UNIT 1

Relations, Functions, and Graphs

Throughout this text, you will see that many real-world phenomena can be modeled by special relations called functions that can be written as equations or graphed. As you work through Unit 1, you will study some of the tools used for mathematical modeling.

Unit 1 interNET Projects

TELECOMMUNICATION

In today's world, there are various forms of communication, some that boggle the mind with their speed and capabilities. In this project, you will use the Internet to help you gather information for investigating various aspects of modern communication. At the end of each chapter, you will work on completing the Unit 1 Internet Project. Here are the topics for each chapter.

CHAPTER 1
(page 61)

Is Anybody Listening? Everyday that you watch television, you are bombarded by various telephone service commercials offering you the best deal for your dollar.
Math Connection: How could you use the Internet and graph data to help determine the best deal for you?

CHAPTER 2
(page 123)

You've Got Mail! The number of homes connected to the Internet and e-mail is on the rise. Use the Internet to find out more information about the types of e-mail and Internet service providers available and their costs.
Math Connection: Use your data and a system of equations to determine if any one product is better for you.

CHAPTER 3
(page 201)

Sorry, You Are Out of Range for Your Telephone Service ... Does your family have a cell phone? Is its use limited to a small geographical area? How expensive is it? Use the Internet to analyze various offers for cellular phone service.
Math Connection: Use graphs to describe the cost of each type of service. Include initial start-up fees or equipment cost, beginning service offers, and actual service fees.

CHAPTER 4
(page 271)

The Pen is Mightier Than the Sword! Does anyone write letters by hand anymore? Maybe fewer people are writing by pen, but most people use computers to write letters, reports, and books. Use the Internet to discover various types of word processing, graphics, spreadsheet, and presentation software that would help you prepare your Unit 1 presentation.
Math Connection: Create graphs using computer software to include in your presentation.

interNET CONNECTION

For more information on the Unit Project, visit:
www.amc.glencoe.com

Linear Relations and Functions

CHAPTER OBJECTIVES

- **Determine whether a given relation is a function and perform operations with functions.** *(Lessons 1-1, 1-2)*
- **Evaluate and find zeros of linear functions using functional notation.** *(Lesson 1-1, 1-3)*
- **Graph and write functions and inequalities.** *(Lessons 1-3, 1-4, 1-7, 1-8)*
- **Write equations of parallel and perpendicular lines.** *(Lesson 1-5)*
- **Model data using scatter plots and write prediction equations.** *(Lesson 1-6)*

1-1

Relations and Functions

OBJECTIVES

- Determine whether a given relation is a function.
- Identify the domain and range of a relation or function.
- Evaluate functions.

Real World Application

METEOROLOGY Have you ever wished that you could change the weather? One of the technologies used in weather management is cloud seeding. In cloud seeding, microscopic particles are released in a cloud to bring about rainfall. The data in the table show the number of acre-feet of rain from pairs of similar unseeded and seeded clouds.

An acre-foot is a unit of volume equivalent to one foot of water covering an area of one acre. An acre-foot contains 43,560 cubic feet or about 27,154 gallons.

Acre-Feet of Rain

Unseeded Clouds	Seeded Clouds
1.0	4.1
4.9	17.5
4.9	7.7
11.5	31.4
17.3	32.7
21.7	40.6
24.4	92.4
26.1	115.3
26.3	118.3
28.6	119.0

Source: Wadsworth International Group

We can write the values in the table as a set of ordered pairs. A pairing of elements of one set with elements of a second set is called a **relation.** The first element of an ordered pair is the *abscissa.* The set of abscissas is called the **domain** of the relation. The second element of an ordered pair is the *ordinate.* The set of ordinates is called the **range** of the relation. *Sets D and R are often used to represent domain and range.*

Relation, Domain, and Range

A relation is a set of ordered pairs. The domain is the set of all abscissas of the ordered pairs. The range is the set of all ordinates of the ordered pairs.

Example **1** **METEOROLOGY** **State the relation of the rain data above as a set of ordered pairs. Also state the domain and range of the relation.**

Real World Application

Relation: {(28.6, 119.0), (26.3, 118.3), (26.1, 115.3), (24.4, 92.4), (21.7, 40.6), (17.3, 32.7), (11.5, 31.4), (4.9, 17.5), (4.9, 7.7), (1.0, 4.1)}

Domain: {1.0, 4.9, 11.5, 17.3, 21.7, 24.4, 26.1, 26.3, 28.6}

Range: {4.1, 7.7, 31.4, 17.5, 32.7, 40.6, 92.4, 115.3, 118.3, 119.0}

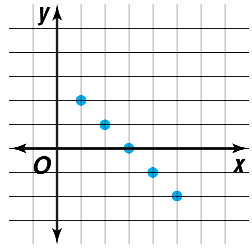
There are multiple representations for each relation. You have seen that a relation can be expressed as a set of ordered pairs. Those ordered pairs can also be expressed as a table of values. The ordered pairs can be graphed for a pictorial representation of the relation. Some relations can also be described by a rule or equation relating the first and second coordinates of each ordered pair.

Example 2 **The domain of a relation is all positive integers less than 6. The range y of the relation is 3 less x, where x is a member of the domain. Write the relation as a table of values and as an equation. Then graph the relation.**

Table:

x	y
1	2
2	1
3	0
4	−1
5	−2

Graph:

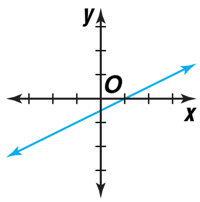


Equation: $y = 3 - x$

You can use the graph of a relation to determine its domain and range.

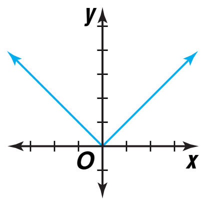
Example 3 **State the domain and range of each relation.**

a.



It appears from the graph that all real numbers are included in the domain and range of the relation.

b.



It appears from the graph that all real numbers are included in the domain. The range includes the non-negative real numbers.

The relations in Example 3 are a special type of relation called a **function.**

Function A function is a relation in which each element of the domain is paired with exactly one element in the range.

Example 4 **State the domain and range of each relation. Then state whether the relation is a function.**

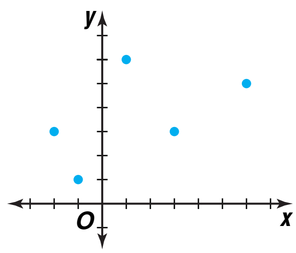
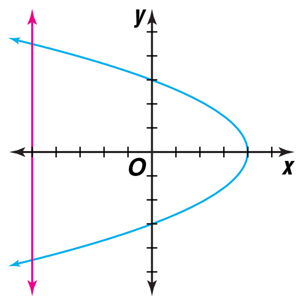
a. $\{(-3, 0), (4, -2), (2, -6)\}$

The domain is $\{-3, 2, 4\}$, and the range is $\{-6, -2, 0\}$. Each element of the domain is paired with exactly one element of the range, so this relation is a function.

b. $\{(4, -2), (4, 2), (9, -3), (-9, 3)\}$

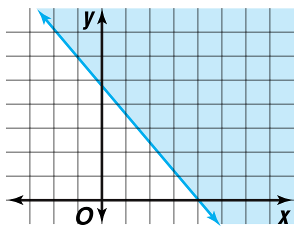
For this relation, the domain is $\{-9, 4, 9\}$, and the range is $\{-3, -2, 2, 3\}$. In the domain, 4 is paired with two elements of the range, −2 and 2. Therefore, this relation is *not* a function.

An alternate definition of a function is a set of ordered pairs in which no two pairs have the same first element. This definition can be applied when a relation is represented by a graph. If every vertical line drawn on the graph of a relation passes through no more than one point of the graph, then the relation is a function. This is called the **vertical line test.**

a relation that is a function	a relation that is not a function
	

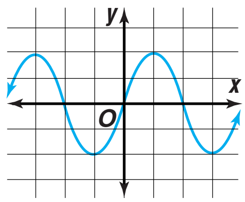
Example 5 **Determine if the graph of each relation represents a function. Explain.**

a.



No, the graph does not represent a function. A vertical line at $x = 1$ would pass through infinitely many points.

b.



Every element of the domain is paired with exactly one element of the range. Thus, the graph represents a function.

x is called the independent variable, and y is called the dependent variable.

Any letter may be used to denote a function. In **function notation,** the symbol $f(x)$ is read "f of x" and should be interpreted as the value of the function f at x. Similarly, $h(t)$ is the value of function h at t. The expression $y = f(x)$ indicates that for each element in the domain that replaces x, the function assigns one and only one replacement for y. The ordered pairs of a function can be written in the form (x, y) or $(x, f(x))$.

Every function can be evaluated for each value in its domain. For example, to find $f(-4)$ if $f(x) = 3x^3 - 7x^2 - 2x$, evaluate the expression $3x^3 - 7x^2 - 2x$ for $x = -4$.

Example 6 **Evaluate each function for the given value.**

a. $f(-4)$ **if** $f(x) = 3x^3 - 7x^2 - 2x$

$$f(-4) = 3(-4)^3 - 7(-4)^2 - 2(-4)$$
$$= -192 - 112 - (-8) \text{ or } -296$$

b. $g(9)$ **if** $g(x) = |6x - 77|$

$$g(9) = |6(9) - 77|$$
$$= |-23| \text{ or } 23$$

Functions can also be evaluated for another variable or an expression.

Example 7 **Evaluate each function for the given value.**

a. $h(a)$ **if** $h(x) = 3x^7 - 10x^4 + 3x - 11$

$$h(a) = 3(a)^7 - 10(a)^4 + 3(a) - 11 \quad x = a$$
$$= 3a^7 - 10a^4 + 3a - 11$$

b. $j(c - 5)$ **if** $j(x) = x^2 - 7x + 4$

$$j(c - 5) = (c - 5)^2 - 7(c - 5) + 4 \quad x = c - 5$$
$$= c^2 - 10c + 25 - 7c + 35 + 4$$
$$= c^2 - 17c + 64$$

When you are given the equation of a function but the domain is not specified, the domain is all real numbers for which the corresponding values in the range are also real numbers.

Example 8 **State the domain of each function.**

a. $f(x) = \dfrac{x^3 + 5x}{x^2 - 4x}$

Any value that makes the denominator equal to zero must be excluded from the domain of f since division by zero is undefined. To determine the excluded values, let $x^2 - 4x = 0$ and solve.

$$x^2 - 4x = 0$$
$$x(x - 4) = 0$$
$$x = 0 \text{ or } x = 4$$

Therefore, the domain includes all real numbers except 0 and 4.

b. $g(x) = \dfrac{1}{\sqrt{x - 4}}$

Any value that makes the radicand negative must be excluded from the domain of g since the square root of a negative number is not a real number. Also, the denominator cannot be zero. Let $x - 4 \le 0$ and solve for the excluded values.

$$x - 4 \le 0$$
$$x \le 4$$

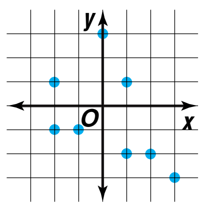
The domain excludes numbers less than or equal to 4. The domain is written as $\{x \mid x > 4\}$, which is read "the set of all x such that x is greater than 4."

CHECK FOR UNDERSTANDING

Communicating Mathematics

Read and study the lesson to answer each question.

1. **Represent** the relation $\{(-4, 2), (6, 1), (0, 5), (8, -4), (2, 2), (-4, 0)\}$ in two other ways.
2. **Draw** the graph of a relation that is not a function.
3. **Describe** how to use the vertical line test to determine whether the graph at the right represents a function.



4. **You Decide** Keisha says that all functions are relations but not all relations are functions. Kevin says that all relations are functions but not all functions are relations. Who is correct and why?

Guided Practice

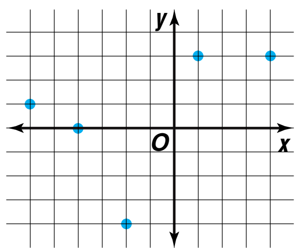
5. The domain of a relation is all positive integers less than 8. The range y of the relation is x less 4, where x is a member of the domain. Write the relation as a table of values and as an equation. Then graph the relation.

State each relation as a set of ordered pairs. Then state the domain and range.

6.

x	y
−3	4
0	0
3	−4
6	−8

7.



Given that x is an integer, state the relation representing each equation by making a table of values. Then graph the ordered pairs of the relation.

8. $y = 3x + 5$ and $-4 \leq x \leq 4$

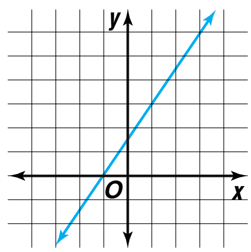
9. $y = -5$ and $1 \leq x \leq 8$

State the domain and range of each relation. Then state whether the relation is a function. Write *yes* or *no*. Explain.

10. $\{(1, 2), (2, 4), (-3, -6), (0, 0)\}$

11. $\{(6, -2), (3, 4), (6, -6), (-3, 0)\}$

12. Study the graph at the right.
   a. State the domain and range of the relation.
   b. State whether the graph represents a function. Explain.



Evaluate each function for the given value.

13. $f(-3)$ if $f(x) = 4x^3 + x^2 - 5x$

14. $g(m + 1)$ if $g(x) = 2x^2 - 4x + 2$

15. State the domain of $f(x) = \sqrt{x + 1}$.

16. **Sports** The table shows the heights and weights of members of the Los Angeles Lakers basketball team during a certain year.
   a. State the relation of the data as a set of ordered pairs. Also state the domain and range of the relation.
   b. Graph the relation.
   c. Determine whether the relation is a function.

Los Angeles Lakers

Height (in.)	Weight (lb)
83	240
81	220
82	245
78	200
83	255
73	200
80	215
77	210
78	190
73	180
86	300
77	220
82	260

Source: Preview Sports

EXERCISES

Practice

Write each relation as a table of values and as an equation. Graph the relation.

17. the domain is all positive integers less than 10, the range is 3 times x, where x is a member of the domain

18. the domain is all negative integers greater than -7, the range is x less 5, where x is a member of the domain

19. the domain is all integers greater than -5 and less than or equal to 4, the range is 8 more than x, where x is a member of the domain

State each relation as a set of ordered pairs. Then state the domain and range.

20.

x	y
−5	−5
−3	−3
−1	−1
1	1

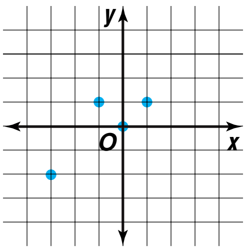
21.

x	y
−10	0
−5	0
0	0
5	0

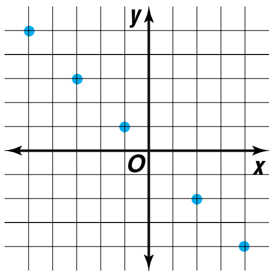
22.

x	y
4	0
5	1
8	0
13	1

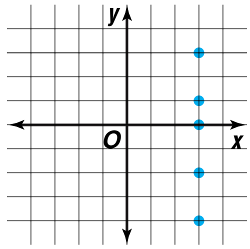
23.



24.



25.



interNET CONNECTION

Graphing Calculator Programs
For a graphing calculator program that plots points in a relation, visit **www.amc.glencoe.com**

Given that x is an integer, state the relation representing each equation by making a table of values. Then graph the ordered pairs of the relation.

26. $y = x - 5$ and $-4 \le x \le 1$

27. $y = -x$ and $1 \le x < 7$

28. $y = |x|$ and $-5 \le x \le 1$

29. $y = 3x - 3$ and $0 < x < 6$

30. $y^2 = x - 2$ and $x = 11$

31. $|2y| = x$ and $x = 4$

State the domain and range of each relation. Then state whether the relation is a function. Write *yes* or *no*. Explain.

32. $\{(4, 4), (5, 4), (6, 4)\}$

33. $\{(1, -2), (1, 4), (1, -6), (1, 0)\}$

34. $\{(4, -2), (4, 2), (1, -1), (1, 1), (0, 0)\}$

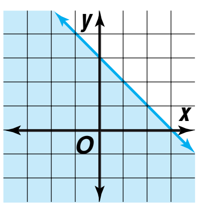
35. $\{(0, 0), (2, 2), (2, -2), (5, 8), (5, -8)\}$

36. $\{(-1.1, -2), (-0.4, -1), (-0.1, -1)\}$

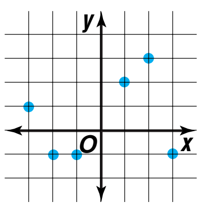
37. $\{(2, -3), (9, 0), (8, -3), (-9, 8)\}$

For each graph, state the domain and range of the relation. Then explain whether the graph represents a function.

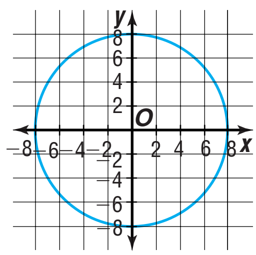
38.



39.



40.



Evaluate each function for the given value.

41. $f(3)$ if $f(x) = 2x + 3$

42. $g(-2)$ if $g(x) = 5x^2 + 3x - 2$

43. $h(0.5)$ if $h(x) = \frac{1}{x}$

44. $j(2a)$ if $j(x) = 1 - 4x^3$

45. $f(n - 1)$ if $f(x) = 2x^2 - x + 9$

46. $g(b^2 + 1)$ if $g(x) = \frac{3 - x}{5 + x}$

47. Find $f(5m)$ if $f(x) = |x^2 - 13|$.

State the domain of each function.

48. $f(x) = \frac{3x}{x^2 - 5}$

49. $g(x) = \sqrt{x^2 - 9}$

50. $h(x) = \frac{x + 2}{\sqrt{x^2 - 7}}$

Graphing Calculator

51. You can use the table feature of a graphing calculator to find the domain of a function. Enter the function into the Y= list. Then observe the y-values in the table. An error indicates that an x-value is excluded from the domain. Determine the domain of each function.

a. $f(x) = \frac{3}{x - 1}$ **b.** $g(x) = \frac{3 - x}{5 + x}$ **c.** $h(x) = \frac{x^2 - 12}{x^2 - 4}$

Applications and Problem Solving

Real World Application

52. Education The table shows the number of students who applied and the number of students attending selected universities.

a. State the relation of the data as a set of ordered pairs. Also state the domain and range of the relation.

b. Graph the relation.

c. Determine whether the relation is a function. Explain.

University	Number Applied	Number Attending
Auburn University	9244	3166
University of California, Davis	18,584	3697
University of Illinois-Champaign-Urbana	18,140	5805
University of Maryland	16,182	3999
State University of New York – Stony Brook	13,589	2136
The Ohio State University	18,912	5950
Texas A&M University	13,877	6233

Source: *Newsweek, "How to get into college, 1998"*

53. Critical Thinking If $f(2m + 1) = 24m^3 + 36m^2 + 26m$, what is $f(x)$? (*Hint:* Begin by solving $x = 2m + 1$ for m.)

54. Aviation The temperature of the atmosphere decreases about 5°F for every 1000 feet that an airplane ascends. Thus, if the ground-level temperature is 95°F, the temperature can be found using the function $t(d) = 95 - 0.005d$, where $t(d)$ is the temperature at a height of d feet. Find the temperature outside of an airplane at each height.

a. 500 ft **b.** 750 ft **c.** 1000 ft **d.** 5000 ft **e.** 30,000 ft

55. Geography A global positioning system, GPS, uses satellites to allow a user to determine his or her position on Earth. The system depends on satellite signals that are reflected to and from a hand-held transmitter. The time that the signal takes to reflect is used to determine the transmitter's position. Radio waves travel through air at a speed of 299,792,458 meters per second. Thus, the function $d(t) = 299{,}792{,}458t$ relates the time t in seconds to the distance traveled $d(t)$ in meters.

a. Find the distance a sound wave will travel in 0.05, 0.2, 1.4, and 5.9 seconds.

b. If a signal from a GPS satellite is received at a transmitter in 0.08 seconds, how far from the transmitter is the satellite?

Extra Practice See p. A26.

56. **Critical Thinking** $P(x)$ is a function for which $P(1) = 1$, $P(2) = 2$, $P(3) = 3$, and $P(x + 1) = \frac{P(x - 2)\,P(x - 1) + 1}{P(x)}$ for $x \geq 3$. Find the value of $P(6)$.

57. **SAT Practice** What is the value of $7^2 - (3^2 + 4^2)$?
A 56
B 24
C 0
D -24
E -56

CAREER CHOICES

Veterinary Medicine

If you like working with animals and have a strong interest in science, you may want to consider a career in veterinary medicine. Many veterinarians work with small animals, such as pets, maintaining their good health and treating illnesses and injuries. Some veterinarians work with large animals, such as farm animals, to ensure the health of animals that we depend upon for food. Still other veterinarians work to control diseases in wildlife.

Duties of veterinarians can include administering medications to the animals, performing surgeries, instructing people in the care of animals, and researching genetics, prevention of disease, and better animal nutrition.

Many veterinarians work in private practice, but jobs are also available in industry and governmental agencies.

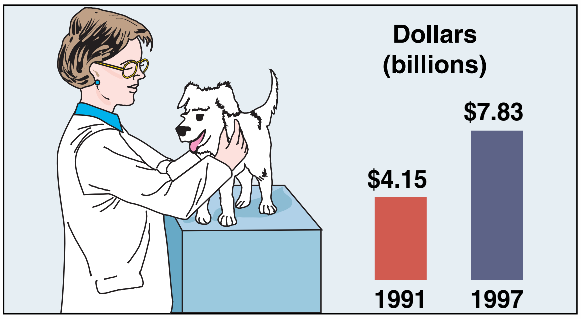
Career Overview

Degree Preferred:
D.V.M. (doctor of veterinary medicine) consisting of six years of college

Related Courses:
biology, chemistry, mathematics

Outlook:
number of jobs expected to increase through 2006

Spending on Pet Health Care



Source: American Veterinary Medical Association

interNET CONNECTION For more information on careers in veterinary medicine, visit: **www.amc.glencoe.com**

1-2 Composition of Functions

OBJECTIVES

- Perform operations with functions.
- Find composite functions.
- Iterate functions using real numbers.

Real World Application

BUSINESS Each year, thousands of people visit Yellowstone National Park in Wyoming. Audiotapes for visitors include interviews with early settlers and information about the geology, wildlife, and activities of the park. The revenue $r(x)$ from the sale of x tapes is $r(x) = 9.5x$. Suppose that the function for the cost of manufacturing x tapes is $c(x) = 0.8x + 1940$. What function could be used to find the profit on x tapes? *This problem will be solved in Example 2.*

To solve the profit problem, you can subtract the cost function $c(x)$ from the revenue function $r(x)$. If you have two functions, you can form new functions by adding, subtracting, multiplying, or dividing the functions.

GRAPHING CALCULATOR EXPLORATION

Use a graphing calculator to explore the sum of two functions.

- Enter the functions $f(x) = 2x - 1$ and $f(x) = 3x + 2$ as **Y1** and **Y2**, respectively.
- Enter **Y1** + **Y2** as the function for **Y3**. To enter **Y1** and **Y2**, press VARS, then select **Y-VARS.** Then choose the equation name from the menu.
- Use **TABLE** to compare the function values for **Y1**, **Y2**, and **Y3**.

TRY THESE

Use the functions $f(x) = 2x - 1$ and $f(x) = 3x + 2$ as Y1 and Y2. Use TABLE to observe the results for each definition of Y3.

1. Y3 = Y1 − Y2

2. Y3 = Y1 · Y2

3. Y3 = Y1 ÷ Y2

WHAT DO YOU THINK?

4. Repeat the activity using functions $f(x) = x^2 - 1$ and $f(x) = 5 - x$ as **Y1** and **Y2**, respectively. What do you observe?

5. Make conjectures about the functions that are the sum, difference, product, and quotient of two functions.

The Graphing Calculator Exploration leads us to the following definitions of operations with functions.

Operations with Functions

Sum: $(f + g)(x) = f(x) + g(x)$

Difference: $(f - g)(x) = f(x) - g(x)$

Product: $(f \cdot g)(x) = f(x) \cdot g(x)$

Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \; g(x) \neq 0$

For each new function, the domain consists of those values of x common to the domains of f and g. The domain of the quotient function is further restricted by excluding any values that make the denominator, $g(x)$, zero.

Example 1 **Given $f(x) = 3x^2 - 4$ and $g(x) = 4x + 5$, find each function.**

a. $(f + g)(x)$

$$\begin{aligned}(f + g)(x) &= f(x) + g(x)\\ &= 3x^2 - 4 + 4x + 5\\ &= 3x^2 + 4x + 1\end{aligned}$$

b. $(f - g)(x)$

$$\begin{aligned}(f - g)(x) &= f(x) - g(x)\\ &= 3x^2 - 4 - (4x + 5)\\ &= 3x^2 - 4x - 9\end{aligned}$$

c. $(f \cdot g)(x)$

$$\begin{aligned}(f \cdot g)(x) &= f(x) \cdot g(x)\\ &= (3x^2 - 4)(4x + 5)\\ &= 12x^3 + 15x^2 - 16x - 20\end{aligned}$$

d. $\left(\frac{f}{g}\right)(x)$

$$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}\\ &= \frac{3x^2 - 4}{4x + 5}, x \neq -\frac{5}{4}\end{aligned}$$

You can use the difference of two functions to solve the application problem presented at the beginning of the lesson.

Example 2 **BUSINESS** **Refer to the application at the beginning of the lesson.**

Real World Application

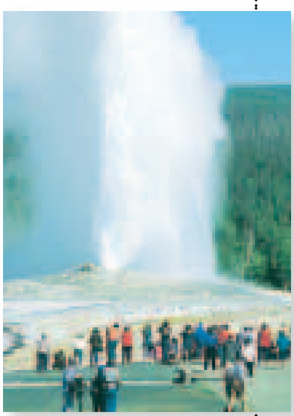
a. Write the profit function.

b. Find the profit on 500, 1000, and 5000 tapes.

a. Profit is revenue minus cost. Thus, the profit function $p(x)$ is $p(x) = r(x) - c(x)$.

The revenue function is $r(x) = 9.5x$. The cost function is $c(x) = 0.8x + 1940$.

$$\begin{aligned}p(x) &= r(x) - c(x)\\ &= 9.5x - (0.8x + 1940)\\ &= 8.7x - 1940\end{aligned}$$



b. To find the profit on 500, 1000, and 5000 tapes, evaluate $p(500)$, $p(1000)$, and $p(5000)$.

$p(500) = 8.7(500) - 1940$ or 2410

$p(1000) = 8.7(1000) - 1940$ or 6760

$p(5000) = 8.7(5000) - 1940$ or $41{,}560$

The profit on 500, 1000, and 5000 tapes is \$2410, \$6760, and \$41,560, respectively. *Check by finding the revenue and the cost for each number of tapes and subtracting to find profit.*

Functions can also be combined by using **composition.** In a composition, a function is performed, and then a second function is performed on the result of the first function. You can think of composition in terms of manufacturing a product. For example, fiber is first made into cloth. Then the cloth is made into a garment.

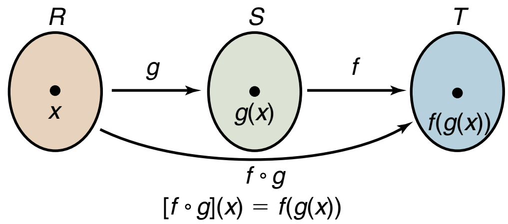
In composition, a function g maps the elements in set R to those in set S. Another function f maps the elements in set S to those in set T. Thus, the range of function g is the same as the domain of function f. A diagram is shown below.

R	**S**
x	$g(x) = \frac{1}{4}x$
4	1
8	2
12	3

S	**T**
x	$f(x) = 6 - 2x$
1	4
2	2
3	0

domain of $g(x)$ *The range of $g(x)$ is the domain of $f(x)$.* *range of $f(x)$*

The function formed by composing two functions f and g is called the **composite** of f and g. It is denoted by $f \circ g$, which is read as "*f composition g*" or "*f of g*."



Composition of Functions

Given functions f and g, the composite function $f \circ g$ can be described by the following equation.

$$[f \circ g](x) = f[g(x)]$$

The domain of $f \circ g$ includes all of the elements x in the domain of g for which $g(x)$ is in the domain of f.

Example 3 **Find $[f \circ g](x)$ and $[g \circ f](x)$ for $f(x) = 2x^2 - 3x + 8$ and $g(x) = 5x - 6$.**

$[f \circ g](x) = f(g(x))$

$= f(5x - 6)$ *Substitute $5x - 6$ for $g(x)$.*

$= 2(5x - 6)^2 - 3(5x - 6) + 8$ *Substitute $5x - 6$ for x in $f(x)$.*

$= 2(25x^2 - 60x + 36) - 15x + 18 + 8$

$= 50x^2 - 135x + 98$

$[g \circ f](x) = g(f(x))$

$= g(2x^2 - 3x + 8)$ *Substitute $2x^2 - 3x + 8$ for $f(x)$.*

$= 5(2x^2 - 3x + 8) - 6$ *Substitute $2x^2 - 3x + 8$ for x in $g(x)$.*

$= 10x^2 - 15x + 34$

The domain of a composed function $[f \circ g](x)$ is determined by the domains of both $f(x)$ and $g(x)$.

Example 4 **State the domain of $[f \circ g](x)$ for $f(x) = \sqrt{x - 4}$ and $g(x) = \frac{1}{x^2}$.**

$f(x) = \sqrt{x - 4}$ *Domain: $x \geq 4$*

$g(x) = \frac{1}{x^2}$ *Domain: $x \neq 0$*

If $g(x)$ is undefined for a given value of x, then that value is excluded from the domain of $[f \circ g](x)$. Thus, 0 is excluded from the domain of $[f \circ g](x)$.

The domain of $f(x)$ is $x \geq 4$. So for x to be in the domain of $[f \circ g](x)$, it must be true that $g(x) \geq 4$.

$g(x) \geq 4$

$\frac{1}{x^2} \geq 4$ *$g(x) = \frac{1}{x^2}$*

$1 \geq 4x^2$ *Multiply each side by x^2.*

$\frac{1}{4} \geq x^2$ *Divide each side by 4.*

$\frac{1}{2} \geq |x|$ *Take the square root of each side.*

$-\frac{1}{2} \leq x \leq \frac{1}{2}$ *Rewrite the inequality.*

Therefore, the domain of $[f \circ g](x)$ is $-\frac{1}{2} \leq x \leq \frac{1}{2}$, $x \neq 0$.

The composition of a function and itself is called iteration. Each output of an iterated function is called an iterate. To iterate a function $f(x)$, find the function value $f(x_0)$, of the initial value x_0. The value $f(x_0)$ is the first iterate, x_1. The second iterate is the value of the function performed on the output; that is, $f(f(x_0))$ or $f(x_1)$. Each iterate is represented by x_n, where n is the iterate number. For example, the third iterate is x_3.

Example 5 **Find the first three iterates, x_1, x_2, and x_3, of the function $f(x) = 2x - 3$ for an initial value of $x_0 = 1$.**

To obtain the first iterate, find the value of the function for $x_0 = 1$.

$$x_1 = f(x_0) = f(1)$$
$$= 2(1) - 3 \text{ or } -1$$

To obtain the second iterate, x_2, substitute the function value for the first iterate, x_1, for x.

$$x_2 = f(x_1) = f(-1)$$
$$= 2(-1) - 3 \text{ or } -5$$

Now find the third iterate, x_3, by substituting x_2 for x.

$$x_3 = f(x_2) = f(-5)$$
$$= 2(-5) - 3 \text{ or } -13$$

Thus, the first three iterates of the function $f(x) = 2x - 3$ for an initial value of $x_0 = 1$ are -1, -5, and -13.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Read and study the lesson to answer each question.

1. **Write** two functions $f(x)$ and $g(x)$ for which $(f \cdot g)(x) = 2x^2 + 11x - 6$. Tell how you determined $f(x)$ and $g(x)$.
2. **Explain** how iteration is related to composition of functions.
3. **Determine** whether $[f \circ g](x)$ is always equal to $[g \circ f](x)$ for two functions $f(x)$ and $g(x)$. Explain your answer and include examples or counterexamples.
4. *Math Journal* Write an explanation of function composition. Include an everyday example of two composed functions and an example of a realworld problem that you would solve using composed functions.

Guided Practice

5. Given $f(x) = 3x^2 + 4x - 5$ and $g(x) = 2x + 9$, find $f(x) + g(x)$, $f(x) - g(x)$, $f(x) \cdot g(x)$, and $\left(\frac{f}{g}\right)(x)$.

Find $[f \circ g](x)$ and $[g \circ f](x)$ for each $f(x)$ and $g(x)$.

6. $f(x) = 2x + 5$
   $g(x) = 3 + x$
7. $f(x) = 2x - 3$
   $g(x) = x^2 - 2x$
8. State the domain of $[f \circ g](x)$ for $f(x) = \dfrac{1}{(x-1)^2}$ and $g(x) = x + 3$.
9. Find the first three iterates of the function $f(x) = 2x + 1$ using the initial value $x_0 = 2$.
10. **Measurement** In 1954, the Tenth General Conference on Weights and Measures adopted the kelvin K as the basic unit for measuring temperature for all international weights and measures. While the kelvin is the standard unit, degrees Fahrenheit and degrees Celsius are still in common use in the United States. The function $C(F) = \frac{5}{9}(F - 32)$ relates Celsius temperatures and Fahrenheit temperatures. The function $K(C) = C + 273.15$ relates Celsius temperatures and Kelvin temperatures.
    a. Use composition of functions to write a function to relate degrees Fahrenheit and kelvins.
    b. Write the temperatures $-40°$F, $-12°$F, $0°$F, $32°$F, and $212°$F in kelvins.

EXERCISES

Practice

Find $f(x) + g(x)$, $f(x) - g(x)$, $f(x) \cdot g(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

11. $f(x) = x^2 - 2x$
    $g(x) = x + 9$
12. $f(x) = \dfrac{x}{x+1}$
    $g(x) = x^2 - 1$
13. $f(x) = \dfrac{3}{x-7}$
    $g(x) = x^2 + 5x$
14. If $f(x) = x + 3$ and $g(x) = \dfrac{2x}{x-5}$, find $f(x) + g(x)$, $f(x) - g(x)$, $f(x) \cdot g(x)$, and $\left(\frac{f}{g}\right)(x)$.

Find $[f \circ g](x)$ and $[g \circ f](x)$ for each $f(x)$ and $g(x)$.

15. $f(x) = x^2 - 9$
$g(x) = x + 4$

16. $f(x) = \frac{1}{2}x - 7$
$g(x) = x + 6$

17. $f(x) = x - 4$
$g(x) = 3x^2$

18. $f(x) = x^2 - 1$
$g(x) = 5x^2$

19. $f(x) = 2x$
$g(x) = x^3 + x^2 + 1$

20. $f(x) = 1 + x$
$g(x) = x^2 + 5x + 6$

21. What are $[f \circ g](x)$ and $[g \circ f](x)$ for $f(x) = x + 1$ and $g(x) = \frac{1}{x-1}$?

State the domain of $[f \circ g](x)$ for each $f(x)$ and $g(x)$.

22. $f(x) = 5x$
$g(x) = x^3$

23. $f(x) = \frac{1}{x}$
$g(x) = 7 - x$

24. $f(x) = \sqrt{x - 2}$
$g(x) = \frac{1}{4x}$

Find the first three iterates of each function using the given initial value.

25. $f(x) = 9 - x;\ x_0 = 2$

26. $f(x) = x^2 + 1;\ x_0 = 1$

27. $f(x) = x(3 - x);\ x_0 = 1$

Applications and Problem Solving

Real World Application

28. Retail Sara Sung is shopping and finds several items that are on sale at 25% off the original price. The items that she wishes to buy are a sweater originally at \$43.98, a pair of jeans for \$38.59, and a blouse for \$31.99. She has \$100 that her grandmother gave her for her birthday. If the sales tax in San Mateo, California, where she lives is 8.25%, does Sara have enough money for all three items? Explain.

29. Critical Thinking Suppose the graphs of functions $f(x)$ and $g(x)$ are lines. Must it be true that the graph of $[f \circ g](x)$ is a line? Justify your answer.

30. Physics When a heavy box is being pushed on the floor, there are two different forces acting on the movement of the box. There is the force of the person pushing the box and the force of friction. If W is work in joules, F is force in newtons, and d is displacement of the box in meters, $W_p = F_p d$ describes the work of the person, and $W_f = F_f d$ describes the work created by friction. The increase in kinetic energy necessary to move the box is the difference between the work done by the person W_p and the work done by friction W_f.

a. Write a function in simplest form for net work.

b. Determine the net work expended when a person pushes a box 50 meters with a force of 95 newtons and friction exerts a force of 55 newtons.

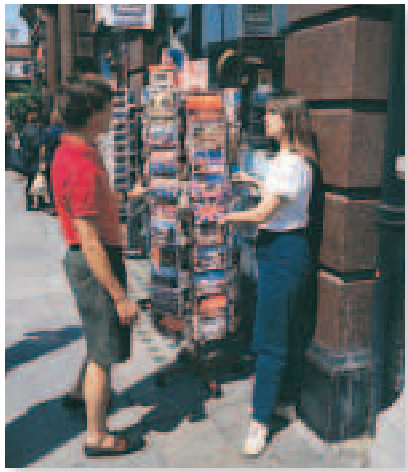
31. Finance A sales representative for a cosmetics supplier is paid an annual salary plus a bonus of 3% of her sales *over* \$275,000. Let $f(x) = x - 275{,}000$ and $h(x) = 0.03x$.

a. If x is greater than \$275,000, is her bonus represented by $f[h(x)]$ or by $h[f(x)]$? Explain.

b. Find her bonus if her sales for the year are \$400,000.

32. Critical Thinking Find $f\left(\frac{1}{2}\right)$ if $[f \circ g](x) = \frac{x^4 + x^2}{1 + x^2}$ and $g(x) = 1 - x^2$.

33. **International Business** Value-added tax, VAT, is a tax charged on goods and services in European countries. Many European countries offer refunds of some VAT to non-resident based businesses. VAT is included in a price that is quoted. That is, if an item is marked as costing \$10, that price includes the VAT.



a. Suppose an American company has operations in The Netherlands, where the VAT is 17.5%. Write a function for the VAT amount paid $v(p)$ if p represents the price including the VAT.

b. In The Netherlands, foreign businesses are entitled to a refund of 84% of the VAT on automobile rentals. Write a function for the refund an American company could expect $r(v)$ if v represents the VAT amount.

c. Write a function for the refund expected on an automobile rental $r(p)$ if the price including VAT is p.

d. Find the refunds due on automobile rental prices of \$423.18, \$225.64, and \$797.05.

Mixed Review

34. **Finance** The formula for the simple interest earned on an investment is $I = prt$, where I is the interest earned, p is the principal, r is the interest rate, and t is the time in years. Assume that \$5000 is invested at an annual interest rate of 8% and that interest is added to the principal at the end of each year. *(Lesson 1-1)*

a. Find the amount of interest that will be earned each year for five years.

b. State the domain and range of the relation.

c. Is this relation a function? Why or why not?

35. State the relation in the table as a set of ordered pairs. Then state the domain and range of the relation. *(Lesson 1-1)*

x	y
−1	8
0	4
2	−6
5	−9

36. What are the domain and the range of the relation {(1, 5), (2, 6), (3, 7), (4, 8)}? Is the relation a function? Explain. *(Lesson 1-1)*

37. Find $g(-4)$ if $g(x) = \frac{x^3 + 5}{4x}$. *(Lesson 1-1)*

38. Given that x is an integer, state the relation representing $y = |-3x|$ and $-2 \le x \le 3$ by making a table of values. Then graph the ordered pairs of the relation. *(Lesson 1-1)*

39. **SAT/ACT Practice** Find $f(n - 1)$ if $f(x) = 2x^2 - x + 9$.

A $2n^2 - n + 9$

B $2n^2 - n + 8$

C $2n^2 - 5n + 12$

D 9

E $2n^2 + 4n + 8$

Extra Practice See p. A26.

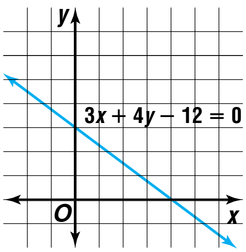
1-3 Graphing Linear Equations

OBJECTIVES

- Graph linear equations.
- Find the x- and y-intercepts of a line.
- Find the slope of a line through two points.
- Find zeros of linear functions.

Real World Application

AGRICULTURE American farmers produce enough food and fiber to meet the needs of our nation and to export huge quantities to countries around the world. In addition to raising grain, cotton and other fibers, fruits, or vegetables, farmers also work on dairy farms, poultry farms, horticultural specialty farms that grow ornamental plants and nursery products, and aquaculture farms that raise fish and shellfish. In 1900, the percent of American workers who were farmers was 37.5%. In 1994, that percent had dropped to just 2.5%. What was the average rate of decline? *This problem will be solved in Example 2.*



The problem above can be solved by using a linear equation. A **linear equation** has the form $Ax + By + C = 0$, where A and B are not both zero. Its graph is a straight line. The graph of the equation $3x + 4y - 12 = 0$ is shown.

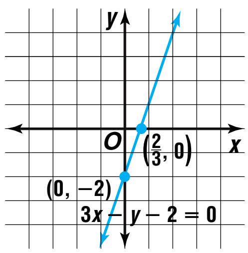
The solutions of a linear equation are the ordered pairs for the points on its graph. An ordered pair corresponds to a point in the coordinate plane. Since two points determine a line, only two points are needed to graph a linear equation. Often the two points that are easiest to find are the ***x*-intercept** and the ***y*-intercept**. The x-intercept is the point where the line crosses the x-axis, and the y-intercept is the point where the graph crosses the y-axis. In the graph above, the x-intercept is at $(4, 0)$, and the y-intercept is at $(0, 3)$. *Usually, the individual coordinates 4 and 3 called the x- and y-intercepts.*

Example 1 **Graph $3x - y - 2 = 0$ using the x-and y-intercepts.**

Substitute 0 for y to find the x-intercept. Then substitute 0 for x to find the y-intercept.

***x*-intercept**	***y*-intercept**
$3x - y - 2 = 0$	$3x - y - 2 = 0$
$3x - (0) - 2 = 0$	$3(0) - y - 2 = 0$
$3x - 2 = 0$	$-y - 2 = 0$
$3x = 2$	$-y = 2$
$x = \frac{2}{3}$	$y = -2$

The line crosses the x-axis at $\left(\frac{2}{3}, 0\right)$ and the y-axis at $(0, -2)$. Graph the intercepts and draw the line.

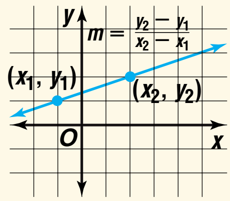


The **slope** of a nonvertical line is the ratio of the change in the ordinates of the points to the corresponding change in the abscissas. The slope of a line is a constant.

Slope

The slope, m, of the line through (x_1, y_1) and (x_2, y_2) is given by the following equation, if $x_1 \neq x_2$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

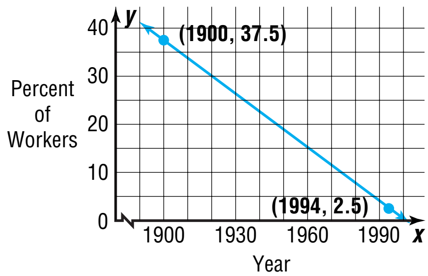


The slope of a line can be interpreted as the rate of change in the y-coordinates for each 1-unit increase in the x-coordinates.

Example 2 Real World Application

AGRICULTURE **Refer to the application at the beginning of the lesson. What was the average rate of decline in the percent of American workers who were farmers?**

The average rate of change is the slope of the line containing the points at (1900, 37.5) and (1994, 2.5). Find the slope of this line.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2.5 - 37.5}{1994 - 1900} \qquad \textit{Let } x_1 = 1900, y_1 = 37.5, x_2 = 1994, \textit{ and } y_2 = 2.5.$$

$$= \frac{-35}{94} \text{ or about } -0.37$$

On average, the number of American workers who were farmers decreased about 0.37% each year from 1900 to 1994.

A linear equation in the form $Ax + By = C$ where A is positive is written in **standard form.** You can also write a linear equation in **slope-intercept form.** Slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept of the line. You can graph an equation in slope-intercept form by graphing the y-intercept and then finding a second point on the line using the slope.

Slope-Intercept Form

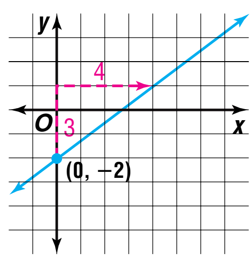
If a line has slope m and y-intercept b, the slope-intercept form of the equation of the line can be written as follows.

$$y = mx + b$$

Example 3 **Graph each equation using the y-intercept and the slope.**

a. $y = \frac{3}{4}x - 2$

The y-intercept is -2. Graph $(0, -2)$.
Use the slope to graph a second point.
Connect the points to graph the line.



Graphing Calculator Appendix

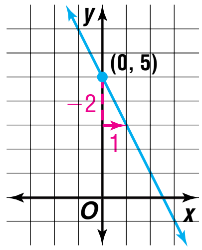
For keystroke instruction on how to graph linear equations, see page A5.

b. $2x + y = 5$

Rewrite the equation in slope-intercept form.

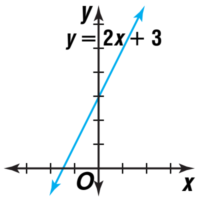
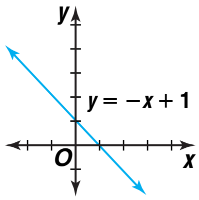
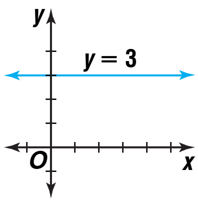
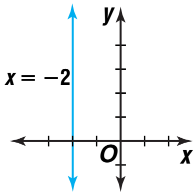
$2x + y = 5 \rightarrow y = -2x + 5$

The y-intercept is 5. Graph (0, 5). Then use the slope to graph a second point. Connect the points to graph the line.



There are four different types of slope for a line. The table below shows a graph with each type of slope.

A line with undefined slope is sometimes described as having "no slope."

Types of Slope			
positive slope	**negative slope**	**0 slope**	**undefined slope**
			

Notice from the graphs that not all linear equations represent functions. A linear function is defined as follows. *When is a linear equation not a function?*

Linear Functions

A linear function is defined by $f(x) = mx + b$, where m and b are real numbers.

Values of x for which $f(x) = 0$ are called **zeros of the function *f*.** For a linear function, the zeros can be found by solving the equation $mx + b = 0$. If $m \neq 0$, then $-\frac{b}{m}$ is the only zero of the function. The zeros of a function are the x-intercepts. Thus, for a linear function, the x-intercept has coordinates $\left(-\frac{b}{m}, 0\right)$.

In the case where $m = 0$, we have $f(x) = b$. This function is called a **constant function** and its graph is a horizontal line. The constant function $f(x) = b$ has no zeros when $b \neq 0$ or every value of x is a zero if $b = 0$.

Example 4 **Find the zero of each function. Then graph the function.**

Graphing Calculator Appendix

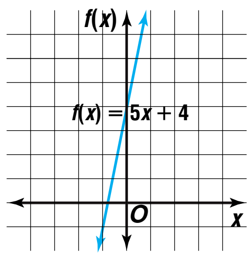
For keystroke instruction on how to find the zeros of a linear function using the **CALC** menu, see page A11.

a. $f(x) = 5x + 4$

To find the zeros of $f(x)$, set $f(x)$ equal to 0 and solve for x.

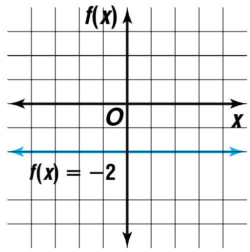
$5x + 4 = 0 \Rightarrow x = -\frac{4}{5}$

$-\frac{4}{5}$ is a zero of the function. So the coordinates of one point on the graph are $\left(-\frac{4}{5}, 0\right)$. Find the coordinates of a second point. When $x = 0$, $f(x) = 5(0) + 4$, or 4. Thus, the coordinates of a second point are (0, 4).



b. $f(x) = -2$

Since $m = 0$ and $b = -2$, this function has no x-intercept, and therefore no zeros. The graph of the function is a horizontal line 2 units below the x-axis.

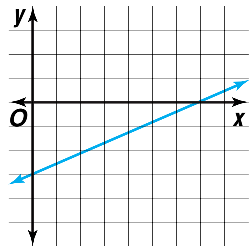


Check for Understanding

Communicating Mathematics

Read and study the lesson to answer each question.

1. **Explain** the significance of m and b in $y = mx + b$.
2. **Name** the zero of the function whose graph is shown at the right. Explain how you found the zero.
3. **Describe** the process you would use to graph a line with a y-intercept of 2 and a slope of -4.
4. **Compare and contrast** the graphs of $y = 5x + 8$ and $y = -5x + 8$.



Guided Practice

Graph each equation using the x- and y-intercepts.

5. $3x - 4y + 2 = 0$
6. $x + 2y - 5 = 0$

Graph each equation using the y-intercept and the slope.

7. $y = x + 7$
8. $y = 5$

Find the zero of each function. If no zero exists, write *none.* Then graph the function.

9. $f(x) = \frac{1}{2}x + 6$
10. $f(x) = 19$

11. **Archaeology** Archaeologists use bones and other artifacts found at historical sites to study a culture. One analysis they perform is to use a function to determine the height of the person from a tibia bone. Typically a man whose tibia is 38.500 centimeters long is 173 centimeters tall. A man with a 44.125-centimeter tibia is 188 centimeters tall.
    a. Write two ordered pairs that represent the function.
    b. Determine the slope of the line through the two points.
    c. Explain the meaning of the slope in this context.

EXERCISES

Practice

Graph each equation.

12. $y = 4x - 9$
13. $y = 3$
14. $2x - 3y + 15 = 0$

15. $x - 4 = 0$
16. $y = 6x - 1$
17. $y = 5 - 2x$

18. $y + 8 = 0$
19. $2x + y = 0$
20. $y = \frac{2}{3}x - 4$

21. $y = 25x + 150$
22. $2x + 5y = 8$
23. $3x - y = 7$

Find the zero of each function. If no zero exists, write *none.* Then graph the function.

24. $f(x) = 9x + 5$
25. $f(x) = 4x - 12$
26. $f(x) = 3x + 1$

27. $f(x) = 14x$
28. $f(x) = 12$
29. $f(x) = 5x - 8$

30. Find the zero for the function $f(x) = 5x - 2$.

31. Graph $y = -\frac{3}{2}x + 3$. What is the zero of the function $f(x) = -\frac{3}{2}x + 3$?

32. Write a linear function that has no zero. Then write a linear function that has infinitely many zeros.

Applications and Problem Solving

Real World Application

33. Electronics The voltage V in volts produced by a battery is a linear function of the current i in amperes drawn from it. The opposite of the slope of the line represents the battery's effective resistance R in ohms. For a certain battery, $V = 12.0$ when $i = 1.0$ and $V = 8.4$ when $i = 10.0$.

a. What is the effective resistance of the battery?

b. Find the voltage that the battery would produce when the current is 25.0 amperes.

34. Critical Thinking A line passes through $A(3, 7)$ and $B(-4, 9)$. Find the value of a if $C(a, 1)$ is on the line.

35. Chemistry According to Charles' Law, the pressure P in pascals of a fixed volume of a gas is linearly related to the temperature T in degrees Celsius. In an experiment, it was found that when $T = 40$, $P = 90$ and when $T = 80$, $P = 100$.

a. What is the slope of the line containing these points?

b. Explain the meaning of the slope in this context.

c. Graph the function.

36. Critical Thinking The product of the slopes of two non-vertical perpendicular lines is always -1. Is it possible for two perpendicular lines to both have positive slope? Explain.

37. Accounting A business's capital costs are expenses for things that last more than one year and lose value or wear out over time. Examples include equipment, buildings, and patents. The value of these items declines, or depreciates over time. One way to calculate depreciation is the straight-line method, using the value and the estimated life of the asset. Suppose $v(t) = 10{,}440 - 290t$ describes the value $v(t)$ of a piece of software after t months.

a. Find the zero of the function. What does the zero represent?

b. Find the slope of the function. What does the slope represent?

c. Graph the function.

38. **Critical Thinking** How is the slope of a linear function related to the number of zeros for the function?

39. **Economics** Economists call the relationship between a nation's disposable income and personal consumption expenditures the marginal propensity to consume or MPC. An MPC of 0.7 means that for each \$1 increase in disposable income, consumption increases \$0.70. That is, 70% of each additional dollar earned is spent and 30% is saved.

   a. Suppose a nation's disposable income, x, and personal consumption expenditures, y, are shown in the table at the right. Find the MPC.

   b. If disposable income were to increase \$1805 in a year, how many additional dollars would the average family spend?

x (billions of dollars)	y (billions of dollars)
56	50
76	67.2

   c. The marginal propensity to save, MPS, is 1 − MPC. Find the MPS.

   d. If disposable income were to increase \$1805 in a year, how many additional dollars would the average family save?

Mixed Review

40. Given $f(x) = 2x$ and $g(x) = x^2 - 4$, find $(f + g)(x)$ and $(f - g)(x)$. *(Lesson 1-2)*

41. **Business** Computer Depot offers a 12% discount on computers sold Labor Day weekend. There is also a \$100 rebate available. *(Lesson 1-2)*

   a. Write a function for the price after the discount $d(p)$ if p represents the original price of a computer.

   b. Write a function for the price after the rebate $r(d)$ if d represents the discounted price.

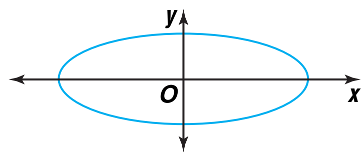
   c. Use composition of functions to write a function to relate the selling price to the original price of a computer.

   d. Find the selling prices of computers with original prices of \$799.99, \$999.99, and \$1499.99.

42. Find $[f \circ g](-3)$ and $[g \circ f](-3)$ if $f(x) = x^2 - 4x + 5$ and $g(x) = x - 2$. *(Lesson 1-2)*

43. Given $f(x) = 4 + 6x - x^3$, find $f(9)$. *(Lesson 1-1)*

44. Determine whether the graph at the right represents a function. Explain. *(Lesson 1-1)*



45. Given that x is an integer, state the relation representing $y = 11 - x$ and $3 \leq x \leq 0$ by listing a set of ordered pairs. Then state whether the relation is a function. *(Lesson 1-1)*

46. **SAT/ACT Practice** What is the sum of four integers whose average is 15?

   A 3.75
   B 15
   C 30
   D 60
   E cannot be determined

Extra Practice See p. A26.

GRAPHING CALCULATOR EXPLORATION

1-3B Analyzing Families of Linear Graphs

An Extension of Lesson 1-3

OBJECTIVE

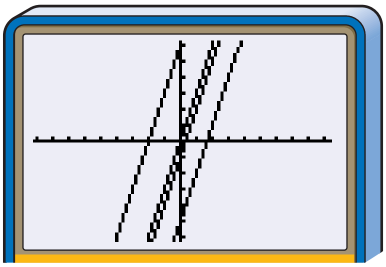
- Investigate the effect of changing the value of m or b in $y = mx + b$.

A **family of graphs** is a group of graphs that displays one or more similar characteristics. For linear functions, there are two types of families of graphs. Using the slope-intercept form of the equation, one family is characterized by having the same slope m in $y = mx + b$. The other type of family has the same y-intercept b in $y = mx + b$.

You can investigate families of linear graphs by graphing several equations on the same graphing calculator screen.

Example **Graph $y = 3x - 5$, $y = 3x - 1$, $y = 3x$, and $y = 3x + 6$. Describe the similarities and differences among the graphs.**

Graph all of the equations on the same screen. Use the viewing window, $[-9.4, 9.4]$ by $[-6.2, 6.2]$.



$[-9.4, 9.4]$ scl:1 by $[-6.2, 6.2]$ scl:1

Notice that the graphs appear to be parallel lines with the same positive slope. They are in the family of lines that have the slope 3.

The slope of each line is the same, but the lines have different y-intercepts. Each of the other three lines are the graph of $y = 3x$ shifted either up or down.

equation	slope	y-intercept	relationship to graph of $y = 3x$
$y = 3x - 5$	3	-5	shifted 5 units down
$y = 3x - 1$	3	-1	shifted 1 unit down
$y = 3x$	3	0	same
$y = 3x + 6$	3	6	shifted 6 units up

TRY THESE

1. Graph $y = 4x - 2$, $y = 2x - 2$, $y = -2$, $y = -x - 2$, and $y = -6x - 2$ on the same graphing calculator screen. Describe how the graphs are similar and different.

WHAT DO YOU THINK?

2. Use the results of the Example and Exercise 1 to predict what the graph of $y = 3x - 2$ will look like.
3. Write a paragraph explaining the effect of different values of m and b on the graph of $y = mx + b$. Include sketches to illustrate your explanation.

1-4

Writing Linear Equations

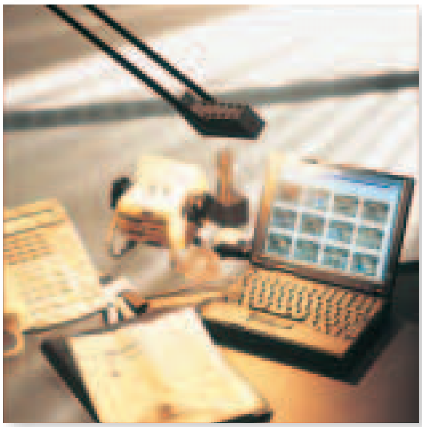
OBJECTIVE
- Write linear equations.

Real World Application

ECONOMICS Each year, the U.S. Department of Commerce publishes its *Survey of Current Business*. Included in the report is the average personal income of U.S. workers.

Years since 1980	Average Personal Income ($)
0	9916
5	13,895
10	18,477
11	19,100
12	19,802
13	20,810
14	21,846
15	23,233
16	24,457
17	25,660

Personal income is one indicator of the health of the U.S. economy. How could you use the data on average personal income for 1980 to 1997 to predict the average personal income in 2010? *This problem will be solved in Example 3.*



A mathematical **model** may be an equation used to approximate a real-world set of data. Often when you work with real-world data, you know information about a line without knowing its equation. You can use characteristics of the graph of the data to write an equation for a line. This equation is a model of the data. Writing an equation of a line may be done in a variety of ways depending upon the information you are given. If one point and the slope of a line are known, the slope-intercept form can be used to write the equation.

Example 1 **Write an equation in slope-intercept form for each line described.**

a. a slope of $-\frac{3}{4}$ and a y-intercept of 7

Substitute $-\frac{3}{4}$ for m and 7 for b in the general slope-intercept form.

$y = mx + b \rightarrow y = -\frac{3}{4}x + 7$

The slope-intercept form of the equation of the line is $y = -\frac{3}{4}x + 7$.

b. a slope of -6 and passes through the point at $(1, -3)$

Substitute the slope and coordinates of the point in the general slope-intercept form of a linear equation. Then solve for b.

$y = mx + b$

$-3 = -6(1) + b$ *Substitute -3 for y, 1 for x, and -6 for m.*

$3 = b$ *Add 6 to each side of the equation.*

The y-intercept is 3. Thus, the equation for the line is $y = -6x + 3$.

Example 2 Real World Application

BUSINESS **Alvin Hawkins is opening a home-based business. He determined that he will need \$6000 to buy a computer and supplies to start. He expects expenses for each following month to be \$700. Write an equation that models the total expense y after x months.**

The initial cost is the y-intercept of the graph. Because the total expense rises \$700 each month, the slope is 700.

$y = mx + b$

$y = 700x + 6000$ *Substitute 700 for m and 6000 for b.*

The total expense can be modeled by $y = 700x + 6000$.

When you know the slope and a point on a line, you can also write an equation for the line in **point-slope form.** Using the definition of slope for points (x, y) and (x_1, y_1), if $\frac{y - y_1}{x - x_1} = m$, then $y - y_1 = m(x - x_1)$.

Point-Slope Form

If the point with coordinates (x_1, y_1) lies on a line having slope m, the point-slope form of the equation of the line can be written as follows.

$$y - y_1 = m(x - x_1)$$

If you know the coordinates of two points on a line, you can find the slope of the line. Then the equation of the line can be written using either the slope-intercept or the point-slope form.

Example 3 Real World Application

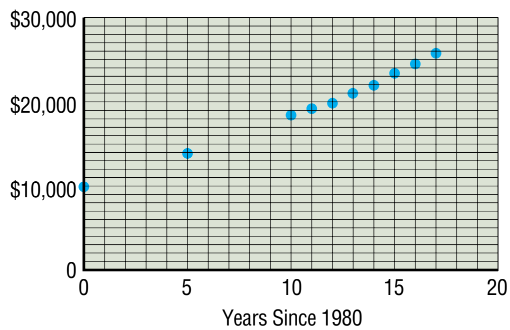
ECONOMICS **Refer to the application at the beginning of the lesson.**

a. Find a linear equation that can be used as a model to predict the average personal income for any year.

b. Assume that the rate of growth of personal income remains constant over time and use the equation to predict the average personal income for individuals in the year 2010.

c. Evaluate the prediction.

a. Graph the data. Then select two points to represent the data set and draw a line that might approximate the data. Suppose we chose (0, 9916) and (17, 25,660). Use the coordinates of those points to find the slope of the line you drew.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{25{,}660 - 9916}{17 - 0} \quad x_1 = 0,\ y_1 = 9916,\ x_2 = 17,\ y_2 = 25{,}660$$

≈ 926 *Thus for each 1-year increase, average personal income increases \$926.*

Use point-slope form.

$y - y_1 = m(x - x_1)$

$y - 9916 = 926(x - 0)$ *Substitute 0 for x_1, 9916 for y_1, and 926 for m.*

$y = 926x + 9916$

The slope-intercept form of the model equation is $y = 926x + 9916$.

b. Evaluate the equation for $x = 2010$ to predict the average personal income for that year. The years since 1980 will be $2010 - 1980$ or 30. So $x = 30$.

$y = 926x + 9916$

$y = 926(30) + 9916$ *Substitute 30 for x.*

$y = 37{,}696$

The predicted average personal income is about \$37,696 for the year 2010.

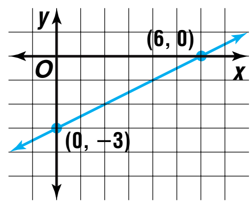
c. Most of the actual data points are close to the graph of the model equation. Thus, the equation and the prediction are probably reliable.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Read and study the lesson to answer each question.

1. **List** all the different sets of information that are sufficient to write the equation of a line.
2. **Demonstrate** two different ways to find the equation of the line with a slope of $\frac{1}{4}$ passing through the point at $(3, -4)$.
3. **Explain** what 55 and 49 represent in the equation $c = 55h + 49$, which represents the cost c of a plumber's service call lasting h hours.
4. **Write** an equation for the line whose graph is shown at the right.



5. *Math Journal* Write a sentence or two to describe when it is easier to use the point-slope form to write the equation of a line and when it is easier to use the slope-intercept form.

Guided Practice

Write an equation in slope-intercept form for each line described.

6. slope $= -\frac{1}{4}$, y-intercept $= -10$
7. slope $= 4$, passes through $(3, 2)$
8. passes through $(5, 2)$ and $(7, 9)$
9. horizontal and passes through $(-9, 2)$
10. **Botany** Do you feel like every time you cut the grass it needs to be cut again right away? Be grateful you aren't cutting the Bermuda grass that grows in Africa and Asia. It can grow at a rate of 5.9 inches per day! Suppose you cut a Bermuda grass plant to a length of 2 inches.
    a. Write an equation that models the length of the plant y after x days.
    b. If you didn't cut it again, how long would the plant be in one week?
    c. Can this rate of growth be maintained indefinitely? Explain.

EXERCISES

Practice

Write an equation in slope-intercept form for each line described.

11. slope = 5, y-intercept = -2

12. slope = 8, passes through $(-7, 5)$

13. slope = $-\frac{3}{4}$, y-intercept = 0

14. slope = -12, y-intercept = $\frac{1}{2}$

15. passes through $A(4, 5)$, slope = 6

16. no slope and passes through $(12, -9)$

17. passes through $A(1, 5)$ and $B(-8, 9)$

18. x-intercept = -8, y-intercept = 5

19. passes through $A(8, 1)$ and $B(-3, 1)$

20. vertical and passes through $(-4, -2)$

21. the y-axis

22. slope = 0.25, x-intercept = 24

23. Line ℓ passes through $A(-2, -4)$ and has a slope of $-\frac{1}{2}$. What is the standard form of the equation for line ℓ?

24. Line m passes through $C(-2, 0)$ and $D(1, -3)$. Write the equation of line m in standard form.

Applications and Problem Solving

Real World Application

25. Sports Skiers, hikers, and climbers often experience altitude sickness as they reach elevations of 8000 feet and more. A good rule of thumb for the amount of time that it takes to become acclimated to high elevations is 2 weeks for the first 7000 feet. After that, it will take 1 week more for each additional 2000 feet of altitude.

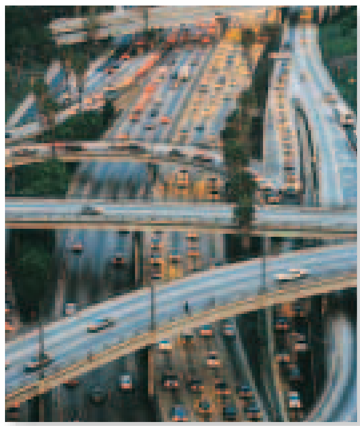
a. Write an equation for the time t to acclimate to an altitude of f feet.

b. Mt. Whitney in California is the highest peak in the contiguous 48 states. It is located in Eastern Sierra Nevada, on the border between Sequoia National Park and Inyo National Forest. About how many weeks would it take a person to acclimate to Mt. Whitney's elevation of 14,494 feet?

26. Critical Thinking Write an expression for the slope of a line whose equation is $Ax + By + C = 0$.

27. Transportation The mileage in miles per gallon (mpg) for city and highway driving of several 1999 models are given in the chart.

Model	City (mpg)	Highway (mpg)
A	24	32
B	20	29
C	20	29
D	20	28
E	23	30
F	24	30
G	27	37
H	22	28



a. Find a linear equation that can be used to find a car's highway mileage based on its city mileage.

b. Model J's city mileage is 19 mpg. Use your equation to predict its highway mileage.

c. Highway mileage for Model J is 26 mpg. How well did your equation predict the mileage? Explain.

28. Economics Research the average personal income for the current year.

a. Find the value that the equation in Example 2 predicts.

b. Is the average personal income equal to the prediction? Explain any difference.

29. Critical Thinking Determine whether the points at (5, 9), (−3, 3), and (1, 6) are collinear. Justify your answer.

Mixed Review

30. Graph $3x - 2y - 5 = 0$. *(Lesson 1-3)*

31. Business In 1995, retail sales of apparel in the United States were $70,583 billion. Apparel sales were $82,805 billion in 1997. *(Lesson 1-3)*

a. Assuming a linear relationship, find the average annual rate of increase.

b. Explain how the rate is related to the graph of the line.

32. If $f(x) = x^3$ and $g(x) = 3x$, find $g[f(-2)]$. *(Lesson 1-2)*

33. Find $(f \cdot g)(x)$ and $\left(\frac{f}{g}\right)(x)$ for $f(x) = x^3$ and $g(x) = x^2 - 3x + 7$. *(Lesson 1-2)*

34. Given that x is an integer, state the relation representing $y = x^2$ and $-4 \leq x \leq -2$ by listing a set of ordered pairs. Then state whether this relation is a function. *(Lesson 1-1)*

35. SAT/ACT Practice If $xy = 1$, then x is the reciprocal of y. Which of the following is the arithmetic mean of x and y?

A $\frac{y^2 + 1}{2y}$ B $\frac{y + 1}{2y}$ C $\frac{y^2 + 2}{2y}$

D $\frac{y^2 + 1}{y}$ E $\frac{x^2 + 1}{y}$

MID-CHAPTER QUIZ

1. What are the domain and the range of the relation {(−2, −3), (−2, 3), (4, 7), (2, −8), (4, 3)}? Is the relation a function? Explain. (Lesson 1-1)

2. Find $f(4)$ for $f(x) = 7 - x^2$. (Lesson 1-1)

3. If $g(x) = \frac{3}{x - 1}$, what is $g(n + 2)$? (Lesson 1-1)

4. Retail Amparo bought a jacket with a gift certificate she received as a birthday present. The jacket was marked 33% off, and the sales tax in her area is 5.5%. If she paid $45.95 for the jacket, use composition of functions to determine the original price of the jacket. (Lesson 1-2)

5. If $f(x) = \frac{1}{x - 1}$ and $g(x) = x + 1$, find $[f \circ g](x)$ and $[g \circ f](x)$. (Lesson 1-2)

Graph each equation. (Lesson 1-3)

6. $2x - 4y = 8$ **7.** $3x = 2y$

8. Find the zero of $f(x) = 5x - 3$. (Lesson 1-3)

9. Points $A(2, 5)$ and $B(7, 8)$ lie on line ℓ. What is the standard form of the equation of line ℓ? (Lesson 1-4)

10. Demographics In July 1990, the population of Georgia was 6,506,416. By July 1997, the population had grown to 7,486,242. (Lesson 1-4)

a. If x represents the year and y represents the population, find the average annual rate of increase of the population.

b. Write an equation to model the population change.

Extra Practice See p. A26.

1-5 Writing Equations of Parallel and Perpendicular Lines

OBJECTIVE

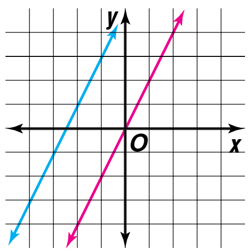
- Write equations of parallel and perpendicular lines.

Real World Application

E-COMMERCE Have you ever made a purchase over the Internet? Electronic commerce, or e-commerce, has changed the way Americans do business. In recent years, hundreds of companies that have no stores outside of the Internet have opened.

Suppose you own shares in two Internet stocks, Bookseller.com and WebFinder. One day these stocks open at \$94.50 and \$133.60 per share, respectively. The closing prices that day were \$103.95 and \$146.96, respectively. If your shares in these companies were valued at \$5347.30 at the beginning of the day, is it possible that the shares were worth \$5882.03 at closing? *This problem will be solved in Example 2.*

This problem can be solved by determining whether the graphs of the equations that describe the situation are parallel or coincide. Two lines that are in the same plane and have no points in common are **parallel lines.** The slopes of two nonvertical parallel lines are equal. The graphs of two equations that represent the same line are said to **coincide.**



Parallel Lines	Two nonvertical lines in a plane are parallel if and only if their slopes are equal and they have no points in common. Two vertical lines are always parallel.

We can use slopes and y-intercepts to determine whether lines are parallel.

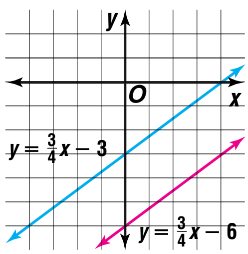
Example 1 **Determine whether the graphs of each pair of equations are *parallel, coinciding,* or *neither.***

a. $\mathbf{3x - 4y = 12}$
$\mathbf{9x - 12y = 72}$

Write each equation in slope-intercept form.

$3x - 4y = 12$ $\qquad$ $9x - 12y = 72$

$y = \frac{3}{4}x - 3$ $\qquad$ $y = \frac{3}{4}x - 6$



The lines have the same slope and different y-intercepts, so they are parallel. The graphs confirm the solution.

b. $\mathbf{15x + 12y = 36}$
$\mathbf{5x + 4y = 12}$

Write each equation in slope-intercept form.

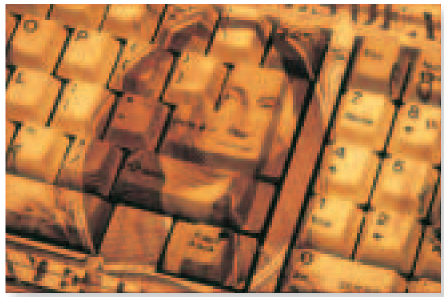
$15x + 12y = 36$	$5x + 4y = 12$
$y = -\frac{5}{4}x + 3$	$y = -\frac{5}{4}x + 3$

The slopes are the same, and the y-intercepts are the same. Therefore, the lines have all points in common. The lines coincide. *Check the solution by graphing.*

You can use linear equations to determine whether real-world situations are possible.

Example 2 Real World Application

FINANCE **Refer to the application at the beginning of the lesson. Is it possible that your shares were worth \$5882.03 at closing? Explain.**



Let x represent the number of shares of Bookseller.com and y represent the number of shares of WebFinder. Then the value of the shares at opening is $94.50x + 133.60y = 5347.30$. The value of the shares at closing is modeled by $103.95x + 146.96y = 5882.03$.

Write each equation in slope-intercept form.

$94.50x + 133.60y = 5347.30$	$103.95x + 146.96y = 5882.03$
$y = \frac{945}{1336}x + \frac{53{,}473}{1336}$	$y = -\frac{945}{1336}x + \frac{53{,}473}{1336}$

Since these equations are the same, their graphs coincide. As a result, any ordered pair that is a solution for the equation for the opening value is also a solution for the equation for the closing value. Therefore, the value of the shares could have been \$5882.03 at closing.

In Lesson 1-3, you learned that any linear equation can be written in standard form. The slope of a line can be obtained directly from the standard form of the equation if B is not 0. Solve the equation for y.

$$Ax + By + C = 0$$
$$By = -Ax - C$$
$$y = \underbrace{-\frac{A}{B}}_{\text{slope}}x \underbrace{- \frac{C}{B}}_{y\text{-intercept}}. \quad B \neq 0$$

So the slope m is $-\frac{A}{B}$, and the y-intercept b is $-\frac{C}{B}$.

Example 3 **Write the standard form of the equation of the line that passes through the point at (4, −7) and is parallel to the graph of $2x - 5y + 8 = 0$.**

Any line parallel to the graph of $2x - 5y + 8 = 0$ will have the same slope. So, find the slope of the graph of $2x - 5y + 8 = 0$.

$$m = -\frac{A}{B}$$
$$= -\frac{2}{(-5)} \text{ or } \frac{2}{5}$$

Use point-slope form to write the equation of the line.

$$y - y_1 = m(x - x_1)$$
$$y - (-7) = \frac{2}{5}(x - 4) \quad \textit{Substitute 4 for } x_1, -7 \textit{ for } y_1, \textit{ and } \frac{2}{5} \textit{ for } m.$$
$$y + 7 = \frac{2}{5}x - \frac{8}{5}$$
$$5y + 35 = 2x - 8 \quad \textit{Multiply each side by 5.}$$
$$2x - 5y - 43 = 0 \quad \textit{Write in standard form.}$$

There is also a special relationship between the slopes of perpendicular lines.

Perpendicular Lines

Two nonvertical lines in a plane are perpendicular if and only if their slopes are opposite reciprocals.

A horizontal and a vertical line are always perpendicular.

You can also use the point-slope form to write the equation of a line that passes through a given point and is perpendicular to a given line.

Example 4 **Write the standard form of the equation of the line that passes through the point at (−6, −1) and is perpendicular to the graph of $4x + 3y - 7 = 0$.**

The line with equation $4x + 3y - 7 = 0$ has a slope of $-\frac{A}{B} = -\frac{4}{3}$. Therefore, the slope of a line perpendicular must be $\frac{3}{4}$.

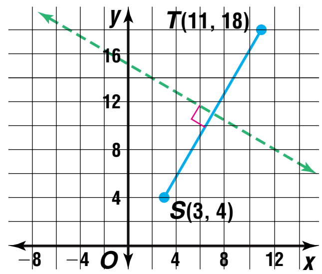
$$y - y_1 = m(x - x_1)$$
$$y - (-1) = \frac{3}{4}[x - (-6)] \quad \textit{Substitute } -6 \textit{ for } x_1, -1 \textit{ for } y_1, \textit{ and } \frac{3}{4} \textit{ for } m.$$
$$y + 1 = \frac{3}{4}x + \frac{9}{2}$$
$$4y + 4 = 3x + 18 \quad \textit{Multiply each side by 4.}$$
$$3x - 4y + 14 = 0 \quad \textit{Write in standard form.}$$

You can use the properties of parallel and perpendicular lines to write linear equations to solve geometric problems.

Example 5 **GEOMETRY** **Determine the equation of the perpendicular bisector of the line segment with endpoints $S(3, 4)$ and $T(11, 18)$.**

Recall that the coordinates of the midpoint of a line segment are the averages of the coordinates of the two endpoints. Let S be (x_1, y_1) and T be (x_2, y_2). Calculate the coordinates of the midpoint.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 + 11}{2}, \frac{4 + 18}{2}\right)$$
$$= (7, 11)$$



The slope of $\overline{ST}$ is $\frac{18 - 4}{11 - 3}$ or $\frac{7}{4}$.

The slope of the perpendicular bisector of $\overline{ST}$ is $-\frac{4}{7}$. The perpendicular bisector of $\overline{ST}$ passes through the midpoint of $\overline{ST}$, (7, 11).

$y - y_1 = m(x - x_1)$	*Point-slope form*
$y - 11 = -\frac{4}{7}(x - 7)$	*Substitute 7 for x_1, 11 for y_1, and $\frac{-4}{7}$ for m.*
$7y - 77 = -4x + 28$	*Multiply each side by 7.*
$4x + 7y - 105 = 0$	*Write in standard form.*

CHECK FOR UNDERSTANDING

Communicating Mathematics

Read and study the lesson to answer each question.

1. **Describe** how you would tell that two lines are parallel or coincide by looking at the equations of the lines in standard form.
2. **Explain** why vertical lines are a special case in the definition of parallel lines.
3. **Determine** the slope of a line that is parallel to the graph of $4x + 3y + 19 = 0$ and the slope of a line that is perpendicular to it.
4. **Write** the slope of a line that is perpendicular to a line that has undefined slope. Explain.

Guided Practice

Determine whether the graphs of each pair of equations are *parallel, coinciding, perpendicular,* or *none of these.*

5. $y = 5x - 5$
   $y = -5x + 2$

6. $y = -6x - 2$
   $y = \frac{1}{6}x - 8$

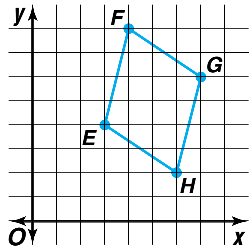
7. $y = x - 6$
   $x - y + 8 = 0$

8. $y = 2x - 8$
   $4x - 2y - 16 = 0$

9. Write the standard form of the equation of the line that passes through $A(5, 9)$ and is parallel to the graph of $y = 5x - 9$.

10. Write the standard form of the equation of the line that passes through $B(-10, -5)$ and is perpendicular to the graph of $6x - 5y = 24$.

11. **Geometry** A quadrilateral is a parallelogram if both pairs of its opposite sides are parallel. A parallelogram is a rectangle if its adjacent sides are perpendicular. Use these definitions to determine if the *EFGH* is a *parallelogram,* a *rectangle,* or *neither.*



EXERCISES

Practice

Determine whether the graphs of each pair of equations are *parallel, coinciding, perpendicular,* or *none of these.*

12. $y = 5x - 18$
$2x + 10y + 10 = 0$

13. $y - 7x + 5 = 0$
$y - 7x - 9 = 0$

14. $y = \frac{1}{3}x + 11$
$y = 3x - 9$

15. $y = -3$
$x = 6$

16. $y = 4x - 3$
$4.8x - 1.2y = 3.6$

17. $4x - 6y = 11$
$3x + 2y = 9$

18. $y = 3x - 2$
$3x + y = 2$

19. $5x + 9y = 14$
$y = -\frac{5}{9}x + \frac{14}{9}$

20. $y + 4x - 2 = 0$
$y + 4x + 1 = 0$

21. Are the graphs of $y = 3x - 2$ and $y = -3x + 2$ *parallel, coinciding, perpendicular,* or *none of these*? Explain.

Write the standard form of the equation of the line that is parallel to the graph of the given equation and passes through the point with the given coordinates.

22. $y = 2x + 10$; $(0, -8)$
23. $4x - 9y = -23$; $(12, -15)$
24. $y = -9$; $(4, -11)$

Write the standard form of the equation of the line that is perpendicular to the graph of the given equation and passes through the point with the given coordinates.

25. $y = 5x + 12$; $(0, -3)$
26. $6x - y = 3$; $(7, -2)$
27. $x = 12$; $(6, -13)$

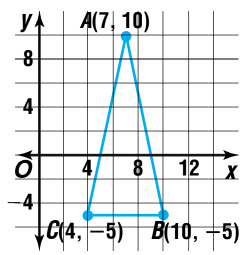
28. The equation of line ℓ is $5y - 4x = 10$. Write the standard form of the equation of the line that fits each description.
   a. parallel to ℓ and passes through the point at $(-15, 8)$
   b. perpendicular to ℓ and passes through the point at $(-15, 8)$

29. The equation of line m is $8x - 14y + 3 = 0$.
   a. For what value of k is the graph of $kx - 7y + 10 = 0$ parallel to line m?
   b. What is k if the graphs of m and $kx - 7y + 10 = 0$ are perpendicular?

Applications and Problem Solving

30. **Critical Thinking** Write equations of two lines that satisfy each description.
   a. perpendicular and one is vertical
   b. parallel and neither has a y-intercept

31. **Geometry** An altitude of a triangle is a segment that passes through one vertex and is perpendicular to the opposite side. Find the standard form of the equation of the line containing each altitude of $\triangle ABC$.



32. Critical Thinking The equations $y = m_1x + b_1$ and $y = m_2x + b_2$ represent parallel lines if $m_1 = m_2$ and $b_1 \neq b_2$. Show that they have no point in common. (*Hint:* Assume that there is a point in common and show that the assumption leads to a contradiction.)

33. Business The Seattle Mariners played their first game at their new baseball stadium on July 15, 1999. The stadium features Internet kiosks, a four-story scoreboard, a retractable roof, and dozens of espresso vendors. Suppose a vendor sells 216 regular espressos and 162 large espressos for a total of \$783 at a Monday night game.

a. On Thursday, 248 regular espressos and 186 large espressos were sold. Is it possible that the vendor made \$914 that day? Explain.

b. On Saturday, 344 regular espressos and 258 large espressos were sold. Is it possible that the vendor made \$1247 that day? Explain.

34. Economics The table shows the closing value of a stock index for one week in February, 1999.

a. Using the day as the x-value and the closing value as the y-value, write equations in slope-intercept form for the lines that represent each value change.

b. What would indicate that the rate of change for two pair of days was the same? Was the rate of change the same for any of the days shown?

c. Use each equation to predict the closing value for the next business day. The actual closing value was 1241.87. Did any equation correctly predict this value? Explain.

Stock Index February, 1999

Day	Closing value
8	1243.77
9	1216.14
10	1223.55
11	1254.04
12	1230.13

Mixed Review

35. Write the slope-intercept form of the equation of the line through the point at (1, 5) that has a slope of -2. *(Lesson 1-4)*

36. Business Knights Screen Printers makes special-order T-shirts. Recently, Knights received two orders for a shirt designed for a symposium. The first order was for 40 T-shirts at a cost of \$295, and the second order was for 80 T-shirts at a cost of \$565. Each order included a standard shipping and handling charge. *(Lesson 1-4)*

a. Write a linear equation that models the situation.

b. What is the cost per T-shirt?

c. What is the standard shipping and handling charge?

37. Graph $3x - 2y - 6 = 0$. *(Lesson 1-3)*

38. Find $[g \circ h](x)$ if $g(x) = x - 1$ and $h(x) = x^2$. *(Lesson 1-2)*

39. Write an example of a relation that is not a function. Tell why it is not a function. *(Lesson 1-1)*

40. SAT Practice **Grid-In** If $2x + y = 12$ and $x + 2y = -6$, what is the value of $2x + 2y$?

Extra Practice See p. A27.

1-6

Modeling Real-World Data with Linear Functions

OBJECTIVES

- Draw and analyze scatter plots.
- Write a prediction equation and draw best-fit lines.
- Use a graphing calculator to compute correlation coefficients to determine goodness of fit.
- Solve problems using prediction equation models.

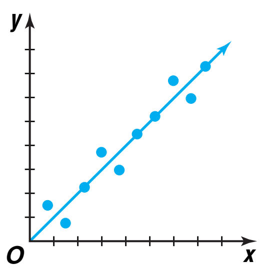
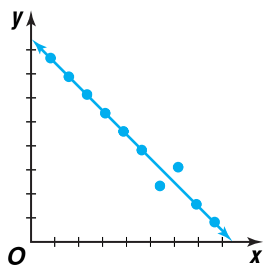
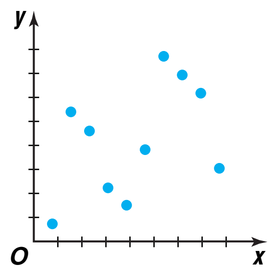
Real World Application

Education The cost of attending college is steadily increasing. However, it can be a good investment since on average, the higher your level of education, the greater your earning potential. The chart shows the average tuition and fees for a full-time resident student at a public four-year college. Estimate the average college cost in the academic year beginning in 2006 if tuition and fees continue at this rate. *This problem will be solved in Example 1.*

Academic Year	Tuition and Fees
1990–1991	2159
1991–1992	2410
1992–1993	2349
1993–1994	2537
1994–1995	2681
1995–1996	2811
1996–1997	2975
1997–1998	3111
1998–1999	3243

Source: The College Board and National Center for Educational Statistics

As you look at the college tuition costs, it is difficult to visualize how quickly the costs are increasing. When real-life data is collected, the data graphed usually does not form a perfectly straight line. However, the graph may approximate a linear relationship. When this is the case, a **best-fit line** can be drawn, and a **prediction equation** that models the data can be determined. Study the **scatter plots** below.

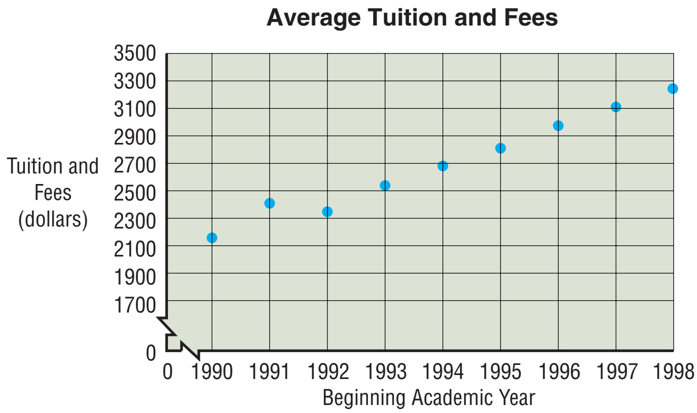
Linear Relationship		No Pattern
		
This scatter plot suggests a linear relationship. Notice that many of the points lie on a line, with the rest very close to it. Since the line has a positive slope, these data have a positive relationship.	This scatter plot also implies a linear relationship. However, the slope of the line suggested by the data is negative.	The points in this scatter plot are very dispersed and do not appear to form a linear pattern.

A prediction equation can be determined using a process similar to determining the equation of a line using two points. The process is dependent upon your judgment. You decide which two points on the line are used to find the slope and intercept. Your prediction equation may be different from someone else's. A prediction equation is used when a rough estimate is sufficient.

Example 1 Real World Application

EDUCATION **Refer to the application at the beginning of the lesson. Predict the average college cost in the academic year beginning in 2006.**

Graph the data. Use the starting year as the independent variable and the tuition and fees as the dependent variable.



Select two points that appear to represent the data. We chose (1992, 2349) and (1997, 3111). Determine the slope of the line.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ *Definition of slope*

$= \dfrac{3111 - 2349}{1997 - 1992}$ *$(x_1, y_1) = (1992, 2349)$, $(x_2, y_2) = (1997, 3111)$*

$= \dfrac{762}{5}$ or 152.4

Now use one of the ordered pairs, such as (1992, 2349), and the slope in the point-slope form of the equation.

$y - y_1 = m(x - x_1)$ *Point-slope form of an equation*

$y - 2349 = 152.4(x - 1992)$ *$(x_1, y_1) = (1992, 2349)$, and $m = 152.4$*

$y = 152.4x - 301{,}231.8$

Thus, a prediction equation is $y = 152.4x - 301{,}231.8$. Substitute 2006 for x to estimate the average tuition and fees for the year 2006.

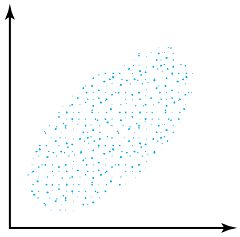
$y = 152.4x - 301{,}231.8$

$y = 152.4(2006) - 301{,}231.8$

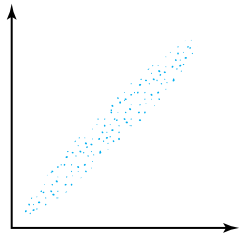
$y = 4482.6$

According to this prediction equation, the average tuition and fees will be \$4482.60 in the academic year beginning in 2006. *Use a different pair of points to find another prediction equation. How does it compare with this one?*

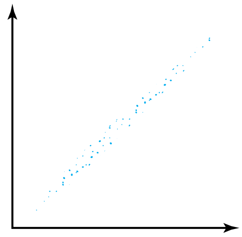
Data that are linear in nature will have varying degrees of **goodness of fit** to the lines of fit. Various formulas are often used to find a **correlation coefficient** that describes the nature of the data. The more closely the data fit a line, the closer the correlation coefficient r approaches 1 or -1. Positive correlation coefficients are associated with linear data having positive slopes, and negative correlation coefficients are associated with negative slopes. Thus, the more linear the data, the more closely the correlation coefficient approaches 1 or -1.



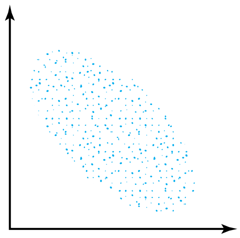
$0 < r \leq 0.5$
positive and weak



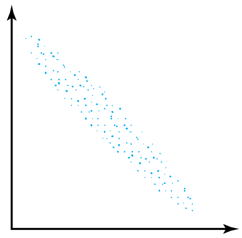
$0.5 \leq r \leq 0.75$
moderately positive



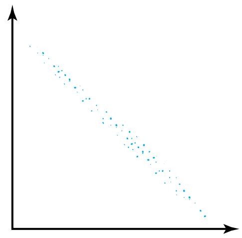
$0.75 \leq r \leq 1$
strongly positive



$-0.5 \leq r < 0$



$-0.75 \leq r \leq -0.5$



$-1 \leq r \leq -0.75$

Statisticians normally use precise procedures, often relying on computers to determine correlation coefficients. The graphing calculator uses the **Pearson product-moment correlation,** which is represented by r. When using these methods, the best fit-line is often called a **regression line.**

Example 2 Real World Application

NUTRITION **The table contains the fat grams and Calories in various fast-food chicken sandwiches.**

a. Use a graphing calculator to find the equation of the regression line and the Pearson product-moment correlation.

b. Use the equation to predict the number of Calories in a chicken sandwich that has 20 grams of fat.

Chicken Sandwich (cooking method)	Fat (grams)	Calories
A (breaded)	28	536
B (grilled)	20	430
C (chicken salad)	33	680
D (broiled)	29	550
E (breaded)	43	710
F (grilled)	12	390
G (breaded)	9	300
H (chicken salad)	5	320
I (breaded)	26	530
J (breaded)	18	440
K (grilled)	8	310

CONTENTS

Graphing Calculator Appendix

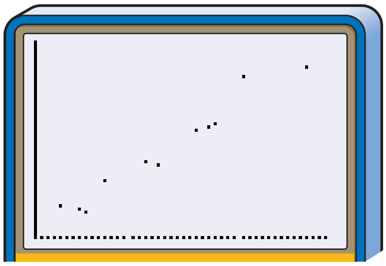
For keystroke instruction on how to enter data, draw a scatter plot, and find a regression equation, see pages A22-A25.

a. Enter the data for fat grams in list L1 and the data for Calories in list L2. Draw a scatter plot relating the fat grams, x, and the Calories, y.

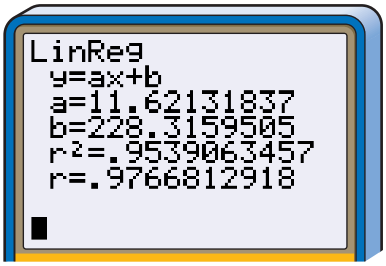
Then use the linear regression statistics to find the equation of the regression line and the correlation coefficient.

The Pearson product-moment correlation is about 0.98. The correlation between grams of fat and Calories is strongly positive. Because of the strong relationship, the equation of the regression line can be used to make predictions.

b. When rounding to the nearest tenth, the equation of the regression line is $y = 11.6x + 228.3$. Thus, there are about $y = 11.6(20) + 228.3$ or 460.3 Calories in a chicken sandwich with 20 grams of fat.



[0, 45] scl: 1 by [250, 750] scl: 50



It should be noted that even when there is a large correlation coefficient, you cannot assume that there is a "cause and effect" relationship between the two related variables.

Check for Understanding

Communicating Mathematics

Read and study the lesson to answer each question.

1. **Explain** what the slope in a best-fit line represents.
2. **Describe** three different methods for finding a best-fit line for a set of data.
3. **Write** about a set of real-world data that you think would show a negative correlation.

Guided Practice

Complete parts a–d for each set of data given in Exercises 4 and 5.

a. Graph the data on a scatter plot.

b. Use two ordered pairs to write the equation of a best-fit line.

c. Use a graphing calculator to find an equation of the regression line for the data. What is the correlation coefficient?

d. If the equation of the regression line shows a moderate or strong relationship, predict the missing value. Explain whether you think the prediction is reliable.

4. **Economics** The table shows the average amount that an American spent on durable goods in several years.

Personal Consumption Expenditures for Durable Goods									
Year	1990	1991	1992	1993	1994	1995	1996	1997	2010
Personal Consumption ($)	1910	1800	1881	2083	2266	2305	2389	2461	?

Source: U.S. Dept. of Commerce

5. **Education** Do you share a computer at school? The table shows the average number of students per computer in public schools in the United States.

Students per Computer															
Academic Year	1983–1984	1984–1985	1985–1986	1986–1987	1987–1988	1988–1989	1989–1990	1990–1991	1991–1992	1992–1993	1993–1994	1994–1995	1995–1996	1996–1997	?
Average	125	75	50	37	32	25	22	20	18	16	14	10.5	10	7.8	1

Source: QED's Technology in Public Schools

EXERCISES

Applications and Problem Solving

Real World Application

Complete parts a–d for each set of data given in Exercises 6–11.

a. **Graph the data on a scatter plot.**

b. **Use two ordered pairs to write the equation of a best-fit line.**

c. **Use a graphing calculator to find an equation of the regression line for the data. What is the correlation coefficient?**

d. **If the equation of the regression line shows a moderate or strong relationship, predict the missing value. Explain whether you think the prediction is reliable.**

6. **Sports** The table shows the number of years coaching and the number of wins as of the end of the 1999 season for selected professional football coaches.

NFL Coach	Years	Wins
Don Shula	33	347
George Halas	40	324
Tom Landry	29	270
Curly Lambeau	33	229
Chuck Noll	23	209
Chuck Knox	22	193
Dan Reeves	19	177
Paul Brown	21	170
Bud Grant	18	168
Steve Owen	23	153
Marv Levy	17	?

Source: *World Almanac*

7. **Economics** Per capita personal income is the average personal income for a nation. The table shows the per capita personal income for the United States for several years.

Year	1990	1991	1992	1993	1994	1995	1996	1997	2005
Personal Income ($)	18,477	19,100	19,802	20,810	21,846	23,233	24,457	25,660	?

Source: U.S. Dept. of Commerce

www.amc.glencoe.com/self_check_quiz
CONTENTS

8. **Transportation** Do you think the weight of a car is related to its fuel economy? The table shows the weight in hundreds of pounds and the average miles per gallon for selected 1999 cars.

Weight (100 pounds)	17.5	20.0	22.5	22.5	22.5	25.0	27.5	35.0	45.0
Fuel Economy (mpg)	65.4	49.0	59.2	41.1	38.9	40.7	46.9	27.7	?

Source: U.S. Environmental Protection Agency

9. **Botany** Acorns were one of the most important foods of the Native Americans. They pulverized the acorns, extracted the bitter taste, and then cooked them in various ways. The table shows the size of acorns and the geographic area covered by different species of oak.

Acorn size (cm^3)	0.3	0.9	1.1	2.0	3.4	4.8	8.1	10.5	17.1
Range (100 km^2)	233	7985	10,161	17,042	7900	3978	28,389	7646	?

Source: *Journal of Biogeography*

10. **Employment** Women have changed their role in American society in recent decades. The table shows the percent of working women who hold managerial or professional jobs.

Percent of Working Women in Managerial or Professional Occupations										
Year	1986	1988	1990	1992	1993	1994	1995	1996	1997	2008
Percent	23.7	25.2	26.2	27.4	28.3	28.7	29.4	30.3	30.8	?

Source: U.S. Dept. of Labor

11. **Demographics** The world's population is growing at a rapid rate. The table shows the number of millions of people on Earth at different years.

World Population							
Year	1	1650	1850	1930	1975	1998	2010
Population (millions)	200	500	1000	2000	4000	5900	?

Source: *World Almanac*

12. **Critical Thinking** Different correlation coefficients are acceptable for different situations. For each situation, give a specific example and explain your reasoning.

a. When would a correlation coefficient of less than 0.99 be considered unsatisfactory?

b. When would a correlation coefficient of 0.6 be considered good?

c. When would a strong negative correlation coefficient be desirable?

13. **Critical Thinking** The table shows the median salaries of American men and women for several years. According to the data, will the women's median salary ever be equal to the men's? If so, predict the year. Explain.

Median Salary ($)					
Year	**Men's**	**Women's**	**Year**	**Men's**	**Women's**
1985	16,311	7217	1991	20,469	10,476
1986	17,114	7610	1992	20,455	10,714
1987	17,786	8295	1993	21,102	11,046
1988	18,908	8884	1994	21,720	11,466
1989	19,893	9624	1995	22,562	12,130
1990	20,293	10,070	1996	23,834	12,815

Source: U.S. Bureau of the Census

Mixed Review

14. **Business** During the month of January, Fransworth Computer Center sold 24 computers of a certain model and 40 companion printers. The total sales on these two items for the month of January was $38,736. In February, they sold 30 of the computers and 50 printers. *(Lesson 1-5)*

   a. Assuming the prices stayed constant during the months of January and February, is it possible that their February sales could have totaled $51,470 on these two items? Explain.

   b. Assuming the prices stayed constant during the months of January and February, is it possible that their February sales could have totaled $48,420 on these two items? Explain.

15. Line ℓ passes through $A(-3, -4)$ and has a slope of -6. What is the standard form of the equation for line ℓ? *(Lesson 1-4)*

16. **Economics** The equation $y = 0.82x + 24$, where $x \geq 0$, models a relationship between a nation's disposable income, x in billions of dollars, and personal consumption expenditures, y in billions of dollars. Economists call this type of equation a *consumption function*. *(Lesson 1-3)*

   a. Graph the consumption function.

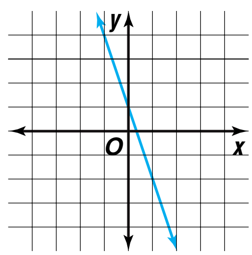
   b. Name the y-intercept.

   c. Explain the significance of the y-intercept and the slope.

17. Find $[f \circ g](x)$ and $[g \circ f](x)$ if $f(x) = x^3$ and $g(x) = x + 1$. *(Lesson 1-2)*

18. Determine if the relation $\{(2, 4), (4, 2), (-2, 4), (-4, 2)\}$ is a function. Explain. *(Lesson 1-1)*

19. **SAT/ACT Practice** Choose the equation that is represented by the graph.



   A $y = 3x - 1$

   B $y = \frac{1}{3}x - 1$

   C $y = 1 - 3x$

   D $y = 1 - \frac{1}{3}x$

   E none of these

Extra Practice See p. A27.

1-7

Piecewise Functions

OBJECTIVE

- Identify and graph piecewise functions including greatest integer, step, and absolute value functions.

Real World Application

ACCOUNTING The Internal Revenue Service estimates that taxpayers who itemize deductions and report interest and capital gains will need an average of almost 24 hours to prepare their returns. The amount that a single taxpayer owes depends upon his or her income. The table shows the tax brackets for different levels of income for a certain year.

Single Individual Income Tax	
Limits of Taxable Income	**Tax Bracket**
\$0 to \$25,350	15%
\$25,351 to \$61,400	28%
\$61,401 to \$128,100	31%
\$128,101 to \$278,450	36%
over \$278,450	39.6%

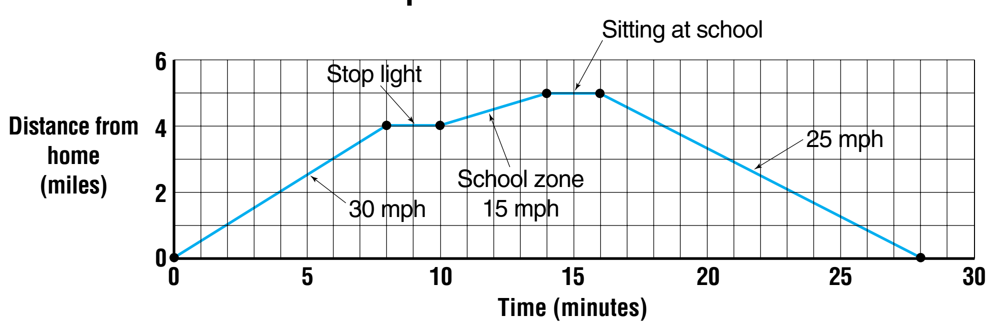
Source: *World Almanac*

A problem related to this will be solved in Example 3.

The tax table defines a special function called a **piecewise function.** For piecewise functions, different equations are used for different intervals of the domain. The graph below shows a piecewise function that models the number of miles from home as a function of time in minutes. Notice that the graph consists of several line segments, each of which is a part of a linear function.

Brittany traveled at a rate of 30 mph for 8 minutes. She stopped at a stoplight for 2 minutes. Then for 4 minutes she traveled 15 mph through the school zone. She sat at the school for 3 minutes while her brother got out of the car. Then she traveled home at 25 mph.

A Trip to and from School



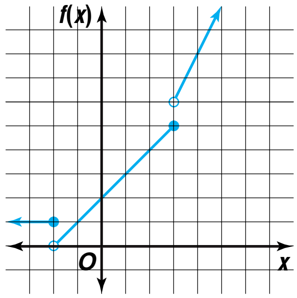
When graphing piecewise functions, the partial graphs over various intervals do not necessarily connect. The definition of the function on the intervals determines if the graph parts connect.

Example 1 Graph $f(x) = \begin{cases} 1 \text{ if } x \leq -2 \\ 2 + x \text{ if } -2 < x \leq 3. \\ 2x \text{ if } x > 3 \end{cases}$

First, graph the constant function $f(x) = 1$ for $x \leq -2$. This graph is part of a horizontal line. Because the point at $(-2, 1)$ is included in the graph, draw a closed circle at that point.

Second, graph the function $f(x) = 2 + x$ for $-2 < x \leq 3$. Because $x = -2$ is not included in this part of the domain, draw an open circle at $(-2, 0)$. $x = 3$ is included in the domain, so draw a closed circle at $(3, 5)$ since for $f(x) = 2 + x$, $f(3) = 5$.

Third, graph the line $y = 2x$ for $x > 3$. Draw an open circle at $(3, 6)$ since for $f(x) = 2x$, $f(3) = 6$.



Graphing Calculator Tip

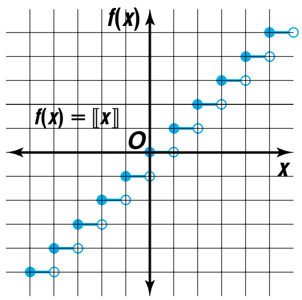
On a graphing calculator, **int(X)** indicates the greatest integer function.

A piecewise function where the graph looks like a set of stairs is called a **step function.** In a step function, there are breaks in the graph of the function. You cannot trace the graph of a step function without lifting your pencil. One type of step function is the **greatest integer function.** The symbol $[\![x]\!]$ means *the greatest integer not greater than x.* This does not mean to round or truncate the number. For example, $[\![8.9]\!] = 8$ because 8 is the greatest integer not greater than 8.9. Similarly, $[\![-3.9]\!] = -4$ because -3 is greater than -3.9. The greatest integer function is given by $f(x) = [\![x]\!]$.

Example 2 Graph $f(x) = [\![x]\!]$.

Make a table of values. The domain values will be intervals for which the greatest integer function will be evaluated.

x	$f(x)$
$-3 \leq x < -2$	-3
$-2 \leq x < -1$	-2
$-1 \leq x < 0$	-1
$0 \leq x < 1$	0
$1 \leq x < 2$	1
$2 \leq x < 3$	2
$3 \leq x < 4$	3
$4 \leq x < 5$	4



Notice that the domain for this greatest integer function is all real numbers and the range is integers.

The graphs of step functions are often used to model real-world problems such as fees for cellular telephones and the cost of shipping an item of a given weight.

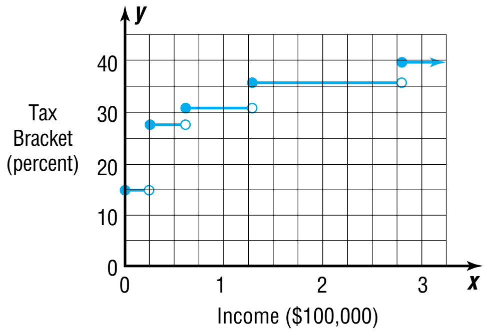
Example 3 Real World Application

Refer to the application at the beginning of the lesson.

a. Graph the tax brackets for the different incomes.

b. What is the tax bracket for a person who makes \$70,000?

a.



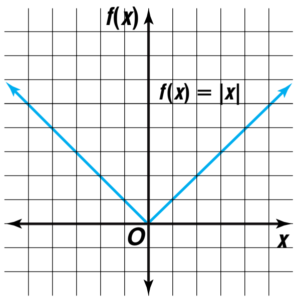
b. \$70,000 falls in the interval \$61,401 to \$128,100. Thus, the tax bracket for \$70,000 is 31%.

The **absolute value function** is another piecewise function. Consider $f(x) = |x|$. The absolute value of a number is always nonnegative. The table lists a specific domain and resulting range values for the absolute value function. Using these points, a graph of the absolute value function can be constructed. Notice that the domain of the graph includes all real numbers. However, the range includes only nonnegative real numbers.

table

$f(x) = \|x\|$	
x	$f(x)$
−3	3
−2.4	2.4
0	0
0.7	0.7
2	2
3.4	3.4

graph



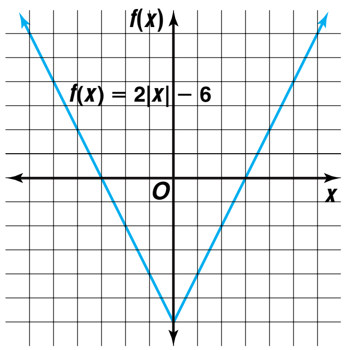
piecewise function

$$f(x) = \begin{cases} -x \text{ if } x < 0 \\ x \text{ if } x \geq 0 \end{cases}$$

Example 4 **Graph $f(x) = 2|x| - 6$.**

Use a table of values to determine points on the graph.

x	$2\|x\| - 6$	$(x, f(x))$
−6	$2\|-6\| - 6 = 6$	(−6, 6)
−3	$2\|-3\| - 6 = 0$	(−3, 0)
−1.5	$2\|-1.5\| - 6 = -3$	(−1.5, −3)
0	$2\|0\| - 6 = -6$	(0, −6)
1	$2\|1\| - 6 = -4$	(1, −4)
2	$2\|2\| - 6 = -2$	(2, −2)



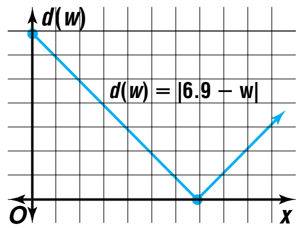
Many real-world situations can be modeled by a piecewise function.

Example 5 Identify the type of function that models each situation. Then write a function for the situation.

Real World Application

a. Manufacturing The stated weight of a box of rice is 6.9 ounces. The company randomly chooses boxes to test to see whether their equipment is dispensing the right amount of product. If the discrepancy is more than 0.2 ounce, the production line is stopped for adjustments.

The situation can be represented with an absolute value function. Let w represent the weight and $d(w)$ represent the discrepancy. Then $d(w) = |6.9 - w|$.

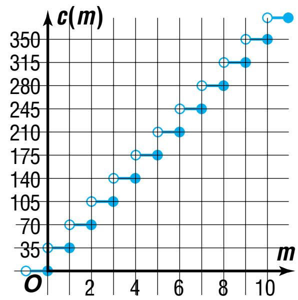


b. Business On a certain telephone rate plan, the price of a cellular telephone call is 35¢ per minute or fraction thereof.

This can be described by a greatest integer function.

Let m represent the number of minutes of the call and $c(m)$ represent the cost in cents.

$$c(m) = \begin{cases} 35m \text{ if } [\![m]\!] = m \\ 35[\![m + 1]\!] \text{ if } [\![m]\!] < m \end{cases}$$

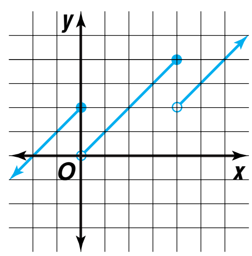


Check for Understanding

Communicating Mathematics

Read and study the lesson to answer each question.

1. **Write** $f(x) = |x|$ as a piecewise function.
2. **State** the domain and range of the function $f(x) = 2[\![x]\!]$.
3. **Write** the function that is represented by the graph.
4. **You Decide** Misae says that a step graph does not represent a function because the graph is not connected. Alex says that it does represent a function because there is only one y for every x. Who is correct and why?



Guided Practice

Graph each function.

5. $f(x) = \begin{cases} 2x \text{ if } 0 \leq x \leq 4 \\ 8 \text{ if } 4 < x \leq 7 \end{cases}$

6. $f(x) = \begin{cases} 6 \text{ if } x \leq -6 \\ |x| \text{ if } -6 < x < 6 \\ 6 \text{ if } x > 6 \end{cases}$

7. $f(x) = -[\![x]\!]$

8. $f(x) = |x - 3|$

9. **Business** Identify the type of function that models the labor cost for repairing a computer if the charge is \$50 per hour or fraction thereof. Then write and graph a function for the situation.

10. **Consumerism** Guillermo Lujan is flying from Denver to Dallas for a convention. He can park his car in the Denver airport long-term parking lot at the terminal or in the shuttle parking facility closeby. In the long-term lot, it costs \$1.00 per hour or any part of an hour with a maximum charge of \$6.00 per day. In shuttle facility, he has to pay \$4.00 for each day or part of a day. Which parking lot is less expensive if Mr. Lujan returns after 2 days and 3 hours?

EXERCISES

Practice

Graph each function.

11. $f(x) = \begin{cases} 2x + 1 \text{ if } x < 0 \\ 2x - 1 \text{ if } x \geq 0 \end{cases}$

12. $g(x) = |x - 5|$

13. $h(x) = [\![x]\!] + 2$

14. $g(x) = |2x + 3|$

15. $f(x) = [\![x - 1]\!]$

16. $h(x) = \begin{cases} 3 \text{ if } -1 \leq x \leq 1 \\ 4 \text{ if } 1 < x \leq 4 \\ x \text{ if } x > 4 \end{cases}$

17. $g(x) = 2|x - 3|$

18. $f(x) = [\![-3x]\!]$

19. $h(x) = \begin{cases} x + 3 \text{ if } x \leq 0 \\ 3 - x \text{ if } 1 < x \leq 3 \\ 3x \text{ if } x > 3 \end{cases}$

20. $f(x) = \begin{cases} -2x \text{ if } x < 1 \\ 3 \text{ if } x = 1 \\ 4x \text{ if } x > 1 \end{cases}$

21. $j(x) = \dfrac{2}{[\![x]\!]}$

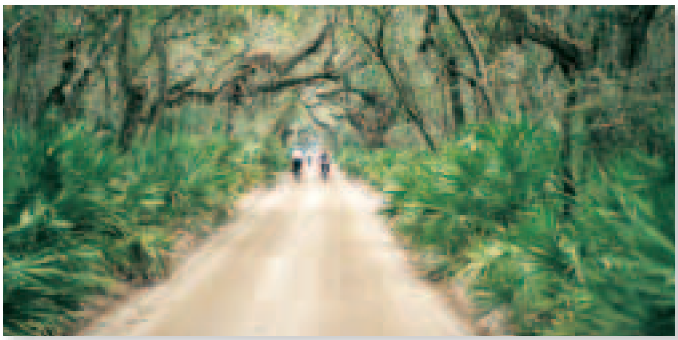
22. $g(x) = |9 - 3|x||$

Identify the type of function that models each situation. Then write and graph a function for the situation.

23. **Tourism** The table shows the charge for renting a bicycle from a rental shop on Cumberland Island, Georgia, for different amounts of time.

Island Rentals

Time	Price
$\frac{1}{2}$ hour	\$6
1 hour	\$10
2 hours	\$16
Daily	\$24



24. **Postage** The cost of mailing a letter is \$0.33 for the first ounce and \$0.22 for each additional ounce or portion thereof.

25. **Manufacturing** A can of coffee is supposed to contain one pound of coffee. How does the actual weight of the coffee in the can compare to 1 pound?

Applications and Problem Solving

26. Retail Sales The table shows the shipping charges that apply to orders placed in a catalog.

a. What type of function is described?

b. Write the shipping charges as a function of the value of the order.

c. Graph the function.

Shipping to or within the United States	
Value of Order	**Shipping, Packing, and Handling Charge**
\$0.00–25.00	\$3.50
\$25.01–75.00	\$5.95
\$75.01–125.00	\$7.95
\$125.01 and up	\$9.95

27. Critical Thinking Describe the values of x and y which are solutions to $[\![x]\!] = [\![y]\!]$.

28. Engineering The degree day is used to measure the demand for heating or cooling. In the United States, 65°F is considered the desirable temperature for the inside of a building. The number of degree days recorded on a given date is equal to the difference between 65 and the mean temperature for that date. If the mean temperature is above 65°F, cooling degree days are recorded. Heating degree days are recorded if the mean temperature is below 65°F.

a. What type of function can be used to model degree days?

b. Write a function to model the number of degree days $d(t)$ for a mean temperature of t°F.

c. Graph the function.

d. The mean temperature is the mean of the high and low temperatures for a day. How many degree days are recorded for a day with a high of temperature of 63°F and a low temperature of 28°F? Are they heating degree days or cooling degree days?

29. Accounting The income tax brackets for the District of Columbia are listed in the tax table.

Income	Tax Bracket
up to \$10,000	6%
more than \$10,000, but no more than \$20,000	8%
more than \$20,000	9.5%

a. What type of function is described by the tax rates?

b. Write the function if x is income and $t(x)$ is the tax rate.

c. Graph the tax brackets for different taxable incomes.

d. Alicia Davis lives in the District of Columbia. In which tax bracket is Ms. Davis if she made \$36,000 last year?

30. Critical Thinking For $f(x) = [\![x]\!]$ and $g(x) = |x|$, are $[f \circ g](x)$ and $[g \circ f](x)$ equivalent? Justify your answer.

Mixed Review

31. **Transportation** The table shows the percent of workers in different cities who use public transportation to get to work. *(Lesson 1-6)*

a. Graph the data on a scatter plot.

b. Use two ordered pairs to write the equation of a best-fit line.

c. Use a graphing calculator to find an equation for the regression line for the data. What is the correlation value?

d. If the equation of the regression line shows a moderate or strong relationship, predict the percent of workers using public transportation in Baltimore, Maryland. Is the prediction reliable? Explain.

City	Workers 16 years and older	Percent who use Public Transportation
New York, NY	3,183,088	53.4
Los Angeles, CA	1,629,096	10.5
Chicago, IL	1,181,677	29.7
Houston, TX	772,957	6.5
Philadelphia, PA	640,577	28.7
San Diego, CA	560,913	4.2
Dallas, TX	500,566	6.7
Phoenix, AZ	473,966	3.3
San Jose, CA	400,932	3.5
San Antonio, TX	395,551	4.9
San Francisco, CA	382,309	33.5
Indianapolis, IN	362,777	3.3
Detroit, MI	325,054	10.7
Jacksonville, FL	312,958	2.7
Baltimore, MD	307,679	22.0

Source: U.S. Bureau of the Census

32. Write the standard form of the equation of the line that passes through the point at (4, 2) and is parallel to the line whose equation is $y = 2x - 4$. *(Lesson 1-5)*

33. **Sports** During a basketball game, the two highest-scoring players scored 29 and 15 points and played 39 and 32 minutes, respectively. *(Lesson 1-3)*

a. Write an ordered pair of the form (minutes played, points scored) to represent each player.

b. Find the slope of the line containing both points.

c. What does the slope of the line represent?

34. **Business** For a company, the revenue $r(x)$ in dollars, from selling x items is $r(x) = 400x - 0.2x^2$. The cost for making and selling x items is $c(x) = 0.1x + 200$. Write the profit function $p(x) = (r - c)(x)$. *(Lesson 1-2)*

35. **Retail** Winston bought a sweater that was on sale 25% off. The original price of the sweater was \$59.99. If sales tax in Winston's area is 6.5%, how much did the sweater cost including sale tax? *(Lesson 1-2)*

36. State the domain and range of the relation $\{(0, 2), (4, -2), (9, 3), (-7, 11), (-2, 0)\}$. Is the relation a function? Explain. *(Lesson 1-1)*

37. **SAT Practice** Which of the following expressions is *not* larger than 5×6^{12}?

A $5 + 6^{12}$

B 7×6^{12}

C 5×8^{12}

D 5×6^{14}

E 10^{13}

Extra Practice See p. A27.

1-8

Graphing Linear Inequalities

OBJECTIVE

- Graph linear inequalities.

Real World Application

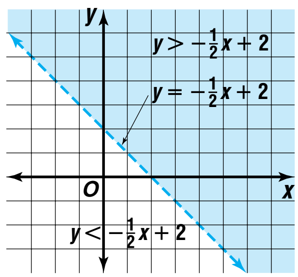
NUTRITION Arctic explorers need endurance and strength. They move sleds weighing more than 1100 pounds each for as much as 12 hours a day. For that reason, Will Steger and members of his exploration team each burn 4000 to 6000 Calories daily!

An *endurance diet* can provide the energy and nutrients necessary for peak performance in the Arctic. An endurance diet has a balance of fat and carbohydrates and protein. Fat is a concentrated energy source that supplies nine calories per gram. Carbohydrates and protein provide four calories per gram and are a quick source of energy. What are some of the combinations of carbohydrates and protein and fat that supply the needed energy for the Arctic explorers?

This problem will be solved in Example 2.

This situation can be described using a **linear inequality.** A linear inequality is not a function. However, you can use the graphs of linear functions to help you graph linear inequalities.

The graph of $y = -\frac{1}{2}x + 2$ separates the coordinate plane into two regions, called **half planes.** The line described by $y = -\frac{1}{2}x + 2$ is called the **boundary** of each region. If the boundary is part of a graph, it is drawn as a solid line. A boundary that is not part of the graph is drawn as a dashed line. The graph of $y > -\frac{1}{2}x + 2$ is the region above the line. The graph of $y < -\frac{1}{2}x + 2$ is the region below the line.



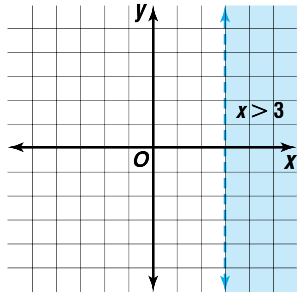
When graphing an inequality, you can determine which half plane to shade by testing a point on either side of the boundary in the original inequality. If it is not on the boundary, the origin (0, 0) is often an easy point to test. If the inequality statement is true for your test point, then shade the half plane that contains the test point. If the inequality statement is false for your test point, then shade the half plane that does not contain the test point.

Example 1 **Graph each inequality.**

a. $x > 3$

The boundary is not included in the graph. So the vertical line $x = 3$ should be a dashed line.

Testing (0, 0) in the inequality yields a false inequality, $0 > 3$. So shade the half plane that does not include (0, 0).



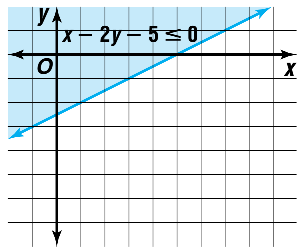
b. $x - 2y - 5 \leq 0$

$$x - 2y - 5 \leq 0$$
$$-2y \leq -x + 5$$
$$y \geq \frac{1}{2}x - \frac{5}{2}$$

Reverse the inequality when you divide or multiply by a negative.

The graph does include the boundary. So the line is solid.

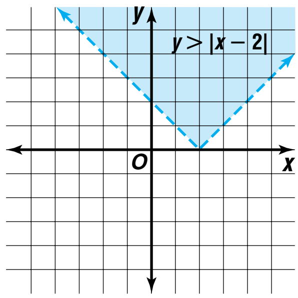
Testing (0, 0) in the inequality yields a true inequality, so shade the half plane that includes (0, 0).



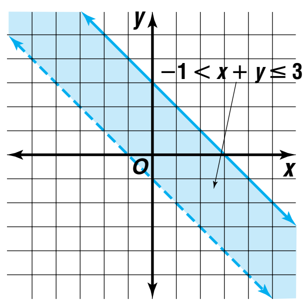
c. $y > |x - 2|$

Graph the equation $y = |x - 2|$ with a dashed boundary.

Testing (0, 0) yields the false inequality $0 > 2$, so shade the region that does not include (0, 0).



You can also graph relations such as $-1 < x + y \leq 3$. The graph of this relation is the intersection of the graph of $-1 < x + y$ and the graph of $x + y \leq 3$. Notice that the boundaries $x + y = 3$ and $x + y = -1$ are parallel lines. The boundary $x + y = 3$ is part of the graph, but $x + y = -1$ is not.



Example 2 Real World Application

NUTRITION **Refer to the application at the beginning of the lesson.**

a. Draw a graph that models the combinations of grams of fat and carbohydrates and protein that the arctic team diet may include to satisfy their daily caloric needs.

Let x represent the number of grams of fat and y represent the number of grams of carbohydrates and protein. The team needs at least 4000, but no more than 6000, Calories each day. Write an inequality.

$4000 \leq 9x + 4y \leq 6000$

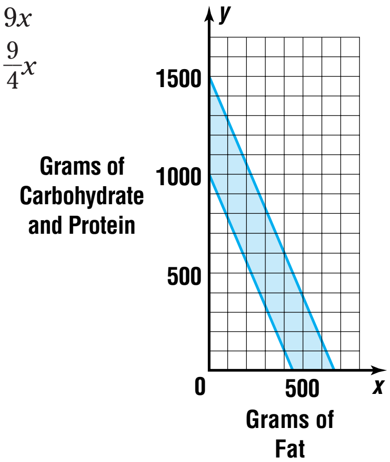
You can write this compound inequality as two inequalities, $4000 \leq 9x + 4y$ and $9x + 4y \leq 6000$. Solve each part for y.

$$4000 \leq 9x + 4y \quad \text{and} \quad 9x + 4y \leq 6000$$

$$4000 - 9x \leq 4y \qquad\qquad 4y \leq 6000 - 9x$$

$$1000 - \frac{9}{4}x \leq y \qquad\qquad y \leq 1500 - \frac{9}{4}x$$

Graph each boundary line and shade the appropriate region. The graph of the compound inequality is the area in which the shading overlaps.



b. Name three combinations of fat or carbohydrates and protein that meet the Calorie requirements.

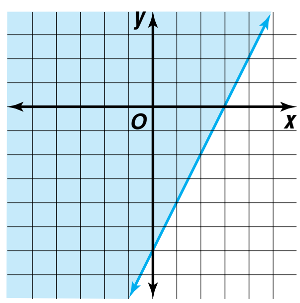
Any point in the shaded region or on the boundary lines meets the requirements. Three possible combinations are (100, 775), (200, 800), and (300, 825). These ordered pairs represent 100 grams of fat and 775 grams of carbohydrate and protein, 200 grams of fat and 800 grams of carbohydrate and protein, and 300 grams of fat and 825 grams of carbohydrate and protein.

CHECK FOR UNDERSTANDING

Communicating Mathematics

Read and study the lesson to answer each question.

1. **Write** the inequality whose graph is shown.
2. **Describe** the process you would use to graph $-3 \leq 2x + y < 7$.
3. **Explain** why you can use a test point to determine which region or regions of the graph of an inequality should be shaded.



Guided Practice Graph each inequality.

4. $x + y < 4$
5. $3x - y \leq 6$
6. $7 < x + y \leq 9$
7. $y < |x + 3|$

8. **Business** Nancy Stone has a small company and has negotiated a special rate for rental cars when she and other employees take business trips. The maximum charge is \$45.00 per day plus \$0.40 per mile. Discounts apply when renting for longer periods of time or during off-peak seasons.
   a. Write a linear inequality that models the total cost of the daily rental $c(m)$ as a function of the total miles driven, m.
   b. Graph the inequality.
   c. Name three combinations of miles and total cost that satisfy the inequality.

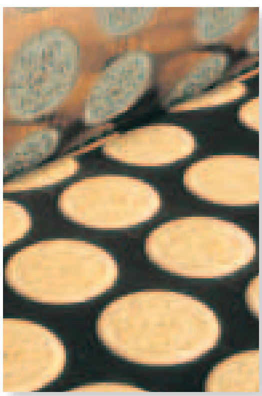
EXERCISES

Practice Graph each inequality.

9. $y < 3$
10. $x - y > -5$
11. $2x + 4y \geq 7$
12. $-y < 2x + 1$
13. $2x - 5y + 19 \leq 0$
14. $-4 \leq x - y \leq 5$
15. $y \geq |x|$
16. $-2 \leq x + 2y \leq 4$
17. $y > |x| + 4$
18. $y < |2x + 3|$
19. $-8 \leq 2x + y < 6$
20. $y - 1 > |x + 3|$
21. Graph the region that satisfies $x \geq 0$ and $y \geq 0$.
22. Graph $2 < |x| \leq 8$.

Applications and Problem Solving

Real World Application



23. **Manufacturing** Many manufacturers use inequalities to solve production problems such as determining how much of each product should be assigned to each machine. Suppose one bakery oven at a cookie manufacturer is being used to bake chocolate cookies and vanilla cookies. A batch of chocolate cookies bakes in 8 minutes, and a batch of vanilla cookies bakes in 10 minutes.
    a. Let x represent the number of batches of chocolate cookies and y represent the number of batches of vanilla cookies. Write a linear inequality for the number of batches of each type of cookie that could be baked in one oven in an 8-hour shift.
    b. Graph the inequality.
    c. Name three combinations of batches of chocolate cookies and vanilla cookies that satisfy the inequality.
    d. Often manufacturers' problems involve as many as 150 products, 218 facilities, 10 plants, and 127 customer zones. Research how problems like this are solved.

24. **Critical Thinking** Graph $|y| \geq x$.

25. Critical Thinking Suppose $xy > 0$.

a. Describe the points whose coordinates are solutions to the inequality.

b. Demonstrate that for points whose coordinates are solutions to the inequality, the equation $|x + y| = |x| + |y|$ holds true.

26. Engineering Mechanics The production cost of a job depends in part on the accuracy required. On most sheet metal jobs, an accuracy of 1, 2, or 0.1 mils is required. A mil is $\frac{1}{1000}$ inch. This means that a dimension must be less than $\frac{1}{1000}$, $\frac{2}{1000}$, or $\frac{1}{10{,}000}$ inch larger or smaller than the blueprint states. Industrial jobs often require a higher degree of accuracy.

a. Write inequalities that models the possible dimensions of a part that is supposed to be 8 inches by $4\frac{1}{4}$ inches if the accuracy required is 2 mils.

b. Graph the region that shows the satisfactory dimensions for the part.

27. Exercise The American College of Sports Medicine recommends that healthy adults exercise at a target level of 60% to 90% of their maximum heart rate. You can estimate your maximum heart rate by subtracting your age from 220.

a. Write a compound inequality that models age, a, and target heart rate, r.

b. Graph the inequality.

Mixed Review

28. Business Gatsby's Automotive Shop charges $55 per hour or any fraction of an hour for labor. *(Lesson 1-7)*

a. What type of function is described?

b. Write the labor charge as a function of the time.

c. Graph the function.

29. The equation of line ℓ is $3x - y = 10$. *(Lesson 1-5)*

a. What is the standard form of the equation of the line that is parallel to ℓ and passes through the point at $(0, -2)$?

b. Write the standard form of the equation of the line that is perpendicular to ℓ and passes through the point at $(0, -2)$.

30. Write the slope-intercept form of the equation of the line through (1, 4) and (5, 7). *(Lesson 1-4)*

31. Temperature The temperature in Indianapolis on January 30 was 23°F at 12:00 A.M. and 48°F at 4:00 P.M. *(Lesson 1-3)*

a. Write two ordered pairs of the form (hours since midnight, temperature) for this date. What is the slope of the line containing these points?

b. What does the slope of the line represent?

32. SAT/ACT Practice Which expression is equivalent to $\frac{9^5 - 9^4}{8}$?

A $\frac{1}{8}$ **B** $\frac{9}{8}$ **C** $\frac{9^3}{8}$

D $\frac{9^9}{8}$ **E** 9^4

Extra Practice See p. A27.

VOCABULARY

abscissa (p. 5)
absolute value function (p.47)
boundary (p. 52)
coinciding lines (p. 32)
composite (p. 15)
composition of functions (pp. 14-15)
constant function (p. 22)
domain (p. 5)
family of graphs (p. 26)
function (p. 6)
function notation (p. 7)
greatest integer function (p. 46)
half plane (p. 52)
iterate (p. 16)
iteration (p. 16)
linear equation (p. 20)
linear function (p. 22)
linear inequality (p. 52)
ordinate (p. 5)
parallel lines (p. 32)
perpendicular lines (p. 34)
piecewise function (p. 45)
point-slope form (p. 28)
range (p. 5)
relation (p. 5)
slope (pp. 20-21)
slope-intercept form (p. 21)
standard form (p. 21)
step function (p. 46)
vertical line test (p. 7)
x-intercept (p. 20)
y-intercept (p. 20)
zero of a function (p. 22)

Modeling

best-fit line (p. 38)
correlation coefficient (p. 40)
goodness of fit (p. 40)
model (p. 27)
Pearson-product moment correlation (p. 40)
prediction equation (p. 38)
regression line (p. 40)
scatter plot (p. 38)

UNDERSTANDING AND USING THE VOCABULARY

Choose the letter of the term that best matches each statement or phrase.

1. for the function f, a value of x for which $f(x) = 0$
2. a pairing of elements of one set with elements of a second set
3. has the form $Ax + By + C = 0$, where A is positive and A and B are not both zero
4. $y - y_1 = m(x - x_1)$, where (x_1, y_1) lies on a line having slope m
5. $y = mx + b$, where m is the slope of the line and b is the y-intercept
6. a relation in which each element of the domain is paired with exactly one element of the range
7. the set of all abscissas of the ordered pairs of a relation
8. the set of all ordinates of the ordered pairs of a relation
9. a group of graphs that displays one or more similar characteristics
10. lie in the same plane and have no points in common

a. function
b. parallel lines
c. zero of a function
d. linear equation
e. family of graphs
f. relation
g. point-slope form
h. domain
i. slope-intercept form
j. range

interNET CONNECTION For additional review and practice for each lesson, visit: **www.amc.glencoe.com**

SKILLS AND CONCEPTS

OBJECTIVES AND EXAMPLES

Lesson 1-1 Evaluate a function.

Find $f(-2)$ if $f(x) = 3x^2 - 2x + 4$.
Evaluate the expression $3x^2 - 2x + 4$ for $x = -2$.

$$\begin{aligned} f(-2) &= 3(-2)^2 - 2(-2) + 4 \\ &= 12 + 4 + 4 \\ &= 20 \end{aligned}$$

REVIEW EXERCISES

Evaluate each function for the given value.

11. $f(4)$ if $f(x) = 5x - 10$

12. $g(2)$ if $g(x) = 7 - x^2$

13. $f(-3)$ if $g(x) = 4x^2 - 4x + 9$

14. $h(0.2)$ if $h(x) = 6 - 2x^3$

15. $g\left(\frac{1}{3}\right)$ if $g(x) = \frac{2}{5x}$

16. $k(4c)$ if $k(x) = x^2 + 2x - 4$

17. Find $f(m + 1)$ if $f(x) = |x^2 + 3x|$.

Lesson 1-2 Perform operations with functions.

Given $f(x) = 4x + 2$ and $g(x) = x^2 - 2x$, find $(f + g)(x)$ and $(f \cdot g)(x)$.

$$\begin{aligned} (f + g)(x) &= f(x) + g(x) \\ &= 4x + 2 + x^2 - 2x \\ &= x^2 + 2x + 2 \end{aligned}$$

$$\begin{aligned} (f \cdot g)(x) &= f(x) \cdot g(x) \\ &= (4x + 2)(x^2 - 2x) \\ &= 4x^3 - 6x^2 - 4x \end{aligned}$$

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

18. $f(x) = 6x - 4$
$g(x) = 2$

19. $f(x) = x^2 + 4x$
$g(x) = x - 2$

20. $f(x) = 4 - x^2$
$g(x) = 3x$

21. $f(x) = x^2 + 7x + 12$
$g(x) = x + 4$

22. $f(x) = x^2 - 1$
$g(x) = x + 1$

23. $f(x) = x^2 - 4x$
$g(x) = \frac{4}{x - 4}$

Lesson 1-2 Find composite functions.

Given $f(x) = 2x^2 + 4x$ and $g(x) = 2x - 1$, find $[f \circ g](x)$ and $[g \circ f](x)$.

$$\begin{aligned} [f \circ g](x) &= f(g(x)) \\ &= f(2x - 1) \\ &= 2(2x - 1)^2 + 4(2x + 1) \\ &= 2(4x^2 - 4x + 1) + 8x + 4 \\ &= 8x^2 + 6 \end{aligned}$$

$$\begin{aligned} [g \circ f](x) &= g(f(x)) \\ &= g(2x^2 + 4x) \\ &= 2(2x^2 + 4x) - 1 \\ &= 4x^2 + 8x - 1 \end{aligned}$$

Find $[f \circ g](x)$ and $[g \circ f](x)$ for each f(x) and g(x).

24. $f(x) = x^2 - 4$
$g(x) = 2x$

25. $f(x) = 0.5x + 5$
$g(x) = 3x^2$

26. $f(x) = 2x^2 + 6$
$g(x) = 3x$

27. $f(x) = 6 + x$
$g(x) = x^2 - x + 1$

28. $f(x) = x^2 - 5$
$g(x) = x + 1$

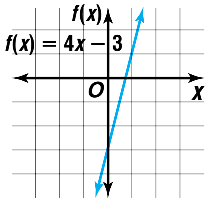
29. $f(x) = 3 - x$
$g(x) = 2x^2 + 10$

30. State the domain of $[f \circ g](x)$ for $f(x) = \sqrt{x - 16}$ and $g(x) = 5 - x$.

OBJECTIVES AND EXAMPLES

Lesson 1-3 Graph linear equations.

- Graph $f(x) = 4x - 3$.



REVIEW EXERCISES

Graph each equation.

31. $y = 3x + 6$

32. $y = 8 - 5x$

33. $y - 15 = 0$

34. $0 = 2x - y - 7$

35. $y = 2x$

36. $y = -8x - 2$

37. $7x + 2y = -5$

38. $y = \frac{1}{4}x - 6$

Lesson 1-4 Write linear equations using the slope-intercept, point-slope, and standard forms of the equation.

- Write the slope-intercept form of the equation of the line that has a slope of 24 and passes through the point at (1, 2).

$$\begin{aligned} y &= mx + b && \textit{Slope-intercept form} \\ 2 &= -4(1) + b && y = 2,\ x = 1,\ m = -4 \\ 6 &= b && \textit{Solve for } b. \end{aligned}$$

The equation for the line is $y = -4x + 6$.

Write an equation in slope-intercept form for each line described.

39. slope = 2, y-intercept = −3

40. slope = −1, y-intercept = 1

41. slope $= \frac{1}{2}$, passes through the point at (−5, 2)

42. passes through $A(-4, 2)$ and $B(2, 5)$

43. x-intercept = 1, y-intercept = −4

44. horizontal and passes through the point at (3, −1)

45. the x-axis

46. slope = 0.1, x-intercept = 1

Lesson 1-5 Write equations of parallel and perpendicular lines.

- Write the standard form of the equation of the line that is parallel to the graph of $y = 2x - 3$ and passes through the point at (1, −1).

$$\begin{aligned} y - y_1 &= m(x - x_1) && \textit{Point-slope form} \\ y - (-1) &= 2(x - 1) && y_1 = -1,\ m = 2,\ x = 1 \\ 2x - y - 3 &= 0 \end{aligned}$$

Write the standard form of the equation of the line that is perpendicular to the graph of $y = 2x - 3$ and passes through the point at (6, −1).

$$\begin{aligned} y - y_1 &= m(x - x_1) && y_1 = -1, \\ y - (-1) &= -\frac{1}{2}(x - 6) && m = -\frac{1}{2},\ x = 6 \\ x + 2y - 2 &= 0 \end{aligned}$$

Write the standard form of the equation of the line that is parallel to the graph of the given equation and passes through the point with the given coordinates.

47. $y = x + 1$; (1, 1)

48. $y = \frac{1}{3}x - 2$; (−1, 6)

49. $2x + y = 1$; (−3, 2)

Write the standard form of the equation of the line that is perpendicular to the graph of the given equation and passes through the point with the given coordinates.

50. $y = -2x + \frac{1}{4}$; (4, −8)

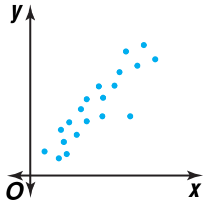
51. $4x - 2y + 2 = 0$; (1, 4)

52. $x = -8$; (4, −6)

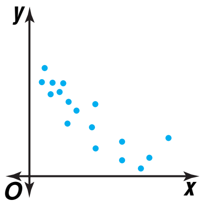
OBJECTIVES AND EXAMPLES

Lesson 1-6 Draw and analyze scatter plots.

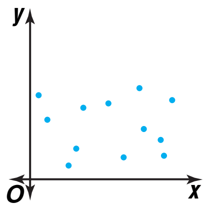
This scatter plot implies a linear relationship. Since data closely fits a line with a positive slope, the scatter plot shows a strong, positive correlation.



This scatter plot implies a linear relationship with a negative slope.



The points in this scatter plot are dispersed and do not form a linear pattern.



REVIEW EXERCISES

53. a. Graph the data below on a scatter plot.

b. Use two ordered pairs to write the equation of a best-fit line.

c. Use a graphing calculator to find an equation of the regression line for the data. What is the correlation value?

d. If the equation of the regression line shows a moderate or strong relationship, predict the number of visitors in 2005. Explain whether you think the prediction is reliable.

Overseas Visitors to the United States (thousands)

Year	1987	1988	1989	1990	1991
Visitors	10,434	12,763	12,184	12,252	12,003
Year	1992	1993	1994	1995	1996
Visitors	11,819	12,024	12,542	12,933	12,909

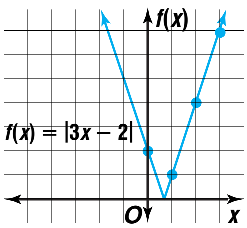
Source: U.S Dept. of Commerce

Lesson 1-7 Identify and graph piecewise functions including greatest integer, step, and absolute value functions.

Graph $f(x) = |3x - 2|$.

This is an absolute value function. Use a table of values to find points to graph.

x	$(x, f(x))$
0	(0, 2)
1	(1, 1)
2	(2, 4)
3	(3, 7)
4	(4, 10)



Graph each function.

54. $f(x) = \begin{cases} x \text{ if } 0 \le x \le 5 \\ 2 \text{ if } 5 < x \le 8 \end{cases}$

55. $h(x) = \begin{cases} -1 \text{ if } -2 \le x \le 0 \\ -3x \text{ if } 0 < x \le 2 \\ 2x \text{ if } 2 < x \le 4 \end{cases}$

56. $f(x) = [\![x]\!] + 1$

57. $g(x) = |4x|$

58. $k(x) = 2|x| + 2$

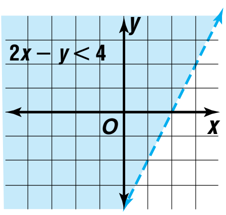
Lesson 1-8 Graph linear inequalities.

Graph the inequality $2x - y < 4$.

$2x - y < 4$

$y > 2x - 4$

The boundary is dashed. Testing (0, 0) yields a true inequality, so shade the region that includes (0, 0).



Graph each inequality.

59. $y > 4$

60. $x \le 5$

61. $x + y \le 1$

62. $2y - x < 4$

63. $y \le |x|$

64. $y - 3x > 2$

65. $y > |x| - 2$

66. $y < |x - 2|$

APPLICATIONS AND PROBLEM SOLVING

67. Aviation A jet plane start from rest on a runway. It accelerates uniformly at a rate of 20 m/s^2. The equation for computing the distance traveled is $d = \frac{1}{2}at^2$. *(Lesson 1-1)*

a. Find the distance traveled at the end of each second for 5 seconds.

b. Is this relation a function? Explain.

68. Finance In 1994, outstanding consumer credit held by commercial banks was about \$463 billion. By 1996, this amount had grown to about \$529 billion. *(Lesson 1-4)*

a. If x represents the year and y represents the amount of credit, find the average annual increase in the amount of outstanding consumer credit.

b. Write an equation to model the annual change in credit.

69. Recreation Juan wants to know the relationship between the number of hours students spend watching TV each week and the number of hours students spend reading each week. A sample of 10 students reveals the following data.

Watching TV	Reading
20	8.5
32	3.0
42	1.0
12	4.0
5	14.0
28	4.5
33	7.0
18	12.0
30	3.0
25	3.0

Find the equation of a regression line for the data. Then make a statement about how representative the line is of the data. *(Lesson 1-6)*

ALTERNATIVE ASSESSMENT

OPEN-ENDED ASSESSMENT

1. If $[f \circ g](x) = 4x^2 - 4$, find $f(x)$ and $g(x)$. Explain why your answer is correct.
2. Suppose two distinct lines have the same x-intercept.
   a. Can the lines be parallel? Explain your answer.
   b. Can the lines be perpendicular? Explain your answer.
3. Write a piecewise function whose graph is the same as each function. The function should not involve absolute value.
   a. $y = x + |4 - x|$
   b. $y = 2x + |x + 1|$

Additional Assessment See p. A56 for Chapter 1 Practice Test.

Unit 1 interNET Project

TELECOMMUNICATION

Is Anybody Listening?

- Research several telephone long-distance services. Write and graph equations to compare the monthly fee and the rate per minute for each service.
- Which service would best meet your needs? Write a paragraph to explain your choice. Use the graphs to support your choice.

PORTFOLIO

Select one of the functions you graphed in this chapter. Write about a real-world situation this type of function can be used to model. Explain what the function shows about the situation that is difficult to show by other means.

Multiple-Choice and Grid-In Questions

At the end of each chapter in this textbook, you will find practice for the SAT and ACT tests. Each group of 10 questions contains nine multiple-choice questions, and one grid-in question.

THE PRINCETON REVIEW

TEST-TAKING TIP

When you take the SAT, bring a calculator that you are used to using. Keep in mind that a calculator is not necessary to solve every question on the test. Also, a graphing calculator may provide an advantage over a scientific calculator on some questions.

MULTIPLE CHOICE

The majority of questions on the SAT are multiple-choice questions. As the name implies, these questions offer five choices from which to choose the correct answer.

The multiple choice sections are arranged in order of difficulty, with the easier questions at the beginning, average difficulty questions in the middle, and more difficult questions at the end.

Every correct answer earns one raw point, while an incorrect answer results in a loss of one fourth of a raw point. Leaving an answer blank results in no penalty.

The test covers topics from numbers and operations (arithmetic), algebra 1, algebra 2, functions, geometry, statistics, probability, and data analysis. Each end-of-chapter practice section in this textbook will cover one of these areas.

Arithmetic

Six percent of 4800 is equal to 12 percent of what number?

A 600

B 800

C 1200

D 2400

E 3000

Write and solve an equation.

$$0.06(4800) = 0.12x$$
$$288 = 0.12x$$
$$\frac{288}{0.12} = x$$
$$2400 = x$$

Choice **D** is correct.

Algebra

If $(p + 2)(p^2 - 4) = (p + 2)^q(p - 2)$ for all values of p, what is the value of q?

A 1

B 2

C 3

D 4

E It cannot be determined from the given information.

Factor the left side.

$$(p + 2)(p^2 - 4) = (p + 2)^q(p - 2)$$
$$(p + 2)(p + 2)(p - 2) = (p + 2)^q(p - 2)$$
$$(p + 2)^2(p - 2) = (p + 2)^q(p - 2)$$
$$(p + 2)^2 = (p + 2)^q$$

If $a^m = a^n$, then $m = n$.

$$2 = q$$

Answer choice **B** is correct.

Geometry

In the figure, what is the value of x?

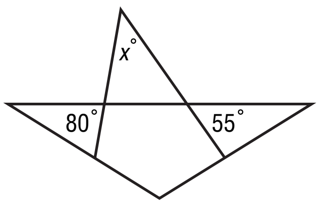
A 25

B 30

C 45

D 90

E 135



This is a multi-step problem. Use vertical angle relationships to determine that the two angles in the triangle with x are 80° and 55°. Then use the fact that the sum of the measures of the angles of a triangle is 180 to determine that x equals 45. The correct answer is choice **C**.

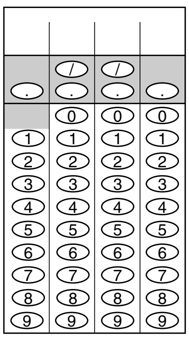
GRID IN

Another section on the SAT includes questions in which you must mark your answer on a grid printed on the answer sheet. These are called *Student Produced Response* questions (or Grid-Ins), because you must create the answer yourself, not just choose from five possible answers.

Every correct answer earns one raw point, but there is no penalty for a wrong answer; it is scored the same as no answer.

These questions are *not* more difficult than the multiple-choice questions, but you'll want to be extra careful when you fill in your answers on the grid, so that you don't make careless errors. Grid-in questions are arranged in order of difficulty.

The instructions for using the grid are printed in the SAT test booklet. *Memorize* these instructions before you take the test.

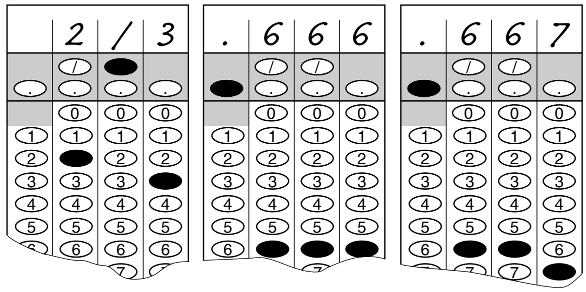


The grid contains a row of four boxes at the top, two rows of ovals with decimal and fraction symbols, and four columns of numbered ovals.

After you solve the problem, always write your answer in the boxes at the top of the grid.

Start with the left column. Write one numeral, decimal point, or fraction line in each box. Shade the oval in each column that corresponds to the numeral or symbol written in the box. Only the shaded ovals will be scored, so work carefully. Don't make any extra marks on the grid.

Suppose the answer is $\frac{2}{3}$ or 0.666 … . You can record the answer as a fraction or a decimal. For the fraction, write $\frac{2}{3}$. For a decimal answer, you must enter the most accurate value that will fit the grid. That is, you must enter as many decimal place digits as space allows. An entry of .66 would not be acceptable.



There is no 0 in bubble column 1. This means that you do *not* enter a zero to the left of the decimal point. For example, enter .25 and not 0.25.

Here are some other helpful hints for successfully completing grid-in questions.

- You don't have to write fractions in simplest form. Any equivalent fraction that fits the grid is counted as correct. If your fraction does not fit (like 15/25), then either write it in simplest form or change it to a decimal before you grid it.
- There is no negative symbol. Grid-in answers are never negative, so if you get a negative answer, you've made an error.
- If a problem has more than one correct answer, enter just one of the answers.
- Do not grid mixed numbers. Change the mixed number to an equivalent fraction or decimal. If you enter 11/2 for $1\frac{1}{2}$, it will be read as $\frac{11}{2}$. Enter it as 3/2 or 1.5.

Chapter 1 SAT & ACT Preparation

Arithmetic Problems

All SAT and ACT tests contain arithmetic problems. Some are easy and some are difficult. You'll need to understand and apply the following concepts.

odd and even	factors	divisibility
positive, negative	integers	fractions
scientific notation	exponents	roots
prime numbers	decimals	inequalities

Several concepts are often combined in a single problem.

THE PRINCETON REVIEW

TEST-TAKING TIP

Know the properties of zero and one. For example, 0 is even, neither positive nor negative, and not prime. 1 is the only integer with only one divisor. 1 is not prime.

SAT EXAMPLE

1. What is the sum of the positive even factors of 12?

HINT Look for words like *positive, even,* and *factor.*

Solution First find all the factors of 12.

1 2 3 4 6 12

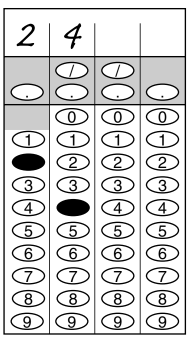
Re-read the question. It asks for the sum of *even* factors. Circle the factors that are even numbers.

1 2 3 4 6 12

Now add these even factors to find the sum.

$2 + 4 + 6 + 12 = 24$ The answer is 24.

This is a grid-in problem. Record your answer on the grid.



ACT EXAMPLE

2. $(-2)^3 + (3)^{-2} + \frac{8}{9}$

A -7 **B** $-1\frac{7}{9}$ **C** $\frac{8}{9}$

D $1\frac{7}{9}$ **E** 12

HINT Analyze what the − (negative) symbol represents each time it is used.

Solution Use the properties of exponents to simplify each term.

$(-2)^3 = (-2)(-2)(-2)$ or -8

$(3)^{-2} = \frac{1}{3^2}$ or $\frac{1}{9}$

Add the terms.

$$(-2)^3 + (3)^{-2} + \frac{8}{9} = -8 + \frac{1}{9} + \frac{8}{9}$$
$$= -8 + 1 \text{ or } -7$$

The answer is choice **A.**

Always look at the answer choices before you start to calculate. In this problem, three (incorrect) answer choices include fractions with denominators of 9. This may be a clue that your calculations may involve ninths.

Never assume that because three answer choices involve ninths and two are integers, that the correct answer is more likely to involve ninths. Also don't conclude that because the expression contains a fraction that the answer will necessarily have a fraction in it.

After you work each problem, record your answer on the answer sheet provided or on a piece of paper.

Multiple Choice

1. Which of the following expresses the prime factorization of 54?

   A 9×6

   B $3 \times 3 \times 6$

   C $3 \times 3 \times 2$

   D $3 \times 3 \times 3 \times 2$

   E 5.4×10

2. If 8 and 12 each divide K without a remainder, what is the value of K?

   A 16

   B 24

   C 48

   D 96

   E It cannot be determined from the information given.

3. After $\dfrac{4\frac{1}{3}}{2\frac{3}{5}}$ has been simplified to a single fraction in lowest terms, what is the denominator?

   A 2 B 3 C 5

   D 9 E 13

4. For a class play, student tickets cost \$2 and adult tickets cost \$5. A total of 30 tickets were sold. If the total sales must exceed \$90, what is the minimum number of adult tickets that must be sold?

   A 7 B 8 C 9

   D 10 E 11

5. $-|-7| - |-5| - 3|-4| = ?$

   A -24 B -11 C 0

   D 13 E 24

6. $(-4)^2 + (2)^{-4} + \frac{3}{4}$

   A $16\frac{13}{16}$ B $16\frac{3}{4}$ C $-15\frac{7}{32}$

   D $15\frac{7}{32}$ E 16

7. Kerri subscribed to four publications that cost \$12.90, \$16.00, \$18.00, and \$21.90 per year. If she made an initial down payment of one half of the total amount and paid the rest in 4 equal monthly payments, how much was each of the 4 monthly payments?

   A \$8.60

   B \$9.20

   C \$9.45

   D \$17.20

   E \$34.40

8. $\sqrt{64 + 36} = ?$

   A 10 B 14 C 28

   D 48 E 100

9. What is the number of distinct prime factors of 60?

   A 12

   B 4

   C 3

   D 2

   E 1

10. **Grid-In** There are 24 fish in an aquarium. If $\frac{1}{8}$ of them are tetras and $\frac{2}{3}$ of the remaining fish are guppies, how many guppies are in the aquarium?

internet CONNECTION **SAT/ACT Practice** For additional test practice questions, visit: **www.amc.glencoe.com**