VOCABULARY

absolute maximum (p.171)
absolute minimum (p. 171)
asymptotes (p. 180)
constant function (pp. 137, 164)
constant of variation (p. 189)
continuous (p. 160)
critical point (p. 171)
decreasing function (p. 164)
direct variation (p. 189)
discontinuous (p. 159)
end behavior (p. 162)
even function (p. 133)
everywhere discontinuous (p. 159)
extremum (p. 171)
horizontal asymptote (p. 180)
horizontal line test (p. 153)
image point (p. 127)
increasing function (p. 164)
infinite discontinuity (p. 159)
inverse function (pp. 152, 155)
inverse process (p. 154)
inversely proportional (p. 191)
inverse relations (p. 152)
joint variation (p. 192)
jump discontinuity (p. 159)
line symmetry (p. 129)
maximum (p. 171)
minimum (p. 171)
monotonicity (p. 163)
odd function (p. 133)
parent graph (p. 137)
point discontinuity (p. 159)
point of inflection (p. 171)
point symmetry (p. 127)
rational function (p. 180)
relative extremum (p. 172)
relative maximum (p. 172)
relative minimum (p. 172)
slant asymptote (p. 183)
symmetry with respect to the origin (p. 128)
vertical asymptote (p. 180)

UNDERSTANDING AND USING THE VOCABULARY

Choose the correct term to best complete each sentence.

1. An (odd, even) function is symmetric with respect to the *y*-axis.
2. If you can trace the graph of a function without lifting your pencil, then the graph is (continuous, discontinuous).
3. When there is a value in the domain for which a function is undefined, but the pieces of the graph match up, then the function has (infinite, point) discontinuity.
4. A function f is (decreasing, increasing) on an interval I if and only if for every a and b contained in I, $f(a) > f(b)$ whenever $a < b$.
5. When the graph of a function is increasing to the left of $x = c$ and decreasing to the right of $x = c$, then there is a (maximum, minimum) at c.
6. A (greatest integer, rational) function is a quotient of two polynomial functions.
7. Two relations are (direct, inverse) relations if and only if one relation contains the element (b, a) whenever the other relation contains the element (a, b).
8. A function is said to be (monotonic, symmetric) on an interval I if and only if the function is increasing on I or decreasing on I.
9. A (horizontal, slant) asymptote occurs when the degree of the numerator of a rational expression is exactly one greater than that of the denominator.
10. (Inverse, Joint) variation occurs when one quantity varies directly as the product of two or more other quantities.

interNET CONNECTION For additional review and practice for each lesson, visit: **www.amc.glencoe.com**

SKILLS AND CONCEPTS

OBJECTIVES AND EXAMPLES

Lesson 3-1 Use algebraic tests to determine if the graph of a relation is symmetrical.

- Determine whether the graph of $f(x) = 4x - 1$ is symmetric with respect to the origin.

$f(-x) = 4(-x) - 1$
$\quad = -4x - 1$
$-f(x) = -(4x - 1)$
$\quad = -4x + 1$

The graph of $f(x) = 4x - 1$ is not symmetric with respect to the origin because $f(-x) \neq -f(x)$.

REVIEW EXERCISES

Determine whether the graph of each function is symmetric with respect to the origin.

11. $f(x) = -2x$
12. $f(x) = x^2 + 2$
13. $f(x) = x^2 - x + 3$
14. $f(x) = x^3 - 6x + 1$

Determine whether the graph of each function is symmetric with respect to the x-axis, y-axis, the line $y = x$, the line $y = -x$, or none of these.

15. $xy = 4$
16. $x + y^2 = 4$
17. $x = -2y$
18. $x^2 = \frac{1}{y}$

Lesson 3-2 Identify transformations of simple graphs.

- Describe how the graphs of $f(x) = x^2$ and $g(x) = x^2 - 1$ are related.

Since 1 is subtracted from $f(x)$, the parent function, $g(x)$ is the graph of $f(x)$ translated 1 unit down.

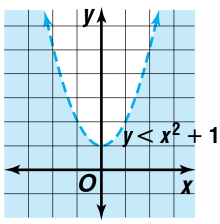
Describe how the graphs of $f(x)$ and $g(x)$ are related.

19. $f(x) = x^4$ and $g(x) = x^4 + 5$
20. $f(x) = |x|$ and $g(x) = |x + 2|$
21. $f(x) = x^2$ and $g(x) = 6x^2$
22. $f(x) = [\![x]\!]$ and $g(x) = \left[\!\left[\frac{3}{4}x\right]\!\right] - 4$

Lesson 3-3 Graph polynomial, absolute value, and radical inequalities in two variables.

- Graph $y < x^2 + 1$.

The boundary of the inequality is the graph of $y = x^2 + 1$. Since the boundary is not included, the parabola is dashed.



Graph each inequality.

23. $y > |x + 2|$
24. $y \leq -2x^3 + 4$
25. $y < (x + 1)^2 + 2$
26. $y \geq \sqrt{2x - 3}$

Solve each inequality.

27. $|4x + 5| > 7$
28. $|x - 3| + 2 \leq 11$

Lesson 3-4 Determine inverses of relations and functions.

- Find the inverse of $f(x) = 4(x - 3)^2$.

$y = 4(x - 3)^2$ *Let $y = f(x)$.*
$x = 4(y - 3)^2$ *Interchange x and y.*
$\frac{x}{4} = (y - 3)^2$ *Solve for y.*
$\pm\sqrt{\frac{x}{4}} = y - 3$
$3 \pm \sqrt{\frac{x}{4}} = y$ So, $f^{-1}(x) = 3 \pm \sqrt{\frac{x}{4}}$.

Graph each function and its inverse.

29. $f(x) = 3x - 1$
30. $f(x) = -\frac{1}{4}x + 5$
31. $f(x) = \frac{2}{x} + 3$
32. $f(x) = (x + 1)^2 - 4$

Find $f^{-1}(x)$. Then state whether $f^{-1}(x)$ is a function.

33. $f(x) = (x - 2)^3 - 8$
34. $f(x) = 3(x + 7)^4$

OBJECTIVES AND EXAMPLES

Lesson 3-5 Determine whether a function is continuous or discontinuous.

Determine whether the function $y = \frac{x}{x+4}$ is continuous at $x = -4$.

Start with the first condition of the continuity test. The function is not defined at $x = -4$ because substituting -4 for x results in a denominator of zero. So the function is discontinuous at $x = -4$.

REVIEW EXERCISES

Determine whether each function is continuous at the given *x*-value. Justify your response using the continuity test.

35. $y = x^2 + 2;\ x = 2$

36. $y = \frac{x-3}{x+1};\ x = -1$

37. $f(x) = \begin{cases} x + 1 \text{ if } x \le 1 \\ 2x \text{ if } x > 1 \end{cases};\ x = 1$

Lesson 3-5 Identify the end behavior of functions.

Describe the end behavior of $f(x) = 3x^4$.

Make a chart investigating the value of f(x) for very large and very small values of x.

x	f(x)
−10,000	3×10^{16}
−1000	3×10^{12}
−100	3×10^{8}
0	0
100	3×10^{8}
1000	3×10^{12}
10,000	3×10^{16}

$y \to \infty$ as $x \to \infty$,
$y \to \infty$ as $x \to -\infty$

Describe the end behavior of each function.

38. $y = 1 - x^3$

39. $f(x) = x^9 + x^7 + 4$

40. $y = \frac{1}{x^2} + 1$

41. $y = 12x^5 + x^3 - 3x^2 + 4$

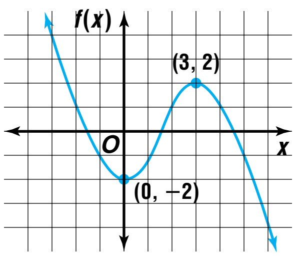
Determine the interval(s) for which the function is increasing and the interval(s) for which the function is decreasing.

42. $y = -2x^3 - 3x^2 + 12x$

43. $f(x) = |x^2 - 9| + 1$

Lesson 3-6 Find the extrema of a function.

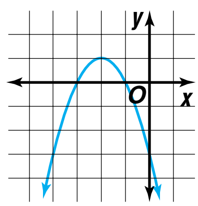
Locate the extrema for the graph of $y = f(x)$. Name and classify the extrema of the function.



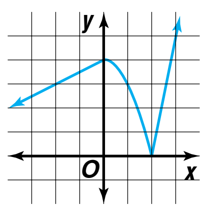
The function has a relative minimum at $(0, -2)$ and a relative maximum at $(3, 2)$.

Locate the extrema for the graph of $y = f(x)$. Name and classify the extrema of the function.

44.



45.



Determine whether the given critical point is the location of a *maximum*, a *minimum*, or a *point of inflection.*

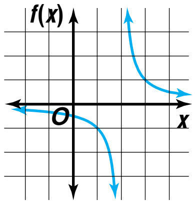
46. $x^3 - 6x^2 + 9x,\ x = 3$

47. $4x^3 + 7,\ x = 0$

OBJECTIVES AND EXAMPLES

Lesson 3-7 Graph rational functions.

- The graph at the right shows a transformation of $f(x) = \frac{1}{x}$. Write an equation of the function.

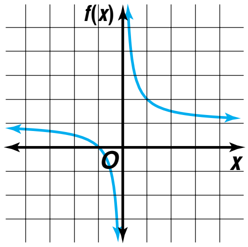


The graph of $f(x) = \frac{1}{x}$ has been translated 2 units to the right. So, the equation of the function is $f(x) = \frac{1}{x - 2}$.

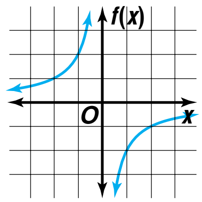
REVIEW EXERCISES

Each graph below shows a transformation of $f(x) = \frac{1}{x}$. Write an equation of each function.

48.



49.



Use the parent graph $f(x) = \frac{1}{x}$ to graph each equation. Describe the transformation(s) that have taken place. Identify the new locations of the asymptotes.

50. $\frac{3}{x + 2}$

51. $\frac{2x - 5}{x - 3}$

Lesson 3-7 Determine vertical, horizontal, and slant asymptotes.

- Determine the equation of the horizontal asymptote of $g(x) = \frac{5x - 9}{x - 3}$.

To find the horizontal asymptote, use division to rewrite the rational expression as a quotient.

$$\begin{array}{r} 5 \\ x - 3\overline{)5x - 9} \\ \underline{5x - 15} \\ 4 \end{array}$$

Therefore, $g(x) = 5 + \frac{4}{x - 3}$. As $x \to \pm\infty$, the value of $\frac{4}{x - 3}$ approaches 0. The value of $5 + \frac{4}{x - 3}$ approaches 5. The line $y = 5$ is the horizontal asymptote.

Determine the equations of the vertical and horizontal asymptotes, if any, of each function.

52. $f(x) = \frac{x}{x - 1}$

53. $g(x) = \frac{x^2 + 1}{x + 2}$

54. $h(x) = \frac{(x - 3)^2}{x^2 - 9}$

55. Does the function $f(x) = \frac{x^2 + 2x + 1}{x}$ have a slant asymptote? If so, find an equation of the slant asymptote. If not, explain.

Lesson 3-8 Solve problems involving direct, inverse, and joint variation.

- If y varies inversely as the square of x and $y = 8$ when $x = 3$, find x when $y = 6$.

$\frac{x_1^n}{y_2} = \frac{x_2^n}{y_1}$

$\frac{3^2}{6} = \frac{x_2^2}{8}$ *$n = 2, x_1 = 3, y_1 = 8, y_2 = 6$*

$6x_2^2 = 72$ *Cross multiply.*

$x_2^2 = 12$ *Divide each side by 6.*

$x_2 = \sqrt{12}$ or $2\sqrt{3}$

Find the constant of variation and use it to write an equation for each statement. Then solve the equation.

56. If y varies jointly as x and z and $y = 5$ when $x = -4$ and $z = -2$, find y when $x = -6$ and $z = -3$.

57. If y varies inversely as the square root of x and $y = 20$ when $x = 49$, find x when $y = 10$.

58. If y varies directly as the square of x and inversely as z and $y = 7.2$ when $x = 0.3$ and $z = 4$, find y when $x = 1$ and $z = 40$.

APPLICATIONS AND PROBLEM SOLVING

59. Manufacturing The length of a part for a bicycle must be 6.5 ± 0.2 centimeters. If the actual length of the part is x, write an absolute value inequality to describe this situation. Then find the range of possible lengths for the part. *(Lesson 3-3)*

60. Consumer Costs A certain copy center charges users \$0.40 for every minute or part of a minute to use their computer scanner. Suppose that you use their scanner for x minutes, where x is any real number greater than 0. *(Lesson 3-4)*

a. Sketch the graph of the function, $C(x)$, that gives the cost of using the scanner for x minutes.

b. What are the domain and range of $C(x)$?

c. Sketch the graph of $C^{-1}(x)$.

d. What are the domain and range of $C^{-1}(x)$?

e. What real-world situation is modeled by $C^{-1}(x)$?

61. Sports One of the most spectacular long jumps ever performed was by Bob Beamon of the United States at the 1968 Olympics.

His jump of 8.9027 meters surpassed the world record at that time by over half a meter! The function $h(t) = 4.6t - 4.9t^2$ describes the height of Beamon's jump (in meters) with respect to time (in seconds). *(Lesson 3-6)*

a. Draw a graph of this function.

b. What was the maximum height of his jump?

ALTERNATIVE ASSESSMENT

OPEN-ENDED ASSESSMENT

1. Write and then graph an equation that exhibits symmetry with respect to the
   a. x-axis.
   b. y-axis.
   c. line $y = x$.
   d. line $y = -x$.
   e. origin.

2. Write the equation of a parent function, other than the identity or constant function, after it has been translated right 4 units, reflected over the x-axis, expanded vertically by a factor of 2, and translated 1 unit up.

3. A graph has one absolute minimum, one relative minimum, and one relative maximum.
   a. Draw the graph of a function for which this is true.
   b. Name and classify the extrema of the function you graphed.

Additional Assessment See page A58 for Chapter 3 practice test.

Unit 1 interNET Project

TELECOMMUNICATION

Sorry, You are Out of Range for Your Telephone Service . . .

- Research several cellular phone services to determine their initial start-up fee, equipment fee, the monthly service charge, the charge per minute for calls, and any other charges.
- Compare the costs of the cellular phone services you researched by writing and graphing equations.

Determine which cellular phone service would best suit your needs. Write a paragraph to explain your choice. Use graphs and area maps to support your choice.

PORTFOLIO

Choose one of the functions you studied in this chapter. Describe the graph of the function and how it can be used to model a real-life situation.

TEST-TAKING TIP

If a problem seems to require lengthy calculations, look for a shortcut. There is probably a quicker way to solve it. Try to eliminate fractions and decimals. Try factoring.

More Algebra Problems

SAT and ACT tests include quadratic expressions and equations. You should be familiar with common factoring formulas, like the difference of two squares or perfect square trinomials.

$a^2 - b^2 = (a + b)(a - b)$
$a^2 + 2ab + b^2 = (a + b)^2$
$a^2 - 2ab + b^2 = (a - b)^2$

Some problems involve systems of equations. Simplify the equations if possible, and then add or subtract them to eliminate one of the variables.

ACT EXAMPLE

1. If $\frac{x^2 - 9}{x + 3} = 12$, then $x = ?$

A 10 **B** 15 **C** 17 **D** 19 **E** 20

HINT Look for factorable quadratics.

Solution Factor the numerator and simplify.

$$\frac{(x + 3)(x - 3)}{x + 3} = 12$$

$x - 3 = 12$ *$\frac{x + 3}{x + 3} = 1$*

$x = 15$ *Add 3 to each side.*

The answer is choice **B.**

Alternate Solution You can also solve this type of problem, with a variable in the question and numbers in the answer choices, with a strategy called "backsolving."

Substitute each answer choice for the variable into the given expression or equation and find which one makes the statement true.

Try choice A. For $x = 10$, $\frac{10^2 - 9}{10 + 3} \neq 12$.

Try choice B. For $x = 15$, $\frac{15^2 - 9}{15 + 3} = 12$.

Therefore, choice **B** is correct.

If the number choices were large, then calculations, even using a calculator, would probably take longer than solving the problem using algebra. In this case, it is *not* a good idea to use the backsolving strategy.

SAT EXAMPLE

2. If $x = y + 1$ and $y \geq 1$, then which of the following must be equal to $x^2 - y^2$?

A $(x - y)^2$ **B** $x^2 - y - 1$ **C** $x + y$

D $x^2 - 1$ **E** $y^2 + 1$

HINT This difficult problem has variables in the answers. It can be solved by using algebra or the "Plug-in" strategy.

Solution Notice the word *must.* This means the relationship is true for all possible values of x and y.

To use the Plug-In strategy, choose a number greater than 1 for y, say 4. Then x must be 5. Since $x^2 - y^2 = 25 - 16$ or 9, check each expression choice to see if it is equal to 9 when $x = 5$ and $y = 4$.

Choice A: $(x - y)^2 = 1$
Choice B: $x^2 - y - 1 = 20$
Choice C: $x + y = 9$

Choice **C** is correct.

Alternate Solution You can also use algebraic substitution to find the answer. Recall that $x^2 - y^2 = (x + y)(x - y)$. The given equation, $x = y + 1$, includes y. Substitute $y + 1$ for x in the second term.

$$\begin{aligned} x^2 - y^2 &= (x + y)(x - y) \\ &= (x + y)[(y + 1) - y] \\ &= (x + y)(1) \\ &= x + y \end{aligned}$$

This is choice **C.**

After you work each problem, record your answer on the answer sheet provided or on a piece of paper.

Multiple Choice

1. For all $y \neq 3$, $\frac{y^2 - 9}{3y - 9} = ?$

   A y B $\frac{y + 1}{8}$

   C $y + 1$ D $\frac{y}{3}$

   E $\frac{y + 3}{3}$

2. If $x + y = z$ and $x = y$, then all of the following are true EXCEPT

   A $2x + 2y = 2z$

   B $x - y = 0$

   C $x - z = y - z$

   D $x = \frac{z}{2}$

   E $z - y = 2x$

3. The Kims drove 450 miles in each direction to Grandmother's house and back again. If their car gets 25 miles per gallon and their cost for gasoline was $1.25 per gallon for the trip to Grandmother's house, but $1.50 per gallon for the return trip, how much *more* money did they spend for gasoline returning from Grandmother's house than they spent going to Grandmother's?

   A $2.25 B $4.50

   C $6.25 D $9.00

   E $27.00

4. If $x + 2y = 8$ and $\frac{x}{2} - y = 10$, then $x = ?$

   A -7 B 0

   C 10 D 14

   E 28

5. $\frac{900}{10} + \frac{90}{100} + \frac{9}{1000} =$

   A 90.09 B 90.099

   C 90.909 D 99.09

   E 999

6. For all x, $(10x^4 - x^2 + 2x - 8) - (3x^4 + 3x^3 + 2x + 9) = ?$

   A $7x^4 - 3x^3 - x^2 - 17$

   B $7x^4 - 4x^2 - 17$

   C $7x^4 + 3x^3 - x^2 + 4x$

   D $7x^4 + 2x^2 + 4x$

   E $13x^4 - 3x^3 + x^2 + 4x$

7. If $\frac{n}{8}$ has a remainder of 5, then which of the following has a remainder of 7?

   A $\frac{n + 1}{8}$ B $\frac{n + 2}{8}$

   C $\frac{n + 3}{8}$ D $\frac{n + 5}{8}$

   E $\frac{n + 7}{8}$

8. If $x > 0$, then $\frac{\sqrt{100x^2 + 600x + 900}}{x + 3} = ?$

   A 9 B 10 C 30 D 40

   E It cannot be determined from the information given.

9. What is the value of c?

   Given: $a + b = c$
   $a - c = 5$
   $b - c = 3$

   A -10
   B -8
   C -5
   D -3
   E 3

10. **Grid-In** If $4x + 2y = 24$ and $\frac{7y}{2x} = 7$, then $x = ?$

interNET CONNECTION **SAT/ACT Practice** For additional test practice questions, visit: **www.amc.glencoe.com**