VOCABULARY

Basic Counting Principle (p. 837)
binomial experiments (p. 876)
circular permutation (p. 847)
combination (p. 841)
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complements (p. 853)
conditional probability (p. 868)
dependent event (p. 837)
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failure (p. 852)
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sample space (p. 852)
simulation (p. 877)
success (p. 852)
theoretical probability (p. 877)
tree diagram (p. 837)

UNDERSTANDING AND USING THE VOCABULARY

Choose the correct term to best complete each sentence.

1. Events that do not affect each other are called (dependent, independent) events.
2. In probability, any outcome other than the desired outcome is called a (failure, success).
3. The sum of the probability of an event and the probability of the complement of the event is always (0, 1).
4. The (odds, probability) of an event occurring is the ratio of the number of ways the event can succeed to the sum of the number of ways the event can succeed and the number of ways the event can fail.
5. The arrangement of objects in a certain order is called a (combination, permutation).
6. A (permutation with repetitions, circular permutation) specifically deals with situations in which some objects that are alike.
7. Two (inclusive, mutually exclusive) events cannot happen at the same time.
8. A (sample space, Venn diagram) is the set of all possible outcomes of an event.
9. The probability of an event A given that event B has occurred is called a (conditional, inclusive) probability.
10. The branch of mathematics that studies different possibilities for the arrangement of objects is called (statistics, combinatorics).

interNET CONNECTION For additional review and practice for each lesson, visit: **www.amc.glencoe.com**

SKILLS AND CONCEPTS

OBJECTIVES AND EXAMPLES

Lesson 13-1 Solve problems related to the Basic Counting Principle.

- How many possible ways can a group of eight students line up to buy tickets to a play?

  There are eight choices for the first spot in line, seven choices for the second spot, six for the third spot, and so on.

  $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$

  There are 40,320 ways for the students to line up.

REVIEW EXERCISES

11. How many different ways can three books be arranged in a row on a shelf?

12. How many different ways can the digits 1, 2, 3, 4, and 5 be arranged to create a password?

13. How many ways can six teachers be assigned to teach six different classes, if each teacher can teach any of the classes?

Lesson 13-1 Solve problems involving permutations and combinations.

- From a choice of 3 meat toppings and 4 vegetable toppings, how many 5-topping pizzas are possible?

  Since order is not important, the selection is a combination of 7 objects taken 5 at a time, or $C(7, 5)$.

  $C(7, 5) = \frac{7!}{(7-5)!\,5!} = 21$

  There are 21 possible 5-topping pizzas.

Find each value.

14. $P(6, 3)$

15. $P(8, 6)$

16. $C(5, 3)$

17. $C(11, 8)$

18. $\frac{P(6, 3)}{P(5, 3)}$

19. $C(5, 5) \cdot C(3, 2)$

20. How many ways can 6 different books be placed on a shelf if the only dictionary must be on an end?

21. From a group of 3 men and 7 women, how many committees of 2 men and 2 women can be formed?

Lesson 13-2 Solve problems involving permutations with repetitions.

- How many ways can the letters of *Tallahassee* be arranged?

  There are 3 *a*'s, 2 *l*'s, 2 *s*'s, and 2 *e*'s. So the number of possible arrangements is $\frac{11!}{3!2!2!2!}$ or 831,600 ways.

How many different ways can the letters of each word be arranged?

22. *level*

23. *Cincinnati*

24. *graduate*

25. *banana*

26. How many different 9-digit Social Security numbers can have the digits 2, 9, 5, 5, 0, 7, 0, 5, and 8.

OBJECTIVES AND EXAMPLES

Lesson 13-3 Find the probability of an event.

Find the probability of randomly selecting 3 red pencils from a box containing 5 red, 3 blue, and 4 green pencils.

There are $C(5, 3)$ ways to select 3 out of 5 red pencils and $C(12, 3)$ ways to select 3 out of 12 pencils.

$$P(\text{3 red pencils}) = \frac{C(5, 3)}{C(12, 3)}$$
$$= \frac{\frac{5!}{2!\,3!}}{\frac{12!}{9!\,3!}}$$
$$= \frac{12}{220} \text{ or } \frac{1}{22}$$

REVIEW EXERCISES

A bag containing 7 pennies, 4 nickels, and 5 dimes. Three coins are drawn at random. Find each probability.

27. P(3 pennies)

28. P(2 pennies and 1 nickel)

29. P(3 nickels)

30. P(1 nickel and 2 dimes)

Lesson 13-3 Find the odds for the success and failure of an event.

Find the odds of randomly selecting 3 red pencils from a box containing 5 red, 3 blue, and 4 green pencils.

$$P(\text{3 red pencils}) = P(s) = \frac{1}{22}$$
$$P(\text{not 3 red pencils}) = P(f) = 1 - \frac{1}{22} \text{ or } \frac{21}{22}$$
$$\text{Odds} = \frac{P(s)}{P(f)} = \frac{\frac{1}{22}}{\frac{21}{22}}$$
$$= \frac{1}{21} \text{ or } 1{:}21$$

Refer to the bag of coins used for Exercises 27-30. Find the odds of each event occurring.

31. 3 pennies

32. 2 pennies and 1 nickel

33. 3 nickels

34. 1 nickel and 2 dimes

Lesson 13-4 Find the probability of independent and dependent events.

Three yellow and 5 black marbles are placed in a bag. What is the probability of drawing a black marble, replacing it, and then drawing a yellow marble?

$$P(\text{black}) = \frac{5}{8} \qquad P(\text{yellow}) = \frac{3}{8}$$
$$P(\text{black and yellow}) = P(\text{black}) \cdot P(\text{yellow})$$
$$= \frac{5}{8} \cdot \frac{3}{8} = \frac{15}{64}$$

Determine if each event is *independent* or *dependent*. Then determine the probability.

35. the probability of rolling a sum of 2 on the first toss of two number cubes and a sum of 6 on the second toss

36. the probability of randomly selecting two yellow markers from a box that contains 4 yellow and 6 pink markers

SKILLS AND CONCEPTS

OBJECTIVES AND EXAMPLES

Lesson 13-4 Find the probability of mutually exclusive and inclusive events.

- On a school board, 2 of the 4 female members are over 40 years of age, and 5 of the 6 male members are over 40. If one person did not attend the meeting, what is the probability that the person was a male or a member over 40?

$$P(\text{male or over } 40) = P(\text{male}) + P(\text{over } 40) - P(\text{male \& over } 40)$$
$$= \frac{6}{10} + \frac{7}{10} - \frac{5}{10} \text{ or } \frac{4}{5}$$

REVIEW EXERCISES

A box contains slips of paper numbered from 1 to 14. One slip of paper is drawn at random. Find each probability.

37. P(selecting a prime number or a multiple of 4)

38. P(selecting a multiple of 2 or a multiple of 3)

39. P(selecting a 3 or a 4)

40. P(selecting an 8 or a number less than 8)

Lesson 13-5 Find the probability of an event given the occurrence of another event.

- A coin is tossed 3 times. What is the probability that at the most 2 heads are tossed given that at least 1 head has been tossed?

Let event A be that at most 2 heads are tossed.
Let event B be that there is at least 1 head.

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$
$$= \frac{\frac{6}{8}}{\frac{7}{8}} \text{ or } \frac{6}{7}$$

Two number cubes are tossed.

41. What is the probability that the sum of the numbers shown on the cubes is less than 5 if exactly one cube shows a 1?

42. What is the probability that the numbers shown on the cubes are different given that their sum is 8?

43. What is the probability that the numbers shown on the cubes match given that their sum is greater than or equal to 5?

Lesson 13-6 Find the probability of an event by using the Binomial Theorem.

- If you guess the answers on all 8 questions of a true/false quiz, what is the probability that exactly 5 of your answers will be correct?

$$(p+q)^8 = \sum_{r=0}^{8} \frac{8!}{r!(8-r)!} p^{8-r}q^r$$
$$= \frac{8!}{5!(8-5)!}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^3$$
$$= \frac{56}{256} \text{ or } \frac{7}{32}$$

Find each probability if a coin is tossed 4 times.

44. P(exactly 1 head)

45. P(no heads)

46. P(2 heads and 2 tails)

47. P(at least 3 tails)

APPLICATIONS AND PROBLEM SOLVING

48. Travel Five people, including the driver, can be seated in Nate's car. Nate and 6 of his friends want to go to a movie. How many different groups of friends can ride in Nate's car on the first trip if the car is full? *(Lesson 13-1)*

49. Sommer has 7 different keys. How many ways can she place these keys on the key ring shown below? *(Lesson 13-2)*

50. Quality Control A collection of 15 memory chips contains 3 chips that are defective. If 2 memory chips are selected at random, what is the probability that at least one of them is good? *(Lesson 13-3)*

51. Gift Exchange The Burnette family is drawing names from a bag for a gift exchange. There are 7 males and 8 females in the family. If someone draws their own name, then they must draw again before replacing their name. *(Lesson 13-4)*

a. Reba draws the first name. What is the probability that Reba will draw a female's name that is not her own?

b. What is the probability that Reba will draw her own name, not replace it, and then draw a male's name?

ALTERNATIVE ASSESSMENT

OPEN-ENDED ASSESSMENT

1. The probability of two independent events occurring is $\frac{1}{12}$. If the probability of one of the events occurring is $\frac{1}{2}$, is it possible to find the probability of the other event? If so, find the probability and give an example of a situation for which this probability could apply. If not, explain why not.

2. Perry says "A permutation is the same as a combination." How would you explain to Perry that his statement is incorrect?

PORTFOLIO

Choose one of the types of probability you studied in this chapter. Describe a situation in which this type of probability would be used. Explain why no other type of probability should be used in this situation.

Unit 4 interNET Project

THE UNITED STATES CENSUS BUREAU

Radically Random!

- Use the Internet to find the population of the United States by age groups or ethnic background for the most recent census.
- Make a table or spreadsheet of the data.
- Suppose that a person was selected at random from all the people in the United States to answer some survey questions. Find the probability that the person was from each one of the age or ethnic groups you used for your table or spreadsheet.
- Write a summary describing how you calculated the probabilities. Include a graph with your summary. Discuss why someone might be interested in your findings.

Additional Assessment See p. A68 for Chapter 13 practice test.

Probability and Combination Problems

Both the ACT and SAT contain probability problems. You'll need to know these concepts:

Combinations	Permutations	Tree Diagram
Outcomes	Probability	

Memorize the definition of the probability of an event:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

THE PRINCETON REVIEW

TEST-TAKING TIP

For problems involving combinations, either use the formula or make a list.

Example:

$C(5, 3) = \frac{5 \cdot 4 \cdot 3}{3!} = 10$

SAT EXAMPLE

1. A bag contains 4 red balls, 10 green balls, and 6 yellow balls. If three balls are removed at random and no ball is returned to the bag after removal, what is the probability that all three balls will be green?

A $\frac{1}{2}$ **B** $\frac{1}{8}$ **C** $\frac{3}{20}$ **D** $\frac{2}{19}$ **E** $\frac{3}{8}$

HINT Calculate the probability of two independent events by multiplying the probability of the first event by the probability of the second event.

Solution Use the definition of the probability of an event. Calculate the probability of getting a green ball each time a ball is removed. The first time a ball is removed there are a total of 20 balls and 10 of them are green. So the probability of removing a green ball as the first ball is $\frac{10}{20}$ or $\frac{1}{2}$. Now there are just 19 balls and 9 of them are green. The probability of removing a green ball is $\frac{9}{19}$. When the third ball is removed, there are 18 balls and 8 of them are green, so the probability of removing a green ball is $\frac{8}{18}$ or $\frac{4}{9}$. To find the probability of removing green balls as the first *and* the second *and* the third balls chosen, multiply the three probabilities.

$$\frac{1}{2} \times \frac{9}{19} \times \frac{4}{19} = \frac{2}{19}$$

The answer is choice **D**.

ACT EXAMPLE

2. If you toss 3 fair coins, what is the probability of getting exactly 2 heads?

A $\frac{1}{3}$ **B** $\frac{3}{8}$

C $\frac{1}{2}$ **D** $\frac{2}{3}$

E $\frac{7}{8}$

HINT Start by listing all the possible outcomes. You can do this since the numbers are small.

Solution Make a list and then count the possible outcomes.

HHH, HHT, HTH, HTT,

TTH, THT, THH, TTT

There are 8 possible outcomes for 3 coins. Since the coins are fair, these are equally likely outcomes.

The favorable outcomes are those that include exactly 2 heads: HHT, HTH, THH. There are 3 favorable outcomes. Give the answer.

$$P(A) = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

$$P(\text{exactly 2 heads}) = \frac{3}{8}$$

The answer is choice **B**.

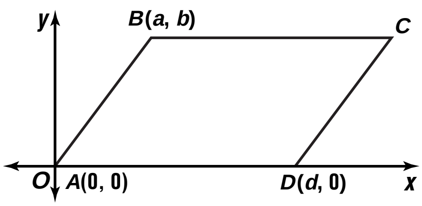
After you work each problem, record your answer on the answer sheet provided or on a piece of paper.

Multiple Choice

1. A coin was flipped 20 times and came up heads 10 times and tails 10 times. If the first and the last flips were both heads, what is the greatest number of consecutive heads that could have occurred?

A 1 **B** 2 **C** 8
D 9 **E** 10

2. In the figure below, *ABCD* is a parallelogram. What must be the coordinates of Point *C*?



A (x, y) **B** $(d + a, y)$ **C** $(d - a, b)$
D $(d + x, b)$ **E** $(d + a, b)$

3. In a plastic jar there are 5 red marbles, 7 blue marbles, and 3 green marbles. How many green marbles need to be added to the jar in order to double the probability of selecting a green marble?

A 2 **B** 3 **C** 5 **D** 6 **E** 7

4. The average of 5 numbers is 20. If one of the numbers is 18, then what is the sum of the other four numbers?

A 2 **B** 20.5 **C** 82
D 90 **E** 100

5. If the sum of x and y is an even number, and the sum of x and z is an even number, and z is an odd number, then which of the following must be true?

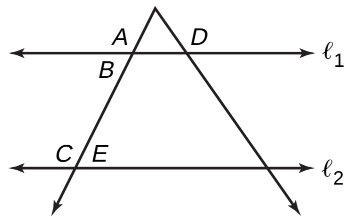
I. y is an even number
II. $y + z$ is an even number
III. y is an odd number

A I only **B** II only **C** III only
D I and II **E** II and III

6. A bag contains only white and blue marbles. The probability of selecting a blue marble is $\frac{1}{5}$. The bag contains 200 marbles. If 100 white marbles are added to the bag, what is the probability of selecting a white marble?

A $\frac{2}{15}$ **B** $\frac{7}{15}$ **C** $\frac{8}{15}$ **D** $\frac{4}{5}$ **E** $\frac{13}{15}$

7. In the figure below, $\ell_1 \parallel \ell_2$. Which of the labeled angles must be equal to each other?



A *A* and *C* **B** *D* and *E* **C** *A* and *B*
D *D* and *B* **E** *C* and *B*

8. What is the probability of drawing a diamond from a well-shuffled standard deck of playing cards?

A $\frac{1}{52}$ **B** $\frac{1}{13}$ **C** $\frac{1}{4}$
D $\frac{4}{13}$ **E** 1

9. A caterer offers 7 different entrees. A customer may choose any 3 of the entrees for a dinner. How many different combinations of entrees can a customer choose?

A 6
B 35
C 84
D 210
E 840

10. Grid-In Six cards are numbered 0 through 5. Two are selected without replacement. What is the probability that the sum of the cards is 4?