VOCABULARY

UNDERSTANDING AND USING THE VOCABULARY

Choose the letter of the term that best matches each equation or phrase.

1. $\sin \frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos \alpha}{2}}$
2. perpendicular to a line, curve, or surface
3. located in Quadrants I and IV for $\sin x$ and $\tan x$
4. $\cos (\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$
5. $\cot \theta = \frac{1}{\tan \theta}$
6. $\frac{\sin \theta}{\cos \theta} = \tan \theta$
7. $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$
8. $\sin (360k^\circ - A) = -\sin A$
9. $1 + \cot^2 \theta = \csc^2 \theta$
10. uses trigonometry to provide information about a line

a. sum identity
b. half-angle identity
c. normal form
d. principal value
e. Pythagorean identity
f. symmetry identity
g. normal line
h. double-angle identity
i. reciprocal identity
j. quotient identity
k. opposite-angle identity

SKILLS AND CONCEPTS

OBJECTIVES AND EXAMPLES

Lesson 7-1 Identify and use reciprocal identities, quotient identities, Pythagorean identities, symmetry identities, and opposite-angle identities.

- If θ is in Quadrant I and $\cos\theta = \frac{1}{3}$, find $\sin\theta$.

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta + \left(\frac{1}{3}\right)^2 = 1$$
$$\sin^2\theta + \frac{1}{9} = 1$$
$$\sin^2\theta = \frac{8}{9}$$
$$\sin\theta = \frac{2\sqrt{2}}{3}$$

REVIEW EXERCISES

Use the given information to determine the trigonometric value. In each case, $0° < \theta < 90°$.

11. If $\sin\theta = \frac{1}{2}$, find $\csc\theta$.

12. If $\tan\theta = 4$, find $\sec\theta$.

13. If $\csc\theta = \frac{5}{3}$, find $\cos\theta$.

14. If $\cos\theta = \frac{4}{5}$, find $\tan\theta$.

15. Simplify $\csc x - \cos^2 x \csc x$.

Lesson 7-2 Use the basic trigonometric identities to verify other identities.

- Verify that $\csc x \sec x = \cot x + \tan x$ is an identity.

$$\csc x \sec x \stackrel{?}{=} \cot x + \tan x$$
$$\frac{1}{\sin x} \cdot \frac{1}{\cos x} \stackrel{?}{=} \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$$
$$\frac{1}{\sin x \cos x} \stackrel{?}{=} \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$$
$$\frac{1}{\sin x \cos x} = \frac{1}{\sin x \cos x}$$

Verify that each equation is an identity.

16. $\cos^2 x + \tan^2 x \cos^2 x = 1$

17. $\dfrac{1 - \cos\theta}{1 + \cos\theta} = (\csc\theta - \cot\theta)^2$

18. $\dfrac{\sec\theta + 1}{\tan\theta} = \dfrac{\tan\theta}{\sec\theta - 1}$

19. $\dfrac{\sin^4 x - \cos^4 x}{\sin^2 x} = 1 - \cot^2 x$

Lesson 7-3 Use the sum and difference identities for the sine, cosine, and tangent functions.

- Find the exact value of $\sin 105°$.

$$\begin{aligned}\sin 105° &= \sin(60° + 45°)\\ &= \sin 60° \cos 45° + \cos 60° \sin 45°\\ &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}\\ &= \frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$

Use sum or difference identities to find the exact value of each trigonometric function.

20. $\cos 195°$

21. $\cos 15°$

22. $\sin\left(-\frac{17\pi}{12}\right)$

23. $\tan\frac{11\pi}{12}$

Find each exact value if $0 < x < \frac{\pi}{2}$ and $0 < y < \frac{\pi}{2}$.

24. $\cos(x - y)$ if $\sin x = \frac{7}{25}$ and $\cos y = \frac{2}{3}$

25. $\tan(x + y)$ if $\tan x = \frac{5}{4}$ and $\sec y = \frac{3}{2}$

OBJECTIVES AND EXAMPLES

Lesson 7-4 Use the double- and half-angle identities for the sine, cosine, and tangent functions.

- If θ is an angle in the first quadrant and $\sin \theta = \frac{3}{4}$, find $\cos 2\theta$.

$$\begin{aligned}\cos 2\theta &= 1 - 2\sin^2 \theta\\ &= 1 - 2\left(\frac{3}{4}\right)^2\\ &= -\frac{1}{8}\end{aligned}$$

REVIEW EXERCISES

Use a half-angle identity to find the exact value of each function.

26. $\cos 75°$

27. $\sin \frac{7\pi}{8}$

28. $\sin 22.5°$

29. $\tan \frac{\pi}{12}$

If θ is an angle in the first quadrant and $\cos \theta = \frac{3}{5}$, find the exact value of each function.

30. $\sin 2\theta$

31. $\cos 2\theta$

32. $\tan 2\theta$

33. $\sin 4\theta$

OBJECTIVES AND EXAMPLES

Lesson 7-5 Solve trigonometric equations and inequalities.

- Solve $2\cos^2 x - 1 = 0$ for $0° \le x < 360°$.

$$\begin{aligned}2\cos^2 x - 1 &= 0\\ \cos^2 x &= \frac{1}{2}\\ \cos x &= \pm\frac{\sqrt{2}}{2}\\ x &= 45°, 135°, 225°, \text{ or } 315°\end{aligned}$$

REVIEW EXERCISES

Solve each equation for $0° \le x < 360°$.

34. $\tan x + 1 = \sec x$

35. $\sin^2 x + \cos 2x - \cos x = 0$

36. $\cos 2x + \sin x = 1$

Solve each equation for all real values of x.

37. $\sin x \tan x - \frac{\sqrt{2}}{2}\tan x = 0$

38. $\sin 2x + \sin x = 0$

39. $\cos^2 x = 2 - \cos x$

OBJECTIVES AND EXAMPLES

Lesson 7-6 Write linear equations in normal form.

- Write $3x + 2y - 6 = 0$ in normal form. Since C is negative, use the positive value of $\sqrt{A^2 + B^2}$. $\sqrt{3^2 + 2^2} = \sqrt{13}$

The normal form is

$$\frac{3}{\sqrt{13}}x + \frac{2}{\sqrt{13}}y - \frac{6}{\sqrt{13}} = 0 \text{ or}$$

$$\frac{3\sqrt{13}}{13}x + \frac{2\sqrt{13}}{13}y - \frac{6\sqrt{13}}{13} = 0.$$

REVIEW EXERCISES

Write the standard form of the equation of each line given p, the length of the normal segment, and ϕ, the angle the normal segment makes with the positive x-axis.

40. $p = 2\sqrt{3}, \phi = \frac{\pi}{3}$

41. $p = 5, \phi = 90°$

42. $p = 3, \phi = \frac{2\pi}{3}$

43. $p = 4\sqrt{2}, \phi = 225°$

Write each equation in normal form. Then find the length of the normal and the angle it makes with the positive x-axis.

44. $7x + 3y - 8 = 0$

45. $6x = 4y - 5$

46. $9x = -5y + 3$

47. $x - 7y = -5$

OBJECTIVES AND EXAMPLES

Lesson 7-7 Find the distance from a point to a line.

- Find the distance between $P(-1, 3)$ and the line with equation $-3x + 4y = -5$.

$$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$$
$$= \frac{-3(-1) + 4(3) + 5}{-\sqrt{(-3)^2 + 4^2}}$$
$$= \frac{20}{-5}$$
$$= -4$$

REVIEW EXERCISES

Find the distance between the point with the given coordinates and the line with the given equation.

48. $(5, 6)$, $2x - 3y + 2 = 0$

49. $(-3, -4)$, $2y = -3x + 6$

50. $(-2, 4)$, $4y = 3x - 1$

51. $(21, 20)$, $y = \frac{1}{3}x + 6$

OBJECTIVES AND EXAMPLES

Lesson 7-7 Write equations of lines that bisect angles formed by intersecting lines.

- Find the equations of the lines that bisect the angles formed by the lines with equations $-4x + 3y = 2$ and $x + 2y = 1$.

$$d_1 = \frac{-4x_1 + 3y_1 - 2}{\sqrt{(-4)^2 + 3^2}}$$
$$= \frac{-4x_1 + 3y_1 - 2}{5}$$

$$d_2 = \frac{x_1 + 2y_1 - 1}{\sqrt{1^2 + 2^2}}$$
$$= \frac{x_1 + 2y_1 - 1}{\sqrt{5}}$$

The origin is in the interior of one of the obtuse angles formed by the given lines.

Bisector of the acute angle: $d_1 = -d_2$

$$\frac{-4x + 3y - 2}{5} = -\frac{x + 2y - 1}{\sqrt{5}}$$
$$\left(-4\sqrt{5} + 5\right)x + \left(3\sqrt{5} + 10\right)y - 2\sqrt{5} - 5 = 0$$

Bisector of the obtuse angle: $d_1 = d_2$

$$\frac{-4x + 3y - 2}{5} = \frac{x + 2y - 1}{\sqrt{5}}$$
$$\left(-4\sqrt{5} - 5\right)x + \left(3\sqrt{5} - 10\right)y - 2\sqrt{5} + 5 = 0$$

REVIEW EXERCISES

Find the distance between the parallel lines with the given equations.

52. $y = \frac{x}{3} - 6$
$y = \frac{x}{3} + 2$

53. $y = \frac{3}{4}x + 3$
$y = \frac{3}{4}x - \frac{1}{2}$

54. $x + y = 1$
$x + y = 5$

55. $2x - 3y + 3 = 0$
$y = \frac{2}{3}x - 2$

Find the equations of the lines that bisect the acute and obtuse angles formed by the lines with the given equations.

56. $y = -3x - 2$
$y = -\frac{x}{2} + \frac{3}{2}$

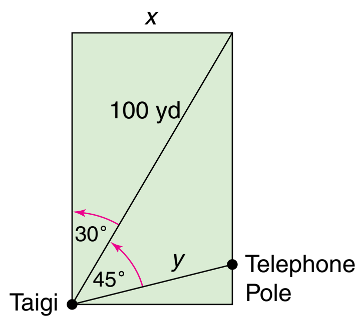
57. $-x + 3y - 2 = 0$
$y = \frac{3}{5}x + 3$

APPLICATIONS AND PROBLEM SOLVING

58. Physics While studying two different physics books, Hector notices that two different formulas are used to find the maximum height of a projectile. One formula is $h = \frac{v_0^2 \sin^2 \theta}{2g}$, and the other is $h = \frac{v_0^2 \tan^2 \theta}{2g \sec^2 \theta}$. Are these two formulas equivalent or is there an error in one of the books? Show your work. *(Lesson 7-2)*

59. Navigation Wanda hikes due east from the edge of the road in front of a lodge into the woods. She stops to eat lunch after she has hiked 1600 feet. In a coordinate system where the positive y-axis points north and the units are hundreds of feet, the equation of the road is $4x - 2y = 0$. How far is Wanda from the road? *(Lesson 7-7)*

60. Surveying Taigi is surveying a rectangular lot for a new office building. She measures the angle between one side of the lot and the line from her position to the opposite corner of the lot as 30°. She then measures the angle between that line and the line to a telephone pole on the edge of the lot as 45°. If Taigi stands 100 yards from the opposite corner of the lot, how far is she from the telephone pole? *(Lesson 7-3)*



ALTERNATIVE ASSESSMENT

OPEN-ENDED ASSESSMENT

1. Give the measure θ of an angle such that trigonometric identities can be used to find exact values for $\sin \theta$, $\cos \theta$, and $\tan \theta$. Find exact values for $\sin \theta$, $\cos \theta$, and $\tan \theta$. Show your work.

2. Write an equation with $\sin x \tan x$ on one side and an expression containing one or more different trigonometric functions on the other side. Verify that your equation is an identity.

PORTFOLIO

Choose one of the identities you studied in this chapter. Explain why it is an important identity and how it is used.

Unit 2 interNET Project

THE CYBERCLASSROOM

That's as clear as mud!

- Search the Internet to find web sites that have lessons involving trigonometric identities. Find at least two types of identities that were not presented in Chapter 7.
- Select one of the types of identities and write at least two of your own examples or sample problems using what you learned on the Internet and in this textbook.
- Design your own web page presenting your examples. Use appropriate software to create the web page. Have another student critique your web page.

Additional Assessment See p. A62 for Chapter 7 practice test.

Chapter 7 SAT & ACT Preparation

Geometry Problems — Triangles and Quadrilaterals

The ACT and SAT contain problems that deal with triangles, quadrilaterals, lengths, and angles. Be sure to review the properties of isosceles and equilateral triangles. Often several geometry concepts are combined in one problem.

Know the number of degrees in various figures.

- A straight angle measures 180°.
- A right angle measures 90°.
- The sum of the measures of the angles in a triangle is 180°.
- The sum of the measures of the angles in a quadrilateral is 360°.

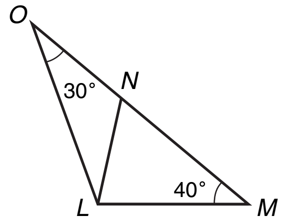
THE PRINCETON REVIEW

TEST-TAKING TIP

The third side of a triangle cannot be longer than the sum of the other two sides. The third side cannot be shorter than the difference between the other two sides.

ACT EXAMPLE

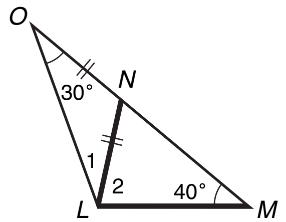
1. In the figure below, O, N, and M are collinear. If the lengths of $\overline{ON}$ and $\overline{NL}$ are the same, the measure of $\angle LON$ is 30°, and $\angle LMN$ is 40°, what is the measure of $\angle NLM$?



A 40° **B** 80° **C** 90° **D** 120° **E** 130°

HINT Look at *all* the triangles in a figure—large and small.

Solution Mark the angle you need to find. You may want to label the missing angles and congruent sides, as shown.



Since two sides of $\triangle ONL$ are the same length, it is isosceles. So $m\angle 1 = 30°$. Since the angles in any triangle total 180°, you can write the following equation for $\triangle OML$.

$180° = 30° + 40° + (30° + m\angle 2)$
$180° = 100° + m\angle 2$
$80° = m\angle 2$

The answer is choice **B.**

SAT EXAMPLE

2. If the average measure of two angles in a parallelogram is $y°$, what is the average degree measure of the other two angles?

A $180 - y$ **B** $180 - \frac{y}{2}$ **C** $360 - 2y$

D $360 - y$ **E** y

HINT Underline important words in the problem and the quantity you must find. Look at the answer choices.

Solution Look for key words in the problem—*average* and *parallelogram.* You need to find the average of two angles. The answer choices are expressions with the variable y. Recall that if the average of two numbers is y, then the sum of the numbers is $2y$.

So the sum of two angle measures is $2y$. Let the sum of the other two angle measures be $2x$. Find x.

The sum of the angle measures in a parallelogram is 360.

$360 = 2y + 2x$
$360 = 2(y + x)$
$180 = y + x$ *Divide each side by 2.*
$x = 180 - y$ *Solve for x.*

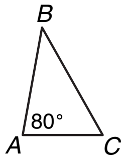
The answer is choice **A.**

SAT AND ACT PRACTICE

After you work each problem, record your answer on the answer sheet provided or on a piece of paper.

Multiple Choice

1. In the figure, the measure of $\angle A$ is $80°$. If the measure of $\angle B$ is half the measure of $\angle A$, what is the measure of $\angle C$?



A $40°$ **B** $60°$ **C** $80°$

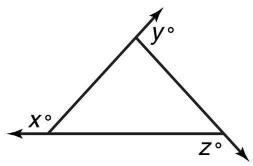
D $100°$ **E** $120°$

2. At what point (x, y) do the two lines with equations $y = 2x - 2$ and $7x - 3y = 11$ intersect?

A $(5, 8)$ **B** $(8, 5)$ **C** $\left(\frac{5}{8}, -1\right)$

D $\left(\frac{5}{8}, 1\right)$ **E** $\left(\frac{25}{16}, \frac{9}{8}\right)$

3. In the figure below, $x + y + z =$



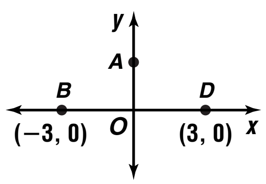
A 90 **B** 180 **C** 270 **D** 360 **E** 450

4. If $x + y = 90°$ and x and y are positive, then $\frac{\sin x}{\cos y} =$

A -1 **B** 0 **C** $\frac{1}{2}$ **D** 1

E It cannot be determined from the information given.

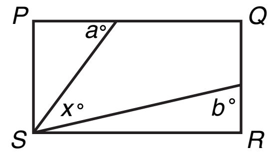
5. In the figure below, what is the sum of the slopes of $\overline{AB}$ and $\overline{AD}$?



A -1 **B** 0 **C** 1 **D** 6

E It cannot be determined from the information given.

6. In the rectangle $PQRS$ below, what is the sum $a + b$ in terms of x?



A $90 + x$

B $190 - x$

C $180 + x$

D $270 - x$

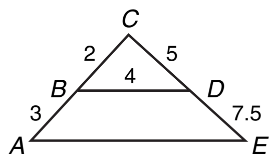
E $360 - x$

7. $$\frac{y - \frac{1}{y}}{1 - \frac{2}{y} + \frac{1}{y^2}} =$$

A $\frac{y^2 + y}{y - 1}$ **B** $\frac{y^2 - y}{(y + 1)^2}$ **C** $\frac{y^2 - y}{y + 1}$

D $\frac{y^2 + y}{(y - 1)^2}$ **E** $\frac{y}{y - 1}$

8. In the figure below, $\triangle ACE$ is similar to $\triangle BCD$. What is the length of $\overline{AE}$?



A 5

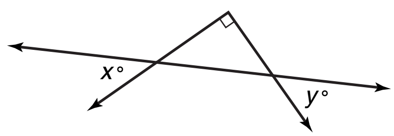
B 6

C 6.5

D 7

E 10

9. The number of degrees in the sum $x° + y°$ would be the same as the number of degrees in which type of angle?



A straight angle

B obtuse angle

C right angle

D acute angle

E It cannot be determined from the information given.

10. **Grid-In** A triangle has a base of 13 units, and the other two sides are congruent. If the side lengths are integers, what is the length of the shortest possible side?

interNET CONNECTION **SAT/ACT Practice** For additional test practice questions, visit: **www.amc.glencoe.com**