VOCABULARY

ambiguous case (p. 320)
angle of depression (p. 300)
angle of elevation (p. 300)
apothem (p. 300)
arccosine relation (p. 305)
arcsine relation (p. 305)
arctangent relation (p. 305)
circular function (p. 292)
cofunctions (p. 287)
cosecant (pp. 286, 292)
cosine (pp. 285, 291)
cotangent (pp. 286, 292)
coterminal angles (p. 279)
degree (p. 277)
Hero's Formula (p. 330)
hypotenuse (p. 284)
initial side (p. 277)
inverse (p. 305)
Law of Cosines (p. 327)
Law of Sines (p. 313)
leg (p. 284)
minute (p. 277)
quadrantal angle (p. 278)
reference angle (p. 280)
secant (pp. 286, 292)
second (p. 277)
side adjacent (p. 284)
side opposite (p. 284)
sine (pp. 285, 291)
solve a triangle (p. 307)
standard position (p. 277)
tangent (pp. 285, 292)
terminal side (p. 277)
trigonometric function (p. 292)
trigonometric ratio (p. 285)
unit circle (p. 291)
vertex (p. 277)

UNDERSTANDING AND USING THE VOCABULARY

State whether each sentence is true or false. If false, replace the underlined word(s) to make a true statement.

1. An angle of <u>elevation</u> is the angle between a horizontal line and the line of sight from the observer to an object at a lower level.
2. The inverse of the cosine function is the <u>arcsine</u> relation.
3. A degree is subdivided into 60 equivalent parts known as <u>minutes</u>.
4. The leg that is a side of an acute angle of a right triangle is called the side <u>opposite</u> the angle.
5. If the terminal side of an angle θ in standard position intersects the unit circle at $P(x, y)$, the relations $\cos \theta = x$ and $\sin \theta = y$ are called <u>circular functions</u>.
6. Two angles in standard position are called <u>reference</u> angles if they have the same terminal side.
7. <u>Trigonometric ratios</u> are defined by the ratios of right triangles.
8. The <u>Law of Sines</u> is derived from the Pythagorean Theorem.
9. The ray that rotates to form an angle is called the <u>initial side</u>.
10. A circle of radius 1 is called a <u>unit circle</u>.

interNET CONNECTION For additional review and practice for each lesson, visit: **www.amc.glencoe.com**

SKILLS AND CONCEPTS

OBJECTIVES AND EXAMPLES

Lesson 5-1 Identify angles that are coterminal with a given angle.

- If a 585° angle is in standard position, determine a coterminal angle that is between 0° and 360°. State the quadrant in which the terminal side lies.

  First, determine the number of complete rotations (k) by dividing 585 by 360.

  $\frac{585}{360} = 1.625$

  Use $\alpha + 360k°$ to find the value of α.

  $\alpha + 360(1)° = 585°$

  $\alpha = 225°$

  The coterminal angle (α) is 225°. Its terminal side lies in the third quadrant.

REVIEW EXERCISES

Change each measure to degrees, minutes, and seconds.

11. 57.15° **12.** −17.125°

If each angle is in standard position, determine a coterminal angle that is between 0° and 360°. State the quadrant in which the terminal side lies.

13. 860° **14.** 1146°

15. −156° **16.** 998°

17. −300° **18.** 1072°

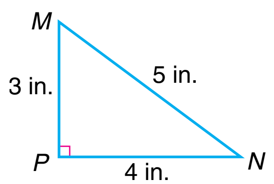
19. 654° **20.** −832°

Find the measure of the reference angle for each angle.

21. −284° **22.** 592°

Lesson 5-2 Find the values of trigonometric ratios for acute angles of right triangles.

- Find the values of the six trigonometric ratios for ∠M.



$\sin M = \frac{4}{5}$ $\quad$ *sin M = side opposite / hypotenuse*

$\cos M = \frac{3}{5}$ $\quad$ *cos M = side adjacent / hypotenuse*

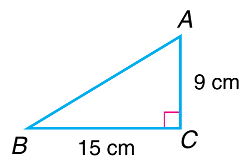
$\tan M = \frac{4}{3}$ $\quad$ *tan M = side opposite / side adjacent*

$\csc M = \frac{5}{4}$ $\quad$ *csc M = hypotenuse / side opposite*

$\sec M = \frac{5}{3}$ $\quad$ *sec M = hypotenuse / side adjacent*

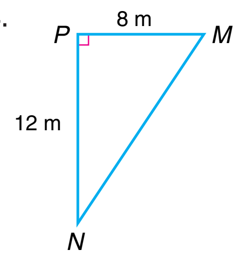
$\cot M = \frac{3}{4}$ $\quad$ *cot M = side adjacent / side opposite*

23. Find the values of the sine, cosine, and tangent for ∠A.

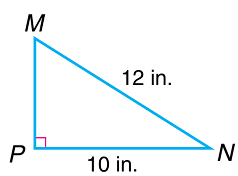


Find the values of the six trigonometric functions for each ∠*M*.

24.



25.



26. If $\sec\theta = \frac{7}{5}$, find $\cos\theta$.

OBJECTIVES AND EXAMPLES

Lesson 5-3 Find the values of the six trigonometric functions of an angle in standard position given a point on its terminal side.

Find the values of the six trigonometric functions for angle θ in standard position if a point with coordinates (3, 4) lies on its terminal side.

$$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = \sqrt{25} \text{ or } 5$$

$$\sin\theta = \frac{y}{r} = \frac{4}{5} \qquad \cos\theta = \frac{x}{r} = \frac{3}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{4}{3} \qquad \csc\theta = \frac{r}{y} = \frac{5}{4}$$

$$\sec\theta = \frac{r}{x} = \frac{5}{3} \qquad \sin\theta = \frac{x}{y} = \frac{3}{4}$$

REVIEW EXERCISES

Find the values of the six trigonometric functions for each angle θ in standard position if a point with the given coordinates lies on its terminal side.

27. (3, 3)
28. (−5, 12)
29. (8, −2)
30. (−2, 0)
31. (4, 5)
32. (−5, −9)
33. (−4, 4)
34. (5, 0)

Suppose θ is an angle in standard position whose terminal side lies in the given quadrant. For each function, find the values of the remaining five trigonometric functions for θ.

35. $\cos\theta = -\frac{3}{8}$; Quadrant II

36. $\tan\theta = 3$; Quadrant III

Lesson 5-4 Use trigonometry to find the measures of the sides of right triangles.

Refer to $\triangle ABC$ at the right. If $A = 25°$ and $b = 12$, find c.

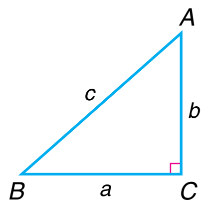
$$\cos A = \frac{b}{c}$$

$$\cos 25° = \frac{12}{c}$$

$$c = \frac{12}{\cos 25°}$$

$$c \approx 13.2$$

Solve each problem. Round to the nearest tenth.



37. If $B = 42°$ and $c = 15$, find b.
38. If $A = 38°$ and $a = 24$, find c.
39. If $B = 67°$ and $b = 24$, find a.

Lesson 5-5 Solve right triangles.

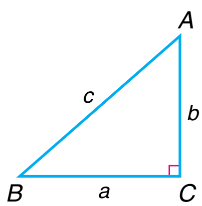
If $c = 10$ and $a = 9$, find A.

$$\sin A = \frac{a}{c}$$

$$\sin A = \frac{9}{10}$$

$$A = \sin^{-1}\frac{9}{10}$$

$$A \approx 64.2°$$



Solve each equation if $0° \leq x \leq 360°$.

40. $\tan\theta = \frac{\sqrt{3}}{3}$
41. $\cos\theta = -1$

Refer to $\triangle ABC$ at the left. Solve each triangle described. Round to the nearest tenth if necessary.

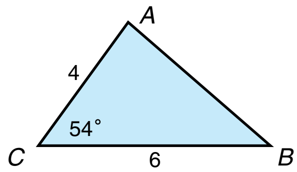
42. $B = 49°$, $a = 16$
43. $b = 15$, $c = 20$
44. $A = 64°$, $c = 28$

OBJECTIVES AND EXAMPLES

Lesson 5-6 Find the area of a triangle if the measures of two sides and the included angle or the measures of two angles and a side are given.

- Find the area of $\triangle ABC$ if $a = 6$, $b = 4$, and $C = 54°$.

Draw a diagram.



$K = \frac{1}{2}ab \sin C$

$K = \frac{1}{2}(6)(4) \sin 54°$

$K \approx 9.708203932$

The area of $\triangle ABC$ is about 9.7 square units.

REVIEW EXERCISES

Solve each triangle. Round to the nearest tenth.

45. $B = 70°$, $C = 58°$, $a = 84$

46. $c = 8$, $C = 49°$, $B = 57°$

Find the area of each triangle. Round to the nearest tenth.

47. $A = 20°$, $a = 19$, $C = 64°$

48. $b = 24$, $A = 56°$, $B = 78°$

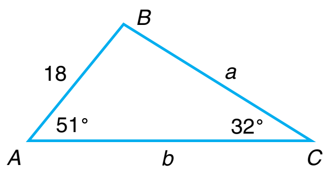
49. $b = 65.5$, $c = 89.4$, $A = 58.2°$

50. $B = 22.6°$, $a = 18.4$, $c = 6.7$

Lesson 5-7 Solve triangles by using the Law of Sines.

- In $\triangle ABC$, if $A = 51°$, $C = 32°$, and $c = 18$, find a.

Draw a diagram.



$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 51°} = \frac{18}{\sin 32°}$$

$$a = \frac{(\sin 51°)18}{\sin 32°}$$

$$a \approx 26.4$$

Find all solutions for each triangle. If no solutions exist, write *none*. Round to the nearest tenth.

51. $A = 38.7°$, $a = 172$, $c = 203$

52. $a = 12$, $b = 19$, $A = 57°$

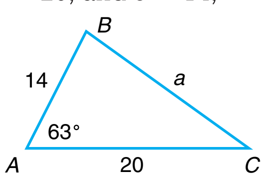
53. $A = 29°$, $a = 12$, $c = 15$

54. $A = 45°$, $a = 83$, $b = 79$

Lesson 5-8 Solve triangles by using the Law of Cosines.

- In $\triangle ABC$, if $A = 63°$, $b = 20$, and $c = 14$, find a.

Draw a diagram.



$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 20^2 + 14^2 - 2(20)(14) \cos 63°$

$a^2 \approx 341.77$

$a \approx 18.5$

Solve each triangle. Round to the nearest tenth.

55. $A = 51°$, $b = 40$, $c = 45$

56. $B = 19°$, $a = 51$, $c = 61$

57. $a = 11$, $b = 13$, $c = 20$

58. $B = 24°$, $a = 42$, $c = 6.5$

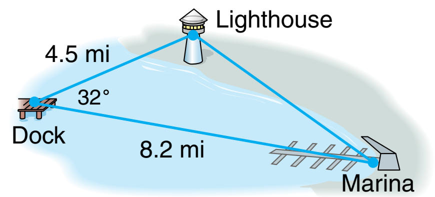
APPLICATIONS AND PROBLEM SOLVING

59. Camping Haloke and his friends are camping in a tent. Each side of the tent forms a right angle with the ground. The tops of two ropes are attached to each side of the tent 8 feet above the ground. The other ends of the two ropes are attached to stakes on the ground. *(Lesson 5-4)*

a. If the rope is 12 feet long, what angle does it make with the level ground?

b. What is the distance between the bottom of the tent and each stake?

60. Navigation Hugo is taking a boat tour of a lake. The route he takes is shown on the map below. *(Lesson 5-8)*



a. How far is it from the lighthouse to the marina?

b. What is the angle between the route from the dock to the lighthouse and the route from the lighthouse to the marina?

ALTERNATIVE ASSESSMENT

OPEN-ENDED ASSESSMENT

1. A triangle has an area of 125 square centimeters and an angle that measures 35°. What are possible lengths of two sides of the triangle?
2. **a.** Give the lengths of two sides and a nonincluded angle so that no triangle exists. Explain why no triangle exists for the measures you give.

   b. Can you change the length of one of the sides you gave in part a so that two triangles exist? Explain.

PORTFOLIO

Explain how you can find the area of a triangle when you know the length of all three sides of the triangle.

Additional Assessment See p. A60 for Chapter 5 practice test.

Unit 2 interNET Project

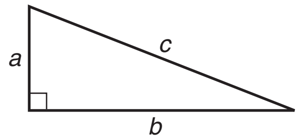
THE CYBERCLASSROOM

Does anybody out there know anything about trigonometry?

- Search the Internet to find at least three web sites that offer lessons on trigonometry. Some possible sites are actual mathematics courses offered on the Internet or webpages designed by teachers.
- Compare the Internet lessons with the lessons from this chapter. Note any similarities or differences.
- Select one topic from Chapter 5. Combine the information from your textbook and the lessons you found on the Internet. Write a summary of this topic using all the information you have gathered.

Pythagorean Theorem

All SAT and ACT tests contain several problems that you can solve using the Pythagorean Theorem. The **Pythagorean Theorem** states that in a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.



$$a^2 + b^2 = c^2$$

THE PRINCETON REVIEW

TEST-TAKING TIP

The 3-4-5 right triangle and its multiples like 6-8-10 and 9-12-15 occur frequently on the SAT and ACT. Other commonly used Pythagorean triples include 5-12-13 and 7-24-25. Memorize them.

SAT EXAMPLE

1. A 25-foot ladder is placed against a vertical wall of a building with the bottom of the ladder standing on concrete 7 feet from the base of the building. If the top of the ladder slips down 4 feet, then the bottom of the ladder will slide out how many feet?

A 4 ft

B 5 ft

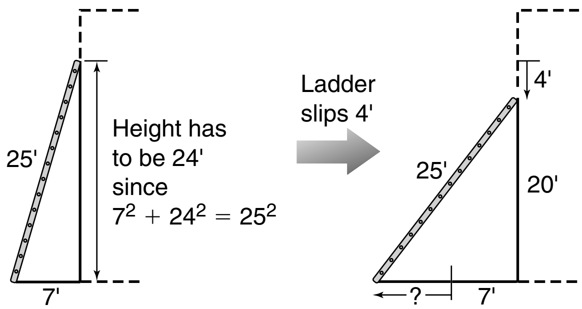
C 6 ft

D 7 ft

E 8 ft

HINT This problem does not have a diagram. So, start by drawing diagrams.

Solution The ladder placed against the wall forms a 7-24-25 right triangle. After the ladder slips down 4 feet, the new right triangle has sides that are multiples of a 3-4-5 right triangle, 15-20-25.



The ladder is now 15 feet from the wall. This means the ladder slipped 15 − 7 or 8 feet. The correct answer is choice **E.**

ACT EXAMPLE

2. In the figure below, right triangles ABC and ACD are drawn as shown. If $AB = 20$, $BC = 15$, and $AD = 7$, then $CD = ?$

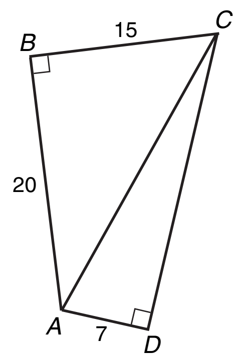
A 21

B 22

C 23

D 24

E 25



HINT Be on the lookout for problems like this one in which the application of the Pythagorean Theorem is not obvious.

Solution Notice that quadrilateral $ABCD$ is separated into two right triangles, $\triangle ABC$ and $\triangle ADC$.

$\triangle ABC$ is a 15-20-25 right triangle (a multiple of the 3-4-5 right triangle). So, side $\overline{AC}$ (the hypotenuse) is 25 units long.

$\overline{AC}$ is also the hypotenuse of $\triangle ADC$. So, $\triangle ADC$ is a 7-24-25 right triangle.

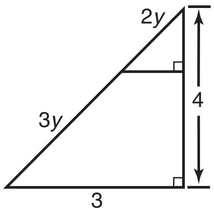
Therefore, $\overline{CD}$ is 24 units long. The correct answer is choice **D.**

SAT AND ACT PRACTICE

After working each problem, record the correct answer on the answer sheet provided or use your own paper.

Multiple Choice

1. In the figure below, $y =$



A 1 B 2 C 3 D 4 E 5

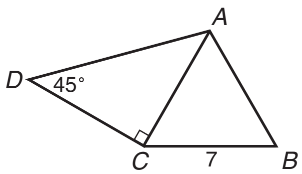
2. What graph would be created if the equation $x^2 + y^2 = 12$ were graphed in the standard (x, y) coordinate plane?

A circle B ellipse
C parabola D straight line
E 2 rays forming a "V"

3. If $999 \times 111 = 3 \times 3 \times n^2$, then which of the following could equal n?

A 9 B 37 C 111
D 222 E 333

4. In the figure below, $\triangle ABC$ is an equilateral triangle with $\overline{BC}$ 7 units long. If $\angle DCA$ is a right angle and $\angle D$ measures 45°, what is the length of $\overline{AD}$ in units?



A 7 B $7\sqrt{2}$ C 14 D $14\sqrt{2}$
E It cannot be determined from the information given.

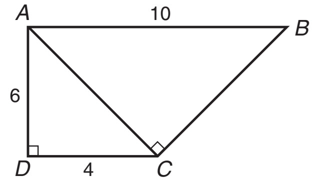
5. If $4 < a < 7 < b < 9$, then which of the following best defines $\frac{a}{b}$?

A $\frac{4}{9} < \frac{a}{b} < 1$ B $\frac{4}{9} < \frac{a}{b} < \frac{7}{9}$
C $\frac{4}{7} < \frac{a}{b} < \frac{7}{9}$ D $\frac{4}{7} < \frac{a}{b} < 1$
E $\frac{4}{7} < \frac{a}{b} < \frac{9}{7}$

6. A swimming pool with a capacity of 36,000 gallons originally contained 9,000 gallons of water. At 10:00 A.M. water begins to flow into the pool at a constant rate. If the pool is exactly three-fourths full at 1:00 P.M. on the same day and the water continues to flow at the same rate, what is the earliest time when the pool will be completely full?

A 1:40 P.M. B 2:00 P.M. C 2:30 P.M.
D 3:00 P.M. E 3:30 P.M.

7. In the figure below, what is the length of $\overline{BC}$?

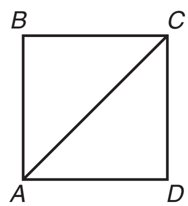


A 6 B $4\sqrt{3}$ C $2\sqrt{13}$
D 8 E $2\sqrt{38}$

8. If $\frac{x^2 + 7x + 12}{x + 4} = 5$, then $x =$

A 1 B 2 C 3 D 5 E 6

9. What is the value of $\frac{AC}{AD}$ if $ABCD$ is a square?



A 1
B $\sqrt{2}$
C $\sqrt{3}$
D 2
E $2\sqrt{2}$

10. **Grid-In** Segment AB is perpendicular to segment BD. Segment AB and segment CD bisect each other at point X. If $AB = 8$ and $CD = 10$, what is the length of BD?

interNET CONNECTION **SAT/ACT Practice** For additional test practice questions, visit: **www.amc.glencoe.com**