VOCABULARY

additive identity matrix (p. 80)
column matrix (p. 78)
consistent (p. 67)
dependent (p. 67)
determinant (p. 98)
dilation (p. 88)
dimensions (p. 78)
element (p. 78)
elimination method (p. 68)
equal matrices (p. 79)
identity matrix for multiplication (p. 99)
image (p. 88)
inconsistent (p. 67)
independent (p. 67)
infeasible (p. 113)
inverse matrix (p. 99)
$m \times n$ matrix (p. 78)
matrix (p. 78)
minor (p. 98)
*n*th order (p. 78)
ordered triple (p. 74)
polygonal convex set (p. 108)
pre-image (p. 88)
reflection (p. 88)
reflection matrix (p. 89)
rotation (p. 88)
rotation matrix (p. 91)
row matrix (p. 78)
scalar (p. 80)
solution (p. 67)
square matrix (p. 78)
substitution method (p. 68)
system of equations (p. 67)
system of linear inequalities (p. 107)
transformation (p. 88)
translation (p. 88)
translation matrix (p. 88)
vertex matrix (p. 88)
Vertex Theorem (p. 108)
zero matrix (p. 80)

Modeling

alternate optimal solutions (p. 114)
constraints (p. 112)
linear programming (p. 112)
unbounded (p. 113)

UNDERSTANDING AND USING THE VOCABULARY

Choose the correct term from the list to complete each sentence.

1. Sliding a polygon from one location to another without changing its size, shape, or orientation is called a(n) __?__ of the figure.
2. Two matrices can be __?__ if they have the same dimensions.
3. The __?__ of $\begin{bmatrix} -1 & 4 \\ 2 & -3 \end{bmatrix}$ is -5.
4. A(n) __?__ system of equations has no solution.
5. The process of multiplying a matrix by a constant is called __?__.
6. The matrices $\begin{bmatrix} 2x \\ 0 \\ 16 \end{bmatrix}$ and $\begin{bmatrix} 8-y \\ y \\ 4x \end{bmatrix}$ are __?__ if $x = 4$ and $y = 0$.
7. A region bounded on all sides by intersecting linear inequalities is called a(n) __?__.
8. A rotation of a figure can be achieved by consecutive __?__ of the figure over given lines.
9. The symbol a_{ij} represents a(n) __?__ of a matrix.
10. Two matrices can be __?__ if the number of columns in the first matrix is the same as the number of rows in the second matrix.

added
consistent
determinant
dilation
divided
element
equal matrices
identity matrices
inconsistent
inverse
multiplied
polygonal convex set
reflections
scalar multiplication
translation

interNET CONNECTION For additional review and practice for each lesson, visit: **www.amc.glencoe.com**

SKILLS AND CONCEPTS

OBJECTIVES AND EXAMPLES

Lesson 2-1 Solve systems of two linear equations.

Solve the system of equations.

$y = -x + 2$

$3x + 4y = 2$

Substitute $-x + 2$ for y in the second equation.

$$\begin{aligned} 3x + 4y &= 2 \\ 3x + 4(-x + 2) &= 2 \\ x &= 6 \end{aligned} \qquad \begin{aligned} y &= -x + 2 \\ y &= -6 + 2 \\ y &= -4 \end{aligned}$$

The solution is $(6, -4)$.

REVIEW EXERCISES

Solve each system of equations algebraically.

11. $2y = -4x$
$y = -x - 2$

12. $y = x - 5$
$6y - x = 0$

13. $2x = 5y$
$3y + x = -1$

14. $y = 6x + 1$
$2y - 15x = -4$

15. $3x - 2y = -1$
$2x + 5y = 12$

16. $x + 5y = 20.5$
$3y - x = 13.5$

Lesson 2-2 Solve systems of equations involving three variables algebraically.

Solve the system of equations.

$x - y + z = 5$

$-3x + 2y - z = -8$

$2x + y + z = 4$

Combine pairs of equations to eliminate z.

$$\begin{array}{rcl} x - y + z &=& 5 \\ -3x + 2y - z &=& -8 \\ \hline -2x + y &=& -3 \end{array} \qquad \begin{array}{rcl} -3x + 2y - z &=& -8 \\ 2x + y + z &=& 4 \\ \hline -x + 3y &=& -4 \end{array}$$

$-5y = 5$

$y = -1$

$-2x + (-1) = -3 \rightarrow x = 1$

$(1) - (-1) + z = 5 \rightarrow z = 3$ *Solve for z.*

The solution is $(1, -1, 3)$.

Solve each system of equations algebraically.

17. $x - 2y - 3z = 2$
$-3x + 5y + 4z = 0$
$x - 4y + 3z = 14$

18. $-x + 2y - 6z = 4$
$x + y + 2z = 3$
$2x + 3y - 4z = 5$

19. $x - 2y + z = 7$
$3x + y - z = 2$
$2x + 3y + 2z = 7$

Lesson 2-3 Add, subtract, and multiply matrices.

Find the difference.

$$\begin{bmatrix} -4 & 7 \\ 6 & -3 \end{bmatrix} - \begin{bmatrix} 8 & -3 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -4 - 8 & 7 - (-3) \\ 6 - 2 & -3 - 0 \end{bmatrix} = \begin{bmatrix} -12 & 10 \\ 4 & -3 \end{bmatrix}$$

Find the product.

$$-2\begin{bmatrix} 1 & -4 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2(1) & -2(-4) \\ -2(0) & -2(3) \end{bmatrix} = \begin{bmatrix} -2 & 8 \\ 0 & -6 \end{bmatrix}$$

Use matrices A, B, and C to find each of the following. If the matrix does not exist, write *impossible*.

$$A = \begin{bmatrix} 7 & 8 \\ 0 & -4 \end{bmatrix} \quad B = \begin{bmatrix} -3 & -5 \\ 2 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$$

20. $A + B$

21. $B - A$

22. $3B$

23. $-4C$

24. AB

25. CB

26. $4A - 4B$

27. $AB - 2C$

OBJECTIVES AND EXAMPLES

Lesson 2-4 Use matrices to determine the coordinates of polygons under a given transformation.

Reflections

$$R_{x\text{-axis}} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$R_{y\text{-axis}} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$R_{y=x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Rotations (counterclockwise about the origin)

$$Rot_{90} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$Rot_{180} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$Rot_{270} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

REVIEW EXERCISES

Use matrices to perform each transformation. Then graph the pre-image and image on the same coordinate grid.

28. $A(-4, 3)$, $B(2, 1)$, $C(5, -3)$ translated 4 units down and 3 unit left
29. $W(-2, -3)$, $X(-1, 2)$, $Y(0, 4)$, $Z(1, -2)$ reflected over the x-axis
30. $D(2, 3)$, $E(2, -5)$, $F(-1, -5)$, $G(-1, 3)$ rotated 180° about the origin.
31. $P(3, -4)$, $Q(1, 2)$, $R(-1, 1)$ dilated by a scale factor of 0.5
32. triangle ABC with vertices at $A(-4, 3)$, $B(2, 1)$, $C(5, -3)$ after $Rot_{90} \circ R_{x\text{-axis}}$
33. What translation matrix would yield the same result on a triangle as a translation 6 units up and 4 units left followed by a translation 3 units down and 5 units right?

OBJECTIVES AND EXAMPLES

Lesson 2-5 Evaluate determinants.

Find the value of $\begin{vmatrix} -7 & 6 \\ 5 & -2 \end{vmatrix}$.

$$\begin{vmatrix} -7 & 6 \\ 5 & -2 \end{vmatrix} = (-7)(-2) - 5(6)$$
$$= -16$$

REVIEW EXERCISES

Find the value of each determinant.

34. $\begin{vmatrix} -3 & 5 \\ -4 & 7 \end{vmatrix}$
35. $\begin{vmatrix} 8 & -4 \\ -6 & 3 \end{vmatrix}$
36. $\begin{vmatrix} 3 & -1 & 4 \\ 5 & -2 & 6 \\ 7 & 3 & -4 \end{vmatrix}$
37. $\begin{vmatrix} 5 & 0 & -4 \\ 7 & 3 & -1 \\ 2 & -2 & 6 \end{vmatrix}$
38. Determine whether $\begin{bmatrix} 2 & -4 & 1 \\ 3 & 8 & -2 \end{bmatrix}$ has a determinant. If so, find the value of the determinant. If not, explain.

OBJECTIVES AND EXAMPLES

Lesson 2-5 Find the inverse of a 2×2 matrix.

Find X^{-1}, if $X = \begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}$.

$$\begin{vmatrix} 2 & -1 \\ 1 & -3 \end{vmatrix} = -6 - (-1) \text{ or } -5 \text{ and } -5 \neq 0$$

$$X^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= -\frac{1}{5}\begin{bmatrix} -3 & 1 \\ -1 & 2 \end{bmatrix}$$

REVIEW EXERCISES

Find the inverse of each matrix, if it exists.

39. $\begin{bmatrix} 3 & 8 \\ -1 & 5 \end{bmatrix}$
40. $\begin{bmatrix} 5 & 2 \\ 10 & 4 \end{bmatrix}$
41. $\begin{bmatrix} -3 & 5 \\ 1 & -4 \end{bmatrix}$
42. $\begin{bmatrix} 3 & 2 \\ 5 & 7 \end{bmatrix}$
43. $\begin{bmatrix} 2 & -5 \\ 6 & 1 \end{bmatrix}$
44. $\begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$

SKILLS AND CONCEPTS

OBJECTIVES AND EXAMPLES

Lesson 2-5 Solve systems of equations by using inverses of matrices.

- Solve the system of equations $3x - 5y = 1$ and $-2x + 2y = -2$ by using a matrix equation.

$$\begin{bmatrix} 3 & -5 \\ -2 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$-\frac{1}{4}\begin{bmatrix} 2 & 5 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -5 \\ -2 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} 2 & 5 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

REVIEW EXERCISES

Solve each system by using a matrix equation.

45. $2x + 5y = 1$
$-x - 3y = 2$

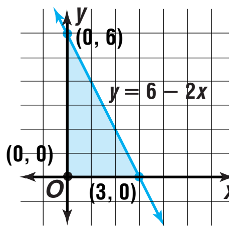
46. $3x + 2y = -3$
$-6x + 4y = 6$

47. $-3x + 5y = 1$
$-2x + 4y = -2$

48. $4.6x - 2.7y = 8.4$
$2.9x + 8.8y = 74.61$

Lesson 2-6 Find the maximum or minimum value of a function defined for a polygonal convex set.

- Find the maximum and minimum values of $f(x, y) = 4y + x - 3$ for the polygonal convex set graphed at the right.



$f(0, 0) = 4(0) + 0 - 3 = -3$ *minimum*
$f(3, 0) = 4(0) + 3 - 3 = 0$
$f(0, 6) = 4(6) + 0 - 3 = 21$ *maximum*

Find the maximum and minimum values of each function for the polygonal convex set determined by the given system of inequalities.

49. $f(x, y) = 2x + 3y$
$x \geq 1$
$y \geq -2$
$y + x \leq 6$
$y \leq 10 - 2x$

50. $f(x, y) = 3x + 2y + 1$
$x \geq 0$
$y \geq 4$
$y + x \leq 11$
$2y + x \leq 18$
$x \leq 6$

Lesson 2-7 Use linear programming procedures to solve applications.

Linear Programming Procedure

1. Define variables.
2. Write the constraints as a system of inequalities.
3. Graph the system and find the coordinates of the vertices of the polygon formed.
4. Write an expression to be maximized or minimized.
5. Substitute values from the coordinates of the vertices into the expression.
6. Select the greatest or least result.

Use linear programming to solve.

51. Transportation Justin owns a truck and a motorcycle. He can buy up to 28 gallons of gasoline for both vehicles. His truck gets 22 miles per gallon and holds up to 25 gallons of gasoline. His motorcycle gets 42 miles per gallon and holds up to 6 gallons of gasoline. How many gallons of gasoline should Justin put in each vehicle if he wants to travel the most miles possible?

APPLICATIONS AND PROBLEM SOLVING

52. Sports In a three-team track meet, the following numbers of first-, second-, and third-place finishes were recorded.

School	First Place	Second Place	Third Place
Broadman	2	5	5
Girard	8	2	3
Niles	6	4	1

Use matrix multiplication to find the final scores for each school if 5 points are awarded for a first place, 3 for second place, and 1 for third place. (Lesson 2-4)

53. Geometry The perimeter of a triangle is 83 inches. The longest side is three times the length of the shortest side and 17 inches more than one-half the sum of the other two sides. Use a system of equations to find the length of each side. (Lesson 2-5)

54. Manufacturing A toy manufacturer produces two types of model spaceships, the *Voyager* and the *Explorer.* Each toy requires the same three operations. Each *Voyager* requires 5 minutes for molding, 3 minutes for machining, and 5 minutes for assembly. Each *Explorer* requires 6 minutes for molding, 2 minutes for machining, and 18 minutes for assembly. The manufacturer can afford a daily schedule of not more than 4 hours for molding, 2 hours for machining, and 9 hours for assembly. (Lesson 2-7)

a. If the profit is \$2.40 on each *Voyager* and \$5.00 on each *Explorer,* how many of each toy should be produced for maximum profit?

b. What is the maximum daily profit?

ALTERNATIVE ASSESSMENT

OPEN-ENDED ASSESSMENT

1. Suppose that a quadrilateral *ABCD* has been rotated 90° clockwise about the origin twice. The resulting vertices are $A'(2, 2)$, $B'(-1, 2)$, $C'(-2, -1)$, and $D'(3, 0)$.

a. State the original coordinates of the vertices of quadrilateral *ABCD* and how you determined them.

b. Make a conjecture about the effect of a double rotation of 90° on any given figure.

2. If the determinant of a coefficient matrix is 0, can you use inverse matrices to solve the system of equations? Explain your answer and illustrate it with such a system of equations.

Additional Assessment See p. A57 for Chapter 2 Practice Test.

Unit 1 interNET Project

TELECOMMUNICATION

You've Got Mail!

- Research several Internet servers and make graphs that reflect the cost of using the server over a year's time period.
- Research various e-mail servers and their costs. Write and graph equations to compare the costs.
- Determine which Internet and e-mail servers best meet your needs. Write a paragraph to explain your choice. Use your graphs to support your choice.

PORTFOLIO

Devise a real-world problem that can be solved by linear programming. Define whether you are seeking a maximum or minimum. Write the inequalities that define the polygonal convex set used to determine the solution. Explain what the solution means.

Algebra Problems

About one third of SAT math problems and many ACT math problems involve algebra. You'll need to simplify algebraic expressions, solve equations, and solve word problems.

The word problems often deal with specific problems.

- consecutive integers
- age
- motion (*distance* = *rate* × *time*)
- investments (*principal* × *rate* = *interest income*)
- work
- coins
- mixtures

THE PRINCETON REVIEW

TEST-TAKING TIP

Review the rules for simplifying algebraic fractions and expressions. Try to simplify expressions whenever possible.

ACT EXAMPLE

1. $\dfrac{(xy)^3z^0}{x^3y^4} =$

A $\dfrac{1}{y}$ **B** $\dfrac{z}{y}$ **C** z **D** xy **E** xyz

HINT Review the properties of exponents.

Solution Simplify the expression. Apply the properties of exponents.

$(xy)^3 = x^3y^3$.

Any number raised to the zero power is equal to 1. Therefore, $z^0 = 1$.

Write y^4 as y^3y^1.

Simplify.

$$\frac{(xy)^3z^0}{x^3y^4} = \frac{x^3y^3 \cdot 1}{x^3y^3y^1} = \frac{1}{y}$$

The answer is choice **A.**

SAT EXAMPLE

2. The sum of two positive consecutive integers is x. In terms of x, what is the value of the smaller of these two integers?

A $\dfrac{x}{2} - 1$ **B** $\dfrac{x-1}{2}$ **C** $\dfrac{x}{2}$

D $\dfrac{x+1}{2}$ **E** $\dfrac{x}{2} + 1$

HINT On multiple-choice questions with variables in the answer choices, you can sometimes use a strategy called "plug-in."

Solution The "plug-in" strategy uses substitution to test the choices. Suppose the two numbers were 2 and 3. Then $x = 2 + 3$ or 5. Substitute 5 for x and see which choice yields 2, the smaller of the two numbers.

Choice A: $\dfrac{5}{2} - 1$ This is not an integer.

Choice B: $\dfrac{5-1}{2} = 2$ This is the answer, but check the rest just to be sure.

Alternate Solution You can solve this problem by writing an algebraic expression for each number. Let a be the first (smallest) positive integer. Then $(a + 1)$ is the next positive integer. Write an equation for "The sum of the two numbers is x" and solve for a.

$$\begin{aligned} a + (a + 1) &= x \\ 2a + 1 &= x \\ 2a &= x - 1 \\ a &= \frac{x-1}{2} \end{aligned}$$

The answer is choice **B.**

After you work each problem, record your answer on the answer sheet provided or on a piece of paper.

Multiple Choice

1. If the product of $(1 + 2)$, $(2 + 3)$, and $(3 + 4)$ is equal to one half the sum of 20 and x, then $x =$

A 10 **B** 85 **C** 105 **D** 190 **E** 1,210

2. $5\frac{1}{3} - 6\frac{1}{4} = ?$

A $-\frac{11}{12}$

B $-\frac{1}{2}$

C $-\frac{2}{7}$

D $\frac{1}{2}$

E $\frac{9}{12}$

3. Mia has a pitcher containing x ounces of root beer. If she pours y ounces of root beer into each of z glasses, how much root beer will remain in the pitcher?

A $\frac{x}{y} + z$

B $xy - z$

C $\frac{x}{yz}$

D $x - yz$

E $\frac{x}{y} - z$

4. Which of the following is equal to 0.064?

A $\left(\frac{1}{80}\right)^2$ **B** $\left(\frac{8}{100}\right)^2$ **C** $\left(\frac{1}{8}\right)^2$

D $\left(\frac{2}{5}\right)^3$ **E** $\left(\frac{8}{10}\right)^3$

5. A plumber charges \$75 for the first thirty minutes of each house call plus \$2 for each additional minute that she works. The plumber charged Mr. Adams \$113 for her time. For what amount of time, in minutes, did the plumber work?

A 38 **B** 44 **C** 49 **D** 59 **E** 64

6. If $\frac{2 + x}{5 + x} = \frac{2}{5} + \frac{2}{5}$, then $x =$

A $\frac{2}{5}$ **B** 1 **C** 2 **D** 5 **E** 10

7. Which of the following must be true?

I. The sum of two consecutive integers is odd.

II. The sum of three consecutive integers is even.

III. The sum of three consecutive integers is a multiple of 3.

A I only

B II only

C I and II only

D I and III only

E I, II, and III

8. Jose has at least one quarter, one dime, one nickel, and one penny in his pocket. If he has twice as many pennies as nickels, twice as many nickels as dimes, and twice as many dimes as quarters, then what is the least amount of money he could have in his pocket?

A \$0.41 **B** \$0.64 **C** \$0.71

D \$0.73 **E** \$2.51

9. Simplify $\dfrac{\frac{3}{2}}{\left(\frac{3}{2}\right)^2}$.

A $\frac{27}{8}$

B $\frac{3}{2}$

C $\frac{2}{3}$

D $\frac{1}{2}$

E $\frac{1}{3}$

10. Grid-In At a music store, the price of a CD is three times the price of a cassette tape. If 40 CDs were sold for a total of \$480 and the combined sales of CDs and cassette tapes totaled \$600, how many cassette tapes were sold?

interNET CONNECTION **SAT/ACT Practice** For additional test practice questions, visit: **www.amc.glencoe.com**