 3 = 11$ **21.** $5x^2 - 60x + 180 = 0$ **22.** $4x^2 + 28x - 15 = 0$

23. $m^2 + 8m = -3$ **24.** $3(p - 9)^2 = 81$ **25.** $6t^2 - 2t + 2 = 4t^2 + t$

26. Find the discriminant of $7x^2 - x + 10 = 0$. What does the discriminant tell you about the number and type of solutions of the equation?

Graph the quadratic inequality.

27. $y \geq x^2 + 1$ **28.** $y \leq -x^2 + 4x + 2$ **29.** $y < 2x^2 + 12x + 15$

Solve the quadratic inequality.

30. $-x^2 + x + 6 \geq 0$ **31.** $2x^2 - 9 > 23$ **32.** $x^2 - 7x < -4$

Write a quadratic function whose graph has the given characteristics.

33. vertex: $(-3, 2)$
point on graph: $(-1, -18)$

34. x-intercepts: 1, 8
point on graph: $(2, -2)$

35. points on graph:
$(1, 7)$, $(4, -2)$, $(5, -1)$

36. **WATERFALLS** Niagara Falls in New York is 167 feet high. How long does it take for water to fall from the top to the bottom of Niagara Falls?

37. **INSURANCE** An insurance company charges a 35-year-old nonsmoker an annual premium of \$118 for a \$100,000 term life insurance policy. The premiums for 45-year-old and 55-year-old nonsmokers are \$218 and \$563, respectively. Write a quadratic model for the premium p as a function of age a.