VOCABULARY

- amplitude (p. 368)
- angular displacement (p. 352)
- angular velocity (p. 352)
- central angle (p. 345)
- circular arc (p. 345)
- compound function (p. 382)
- dimensional analysis (p. 353)
- frequency (p. 372)
- linear velocity (p. 353)
- midline (p. 380)
- period (p. 359)
- periodic (p. 359)
- phase shift (p. 378)
- principal values (p. 406)
- radian (p. 343)
- sector (p. 346)
- sinusoidal function (p. 388)

UNDERSTANDING AND USING THE VOCABULARY

Choose the correct term to best complete each sentence.

1. The (degree, radian) measure of an angle is defined as the length of the corresponding arc on the unit circle.
2. The ratio of the change in the central angle to the time required for the change is known as (angular, linear) velocity.
3. If the values of a function are (different, the same) for each given interval of the domain, the function is said to be periodic.
4. The (amplitude, period) of a function is one-half the difference of the maximum and minimum function values.
5. A central (angle, arc) has a vertex that lies at the center of a circle.
6. A horizontal translation of a trigonometric function is called a (phase, period) shift.
7. The length of a circular arc equals the measure of the radius of the circle times the (degree, radian) measure of the central angle.
8. The period and the (amplitude, frequency) are reciprocals of each other.
9. A function of the form $y = A \sin (k\theta + c) + h$ is a (sinusoidal, compound) function.
10. The values in the (domain, range) of Sine are called principal values.

interNET CONNECTION For additional review and practice for each lesson, visit: **www.amc.glencoe.com**

SKILLS AND CONCEPTS

OBJECTIVES AND EXAMPLES

Lesson 6-1 Change from radian measure to degree measure, and vice versa.

- Change $-\frac{5\pi}{3}$ radians to degree measure.

$$-\frac{5\pi}{3} = \frac{5\pi}{3} \times \frac{180°}{\pi}$$
$$= -300°$$

REVIEW EXERCISES

Change each degree measure to radian measure in terms of π.

11. 60° **12.** −75° **13.** 240°

Change each radian measure to degree measure. Round to the nearest tenth, if necessary.

14. $\frac{5\pi}{6}$ **15.** $-\frac{7\pi}{4}$ **16.** 2.4

Lesson 6-1 Find the length of an arc given the measure of the central angle.

- Given a central angle of $\frac{2\pi}{3}$, find the length of its intercepted arc in a circle of radius 10 inches. Round to the nearest tenth.

$s = r\theta$

$s = 10\left(\frac{2\pi}{3}\right)$

$s \approx 20.94395102$

The length of the arc is about 20.9 inches.

Given the measurement of a central angle, find the length of its intercepted arc in a circle of radius 15 centimeters. Round to the nearest tenth.

17. $\frac{3\pi}{4}$ **18.** 75°

19. 150° **20.** $\frac{\pi}{5}$

Lesson 6-2 Find linear and angular velocity.

- Determine the angular velocity if 5.2 revolutions are completed in 8 seconds. Round to the nearest tenth.

The angular displacement is $5.2 \times 2\pi$ or 10.4π radians.

$\omega = \frac{\theta}{t}$

$\omega = \frac{10.4\pi}{8}$

$\omega \approx 4.08407045$

The angular velocity is about 4.1 radians per second.

Determine each angular displacement in radians. Round to the nearest tenth.

21. 5 revolutions

22. 3.8 revolutions

23. 50.4 revolutions

24. 350 revolutions

Determine each angular velocity. Round to the nearest tenth.

25. 1.8 revolutions in 5 seconds

26. 3.6 revolutions in 2 minutes

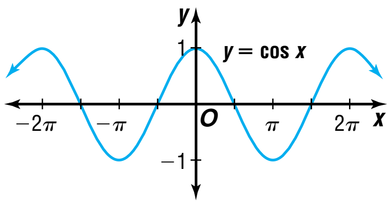
27. 15.4 revolutions in 15 seconds

28. 50 revolutions in 12 minutes

OBJECTIVES AND EXAMPLES

Lesson 6-3 Use the graphs of the sine and cosine functions.

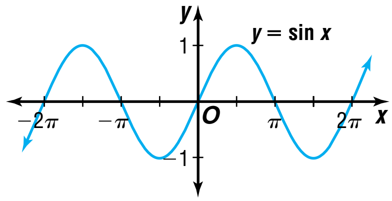
Find the value of $\cos \frac{5\pi}{2}$ by referring to the graph of the cosine function.



$\frac{5\pi}{2} = 2\pi + \frac{\pi}{2}$, so $\cos \frac{5\pi}{2} = \cos \frac{\pi}{2}$ or 0.

REVIEW EXERCISES

Find each value by referring to the graph of the cosine function shown at the left or sine function shown below.



29. $\cos 5\pi$

30. $\sin 13\pi$

31. $\sin \frac{9\pi}{2}$

32. $\cos \left(-\frac{7\pi}{2}\right)$

Lesson 6-4 Find the amplitude and period for sine and cosine functions.

State the amplitude and period for $y = -\frac{3}{4} \cos 2\theta$.

The amplitude of $y = A \cos k\theta$ is $|A|$. Since $A = -\frac{3}{4}$, the amplitude is $\left|-\frac{3}{4}\right|$ or $\frac{3}{4}$.

Since $k = 2$, the period is $\frac{2\pi}{2}$ or π.

State the amplitude and period for each function. Then graph each function.

33. $y = 4 \cos 2\theta$

34. $y = 0.5 \sin 4\theta$

35. $y = -\frac{1}{3} \cos \frac{\theta}{2}$

Lesson 6-5 Write equations of sine and cosine functions, given the amplitude, period, phase shift, and vertical translation.

Write an equation of a cosine function with an amplitude 2, period 2π, phase shift $-\pi$, and vertical shift 2.

A: $|A| = 2$, so $A = 2$ or -2.

k: $\frac{2\pi}{k} = 2\pi$, so $k = 1$.

c: $-\frac{c}{k} = -\pi$, so $-c = -\pi$ or $c = \pi$.

h: $h = 2$

Substituting into $y = A \sin (k\theta + c) + h$, the possible equations are $y = \pm 2 \cos (\theta + \pi) + 2$.

36. Write an equation of a sine function with an amplitude 4, period $\frac{\pi}{2}$, phase shift -2π, and vertical shift -1.

37. Write an equation of a sine function with an amplitude 0.5, period π, phase shift $\frac{\pi}{3}$, and vertical shift 3.

38. Write an equation of a cosine function with an amplitude $\frac{3}{4}$, period $\frac{\pi}{4}$, phase shift 0, and vertical shift 5.

OBJECTIVES AND EXAMPLES

Lesson 6-6 Use sinusoidal functions to solve problems.

- A sinsusoidal function can be any function of the form

  $y = A \sin (k\theta + c) + h$ or

  $y = A \cos (k\theta + c) + h$.

REVIEW EXERCISES

Suppose a person's blood pressure oscillates between the two numbers given. If the heart beats once every second, write a sine function that models this person's blood pressure.

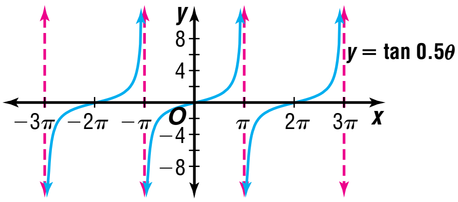
39. 120 and 80

40. 130 and 100

Lesson 6-7 Graph tangent, cotangent, secant, and cosecant functions.

- Graph $y = \tan 0.5\theta$.

  The period of this function is 2π. The phase shift is 0, and the vertical shift is 0.



Graph each function.

41. $y = \frac{1}{3} \csc \theta$

42. $y = 2 \tan \left(3\theta + \frac{\pi}{2}\right)$

43. $y = \sec \theta + 4$

44. $y = \tan \theta - 2$

Lesson 6-8 Find the principal values of inverse trigonometric functions.

- Find $\cos (\text{Tan}^{-1} 1)$.

  Let $\alpha = \text{Tan}^{-1} 1$.

  $\text{Tan } \alpha = 1$

  $\alpha = \frac{\pi}{4}$

  $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

Find each value.

45. $\text{Arctan} \left(-1\right)$

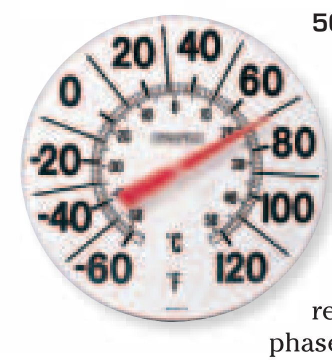
46. $\text{Sin}^{-1} 1$

47. $\text{Cos}^{-1} \left(\tan \frac{\pi}{4}\right)$

48. $\sin \left(\text{Sin}^{-1} \frac{\sqrt{3}}{2}\right)$

49. $\cos \left(\text{Arctan} \sqrt{3} + \text{Arcsin} \frac{1}{2}\right)$

APPLICATIONS AND PROBLEM SOLVING



50. **Meteorology** The mean average temperature in a certain town is 64°F. The temperature fluctuates 11.5° above and below the mean temperature. If $t = 1$ represents January, the phase shift of the sine function is 3. *(Lesson 6-6)*

a. Write a model for the average monthly temperature in the town.

b. According to your model, what is the average temperature in April?

c. According to your model, what is the average temperature in July?

51. **Physics** The strength of a magnetic field is called magnetic induction. An equation for *magnetic induction* is $B = \frac{F}{IL \sin \theta}$, where F is a force on a current I which is moving through a wire of length L at an angle θ to the magnetic field. A wire within a magnetic field is 1 meter long and carries a current of 5.0 amperes. The force on the wire is 0.2 newton, and the magnetic induction is 0.04 newton per ampere-meter. What is the angle of the wire to the magnetic field? *(Lesson 6-8)*

ALTERNATIVE ASSESSMENT

OPEN-ENDED ASSESSMENT

1. The area of a circular sector is about 26.2 square inches. What are possible measures for the radius and the central angle of the sector?

2. a. You are given the graph of a cosine function. Explain how you can tell if the graph has been translated. Sketch two graphs as part of your explanation.

   b. You are given the equation of a cosine function. Explain how you can tell if the graph has been translated. Provide two equations as part of your explanation.

Unit 2 *inter*NET Project

THE CYBERCLASSROOM

What Is Your Sine?

- Search the Internet to find web sites that have applications of the sine or cosine function. Find at least three different sources of information.
- Select one of the applications of the sine or cosine function. Use the Internet to find actual data that can be modeled by a graph that resembles the sine or cosine function.
- Draw a sine or cosine model of the data. Write an equation for a sinusoidal function that fits your data.

PORTFOLIO

Choose a trigonometric function you studied in this chapter. Graph your function. Write three expressions whose values can be found using your graph. Find the values of these expressions.

Additional Assessment See p. A61 for Chapter 6 practice test.

Trigonometry Problems

Each ACT exam contains exactly four trigonometry problems. The SAT has none! You'll need to know the trigonometric functions in a right triangle.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Review the reciprocal functions.

$$\csc \theta = \frac{1}{\sin \theta} \qquad \sec \theta = \frac{1}{\cos \theta} \qquad \cot \theta = \frac{1}{\tan \theta}$$

Review the graphs of trigonometric functions.

THE PRINCETON REVIEW

TEST-TAKING TIP

Use the memory aid SOH-CAH-TOA. Pronounce it as *so-ca-to-a.*

SOH represents **S**ine (is) **O**pposite (over) **H**ypotenuse

CAH represents **C**osine (is) **A**djacent (over) **H**ypotenuse

TOA represents **T**angent (is) **O**pposite (over) **A**djacent

ACT EXAMPLE

1. If $\sin \theta = \frac{1}{2}$ and $90° < \theta < 180°$, then $\theta = ?$

A 100°

B 120°

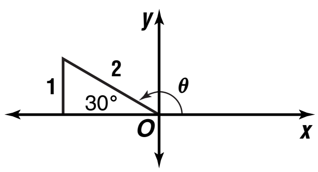
C 130°

D 150°

E 160°

HINT Memorize the sine, cosine, and tangent of special angles 0°, 30°, 45°, 60°, and 90°.

Solution Draw a diagram. Use the quadrant indicated by the size of angle θ.



Recall that the $\sin 30° = \frac{1}{2}$. The angle inside the triangle is 30°. Then $\theta + 30° = 180°$.

If $\theta + 30° = 180°$, then $\theta = 150°$.

The answer is choice **D**.

ACT EXAMPLE

2. What is the least positive value for x where $y = \sin 4x$ reaches its maximum?

A $\frac{\pi}{8}$

B $\frac{\pi}{4}$

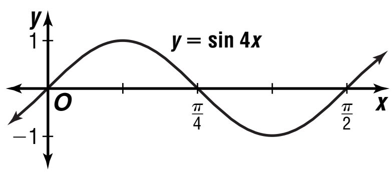
C $\frac{\pi}{2}$

D π

E 2π

HINT Review the graphs of the sine and cosine functions.

Solution The least value for x where $y = \sin x$ reaches its maximum is $\frac{\pi}{2}$. If $4x = \frac{\pi}{2}$, then $x = \frac{\pi}{8}$. The answer is choice **A**.



SAT AND ACT PRACTICE

After you work each problem, record your answer on the answer sheet provided or on a piece of paper.

Multiple Choice

1. What is $\sin \theta$, if $\tan \theta = \frac{4}{3}$?

A $\frac{3}{4}$ **B** $\frac{4}{5}$

C $\frac{5}{4}$ **D** $\frac{5}{3}$

E $\frac{7}{3}$

2. If the sum of two consecutive odd integers is 56, then the greater integer equals:

A 25 **B** 27 **C** 29

D 31 **E** 33

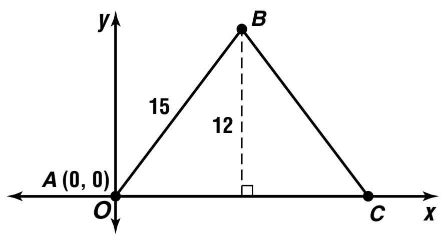
3. For all θ where $\sin \theta - \cos \theta \neq 0$, $\frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta - \cos \theta}$ is equivalent to

A $\sin \theta - \cos \theta$ **B** $\sin \theta + \cos \theta$

C $\tan \theta$ **D** -1

E 1

4. In the figure below, side AB of triangle ABC contains which point?



A (3, 2) **B** (3, 5)

C (4, 6) **D** (4, 10)

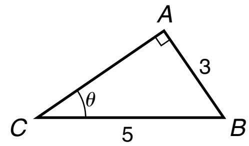
E (6, 8)

5. Which of the following is the sum of both solutions of the equation $x^2 - 2x - 8 = 0$?

A -6 **B** -4 **C** -2

D 2 **E** 6

6. In the figure below, $\angle A$ is a right angle, AB is 3 units long, and BC is 5 units long. If $\angle C = \theta$, what is the value of $\cos \theta$?



A $\frac{3}{5}$ **B** $\frac{3}{4}$ **C** $\frac{4}{5}$ **D** $\frac{5}{4}$ **E** $\frac{5}{3}$

7. The equation $x - 7 = x^2 + y$ represents which conic?

A parabola **B** circle **C** ellipse

D hyperbola **E** line

8. If n is an integer, then which of the following must also be integers?

I. $\frac{16n + 16}{n + 1}$

II. $\frac{16n + 16}{16n}$

III. $\frac{16n^2 + n}{16n}$

A I only **B** II only **C** III only

D I and II **E** II and III

9. For $x > 1$, which expression has a value that is less than 1?

A x^{x-1}

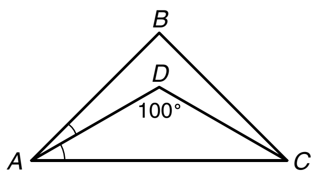
B x^{x+2}

C $(x + 2)^x$

D x^{1-x}

E x^x

10. Grid-In In the figure, segment AD bisects $\angle BAC$, and segment DC bisects $\angle BCA$. If the measure of $\angle ADC = 100°$, then what is the measure of $\angle B$?



interNET CONNECTION **SAT/ACT Practice** For additional test practice questions, visit: **www.amc.glencoe.com**