VOCABULARY

- completing the square (p. 213)
- complex number (p. 206)
- conjugate (p. 216)
- degree (p. 206)
- depressed polynomial (p. 224)
- Descartes' Rule of Signs (p. 231)
- discriminant (p. 215)
- extraneous solution (p. 251)
- Factor Theorem (p. 224)
- Fundamental Theorem of Algebra (p. 207)
- imaginary number (p. 206)
- Integral Root Theorem (p. 230)
- leading coefficient (p. 206)
- Location Principle (p. 236)
- lower bound (p. 238)
- Lower Bound Theorem (p. 238)
- partial fractions (p. 244)
- polynomial equation (p. 206)
- polynomial function (p. 206)
- polynomial in one variable (p. 205)
- pure imaginary number (p. 206)
- Quadratic Formula (p. 215)
- radical equation (p. 251)
- radical inequality (p. 253)
- rational equation (p. 243)
- rational inequality (p. 245)
- Rational Root Theorem (p. 229)
- Remainder Theorem (p. 222)
- root (p. 206)
- synthetic division (p. 223)
- upper bound (p. 238)
- Upper Bound Theorem (p. 238)
- zero (p. 206)

UNDERSTANDING AND USING THE VOCABULARY

Choose the correct term from the list to complete each sentence.

- complex numbers
- complex roots
- discriminant
- extraneous
- Factor Theorem
- Integral Root Theorem
- lower bound
- polynomial function
- quadratic equation
- Quadratic Formula
- radical equation
- synthetic division
- zero

1. The __?__ can be used to solve any quadratic equation.
2. The __?__ states that if the leading coefficient of a polynomial equation a_0 has a value of 1, then any rational root must be factors of a_n.
3. For a rational equation, any possible solution that results with a __?__ in the denominator must be excluded from the list of solutions.
4. The __?__ states that the binomial $x - r$ is a factor of the polynomial $P(x)$ if and only if $P(r) = 0$.
5. Descartes' Rule of Signs can be used to determine the possible number of positive real zeros a __?__ has.
6. A(n) __?__ of the zeros of $P(x)$ can be found by determining an upper bound for the zeros of $P(-x)$.
7. __?__ solutions do not satisfy the original equation.
8. Since the x-axis only represents real numbers, __?__ of a polynomial function cannot be determined by using a graph.
9. The Fundamental Theorem of Algebra states that every polynomial equation with degree greater than zero has at least one root in the set of __?__
10. A __?__ is a special polynomial equation with a degree of two.

interNET CONNECTION For additional review and practice for each lesson, visit: **www.amc.glencoe.com**

SKILLS AND CONCEPTS

OBJECTIVES AND EXAMPLES

Lesson 4-1 Determine roots of polynomial equations.

- Determine whether 2 is a root of $x^4 - 3x^3 - x^2 - x = 0$. Explain.

  $f(2) = 2^4 - 3(2^3) - 2^2 - 2$

  $f(2) = 16 - 24 - 4 - 2$ or -14

  Since $f(2) \neq 0$, 2 is not a root of $x^4 - 3x^3 - x^2 - x = 0$.

REVIEW EXERCISES

Determine whether each number is a root of $a^3 - 3a^2 - 3a - 4 = 0$. Explain.

11. 0 **12.** 4 **13.** -2

14. Is -3 a root of $t^4 - 2t^2 - 3t + 1 = 0$?

15. State the number of complex roots of the equation $x^3 + 2x^2 - 3x = 0$. Then find the roots and graph the related function.

Lesson 4-2 Solve quadratic equations.

- Find the discriminant of $3x^2 - 2x - 5 = 0$ and describe the nature of the roots of the equation. Then solve the equation by using the Quadratic Formula.

  The value of the discriminant, $b^2 - 4ac$, is $(-2)^2 - 4(3)(-5)$ or 64. Since the value of the discriminant is greater than zero, there are two distinct real roots.

  $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

  $$x = \frac{-(-2) \pm \sqrt{64}}{2(3)}$$

  $$x = \frac{2 \pm 8}{6}$$

  $$x = -1 \text{ or } \frac{5}{3}$$

Find the discriminant of each equation and describe the nature of the roots of the equation. Then solve the equation by using the Quadratic Formula.

16. $2x^2 - 7x - 4 = 0$

17. $3m^2 - 10m + 5 = 0$

18. $x^2 - x + 6 = 0$

19. $-2y^2 + 3y + 8 = 0$

20. $a^2 + 4a + 4 = 0$

21. $5r^2 - r + 10 = 0$

Lesson 4-3 Find the factors of polynomials using the Remainder and Factor Theorems.

- Use the Remainder Theorem to find the remainder when $(x^3 + 2x^2 - 5x - 9)$ is divided by $(x + 3)$. State whether the binomial is a factor of the polynomial.

  $f(x) = x^3 + 2x^2 - 5x - 9$

  $f(-3) = (-3)^3 + 2(-3)^2 - 5(-3) - 9$

  $= -27 + 18 + 15 - 9$ or -3

  Since $f(-3) = -3$, the remainder is -3. So the binomial $x + 3$ is not a factor of the polynomial by the Remainder Theorem.

Use the Remainder Theorem to find the remainder for each division. State whether the binomial is a factor of the polynomial.

22. $(x^3 - x^2 - 10x - 8) \div (x + 2)$

23. $(2x^3 - 5x^2 + 7x + 1) \div (x - 5)$

24. $(4x^3 - 7x + 1) \div \left(x + \frac{1}{2}\right)$

25. $(x^4 - 10x^2 + 9) \div (x - 3)$

OBJECTIVES AND EXAMPLES

Lesson 4-4 Identify all possible rational roots of a polynomial equation by using the Rational Root Theorem.

- List the possible rational roots of $4x^3 - x^2 - x - 5 = 0$. Then determine the rational roots.

  If $\frac{p}{q}$ is a root of the equation, then p is a factor of 5 and q is a factor of 4.
  possible values of p: $\pm 1, \pm 5$
  possible values of q: $\pm 1, \pm 2, \pm 4$

  possible rational roots: $\pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{5}{2}, \pm \frac{5}{4}$

  Graphing and substitution show a zero at $\frac{5}{4}$.

REVIEW EXERCISES

List the possible rational roots of each equation. Then determine the rational roots.

26. $x^3 - 2x^2 - x + 2 = 0$
27. $x^4 - x^2 + x - 1 = 0$
28. $2x^3 - 2x^2 - 2x - 4 = 0$
29. $2x^4 + 3x^3 - 6x^2 - 11x - 3 = 0$
30. $x^5 - 7x^3 + x^2 + 12x - 4 = 0$
31. $3x^3 + 7x^2 - 2x - 8 = 0$
32. $4x^3 + x^2 + 8x + 2 = 0$
33. $x^4 + 4x^2 - 5 = 0$

Lesson 4-4 Determine the number and type of real roots a polynomial function has.

- For $f(x) = 3x^4 - 9x^3 + 4x - 6$, there are three sign changes. So there are three or one positive real zeros.

  For $f(-x) = 3x^4 + 9x^3 - 4x - 6$, there is one sign change. So there is one negative real zero.

Find the number of possible positive real zeros and the number of possible negative real zeros for each function. Then determine the rational zeros.

34. $f(x) = x^3 - x^2 - 34x - 56$
35. $f(x) = 2x^3 - 11x^2 + 12x + 9$
36. $f(x) = x^4 - 13x^2 + 36$

Lesson 4-5 Approximate the real zeros of a polynomial function.

- Determine between which consecutive integers the real zeros of $f(x) = x^3 + 4x^2 + x - 2$ are located.
  Use synthetic division.

r	1	4	1	−2	
−4	1	0	1	−6	*change*
−3	1	1	2	4	*in signs*
−2	1	2	−3	4	
−1	1	3	−2	0	*zero*
0	1	4	1	−2	*change*
1	1	5	6	4	*in signs*

  One zero is −1. Another is located between −4 and −3. The other is between 0 and 1.

Determine between which consecutive integers the real zeros of each function are located.

37. $g(x) = 3x^3 + 1$
38. $f(x) = x^2 - 4x + 2$
39. $g(x) = x^2 - 3x - 3$
40. $f(x) = x^3 - x^2 + 1$
41. $g(x) = 4x^3 + x^2 - 11x + 3$
42. $f(x) = -9x^3 + 25x^2 - 24x + 6$
43. Approximate the real zeros of $f(x) = 2x^3 + 9x^2 - 12x - 40$ to the nearest tenth.

OBJECTIVES AND EXAMPLES

Lesson 4-6 Solve rational equations and inequalities.

Solve $\frac{1}{9} + \frac{1}{2a} = \frac{1}{a^2}$.

$$\frac{1}{9} + \frac{1}{2a} = \frac{1}{a^2}$$
$$\left(\frac{1}{9} + \frac{1}{2a}\right)(18a^2) = \left(\frac{1}{a^2}\right)(18a^2)$$
$$2a^2 + 9a = 18$$
$$2a^2 + 9a - 18 = 0$$
$$(2a - 3)(a + 6) = 0$$
$$a = \frac{3}{2} \text{ or } -6$$

REVIEW EXERCISES

Solve each equation or inequality.

44. $n - \frac{6}{n} + 5 = 0$

45. $\frac{1}{x} = \frac{x + 3}{2x^2}$

46. $\frac{5}{6} = \frac{2m}{2m + 2} - \frac{1}{3m - 3}$

47. $\frac{3}{y} - 2 < \frac{5}{y}$

48. $\frac{2}{x + 1} < 1 - \frac{1}{x - 1}$

Lesson 4-7 Solve radical equations and inequalities.

Solve $9 + \sqrt{x - 1} = 1$.

$$9 + \sqrt{x - 1} = 1$$
$$\sqrt{x - 1} = -8$$
$$x - 1 = 64$$
$$x = 65$$

Solve each equation or inequality.

49. $5 - \sqrt{x + 2} = 0$

50. $\sqrt[3]{4a - 1} + 8 = 5$

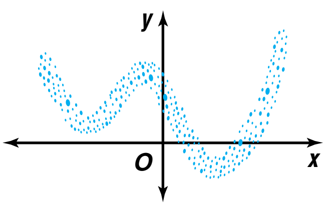
51. $3 + \sqrt{x + 8} = \sqrt{x + 35}$

52. $\sqrt{x - 5} < 7$

53. $4 + \sqrt{2a + 7} \geq 6$

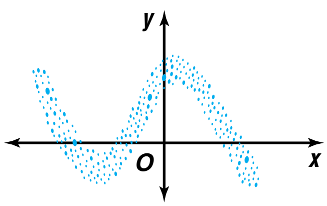
Lesson 4-8 Write polynomial functions to model real-world data.

Determine the type of polynomial function that would best fit the data in the scatter plot.



The scatter plot seems to change direction three times. So a quartic function would best fit the scatter plot.

54. Determine the type of polynomial function that would best fit the scatter plot.



55. Write a polynomial function to model the data.

x	−3	−1	0	2	4	7
f(x)	24	6	3	9	31	94

APPLICATIONS AND PROBLEM SOLVING

56. Entertainment The scenery for a new children's show has a playhouse with a painted window. A special gloss paint covers the area of the window to make them look like glass. If the gloss only covers 315 square inches and the window must be 6 inches taller than it is wide, how large should the scenery painters make the window? *(Lesson 4-1)*

57. Gardening The length of a rectangular flower garden is 6 feet more than its width. A walkway 3 feet wide surrounds the outside of the garden. The total area of the walkway itself is 288 square feet. Find the dimensions of the garden. *(Lesson 4-2)*

58. Medicine Doctors can measure cardiac output in potential heart attack patients by monitoring the concentration of dye after a known amount in injected in a vein near the heart. In a normal heart, the concentration of the dye is given by $g(x) = -0.006x^4 + 0.140x^3 - 0.053x^2 + 1.79x$, where x is the time in seconds. *(Lesson 4-4)*

a. Graph $g(x)$

b. Find all the zeros of this function.

59. Physics The formula $T = 2\pi\sqrt{\frac{\ell}{g}}$ is used to find the period T of a oscillating pendulum. In this formula, ℓ is the length of the pendulum, and g is acceleration due to gravity. Acceleration due to gravity is 9.8 meters per second squared. If a pendulum has an oscillation period of 1.6 seconds, determine the length of the pendulum. *(Lesson 4-7)*

ALTERNATIVE ASSESSMENT

OPEN-ENDED ASSESSMENT

1. Write a rational equation that has at least two solutions, one which is 2. Solve your equation.

2. a. Write a radical equation that has solutions of 3 and 6, one of which is extraneous.

   b. Solve your equation. Identify the extraneous solution and explain why it is extraneous.

3. a. Write a set of data that is best represented by a cubic equation.

   b. Write a polynomial function to model the set of data.

   c. Approximate the real zeros of the polynomial function to the nearest tenth.

PORTFOLIO

Explain how you can use the leading coefficient and the degree of a polynomial equation to determine the number of possible roots of the equation.

Unit 1 interNET Project

TELECOMMUNICATION

The Pen is Mightier than the Sword!

- Gather all materials obtained from your research for the mini-projects in Chapters 1, 2, and 3. Decide what types of software would help you to prepare a presentation.
- Research websites that offer downloads of software including work processing, graphics, spreadsheet, and presentation software. Determine whether the software is a demonstration version or free shareware. Select at least two different programs for each of the four categories listed above.
- Prepare a presentation of your Unit 1 project using the software that you found. Be sure that you include graphs and maps in the presentation.

Additional Assessment See p. A59 for Chapter 4 practice test.

CHAPTER 4 SAT & ACT Preparation

Coordinate Geometry Problems

The ACT test usually includes several coordinate geometry problems. You'll need to know and apply these formulas for points (x_1, y_1) and (x_2, y_2):

Midpoint	Distance	Slope
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$\frac{y_2 - y_1}{x_2 - x_1}$

The SAT test includes problems that involve coordinate points. But they aren't easy!

THE PRINCETON REVIEW

TEST-TAKING TIP

- Draw diagrams.
- Locate points on the grid or number line.
- Eliminate any choices that are clearly incorrect.

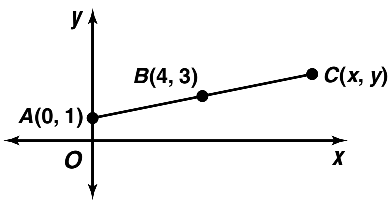
ACT EXAMPLE

1. Point $B(4,3)$ is the midpoint of line segment AC. If point A has coordinates (0,1), then what are the coordinates of point C?

A $(-4, -1)$ **B** (4, 1) **C** (4, 4)
D (8, 5) **E** (8, 9)

HINT Draw a diagram. You may be able to solve the problem without calculations.

Solution Draw a diagram showing the known quantities and the unknown point C.



Since C lies to the right of B and the x-coordinate of A is not 4, any points with an x-coordinate of 4 or less can be eliminated. So eliminate choices A, B, and C.

Use the Midpoint Formula. Consider the x-coordinates. Write an equation in x.

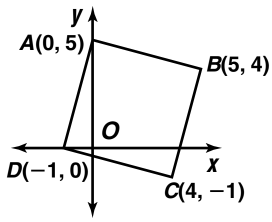
$$\frac{0 + x}{2} = 4$$
$$x = 8$$

Do the same for y.
$$\frac{1 + y}{2} = 3$$
$$y = 5$$

The coordinates of C are (8, 5) The answer is choice **D.**

SAT EXAMPLE

2. What is the area of square $ABCD$ in square units?



A 25 **B** $18\sqrt{2}$ **C** 26
D $25 + \sqrt{2}$ **E** 36

HINT Estimate the answer to eliminate impossible choices and to check your calculations.

Solution First estimate the area. Since the square's side is a little more than 5, the area is a little more than 25. Eliminate choices A and E.

To find the area, find the measure of a side and square it. Choose side AD, because points A and D have simple coordinates. Use the Distance Formula.

$$(\overline{AD})^2 = (-1 - 0)^2 + (0 - 5)^2$$
$$= (-1)^2 + (-5)^2$$
$$= 1 + 25 \text{ or } 26$$

The answer is choice **C.**

Alternate Solution You could also use the Pythagorean Theorem. Draw right triangle DOA, with the right angle at O, the origin. Then DO is 1, OA is 5, and DA is $\sqrt{26}$. So the area is 26 square units.

After you work each problem, record your answer on the answer sheet provided or on a piece of paper.

Multiple Choice

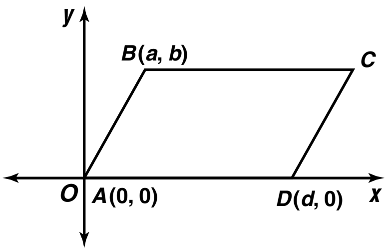
1. What is the length of the line segment whose endpoints are represented on the coordinate axis by points at $(-2, 1)$ and $(1, -3)$?

   A 3 **B** 4 **C** 5
   D 6 **E** 7

2. $\left(\frac{4}{5} \times 3\right)\left(\frac{3}{4} \times 5\right)\left(\frac{5}{3} \times 4\right) =$

   A 1 **B** 3 **C** 6 **D** 20 **E** 60

3. In the figure below, *ABCD* is a parallelogram. What are the coordinates of point C?



   A (x, y) **B** $(d + a, y)$
   C $(d - a, b)$ **D** $(d + x, b)$
   E $(d + a, b)$

4. A rectangular garden is surrounded by a 60-foot long fence. One side of the garden is 6 feet longer than the other. Which equation could be used to find s, the shorter side, of the garden?

   A $8s + s = 60$
   B $4s = 60 + 12$
   C $s(s + 6) = 60$
   D $2(s - 6) + 2s = 60$
   E $2(s + 6) + 2s = 60$

5. What is the slope of a line perpendicular to the line represented by the equation $3x - 6y = 12$?

   A -2 **B** $\frac{-1}{2}$ **C** $\frac{1}{3}$
   D $\frac{1}{2}$ **E** 2

6. If x is an integer, which of the following could be x^3?

   A 2.7×10^{11}
   B 2.7×10^{12}
   C 2.7×10^{13}
   D 2.7×10^{14}
   E 2.7×10^{15}

7. If $0x + 5y = 14$ and $4x - y = 2$, then what is the value of $6x + 6y$

   A 2 **B** 7 **C** 12 **D** 16 **E** 24

8. What is the midpoint of the line segment whose endpoints are represented on the coordinate grid by points at $(3, 5)$ and $(-4, 3)$?

   A $(-2, -5)$ **B** $\left(-\frac{1}{2}, 4\right)$ **C** $(1, 8)$
   D $\left(4, \frac{1}{2}\right)$ **E** $(3, 3)$

9. If $ax \neq 0$, which quantity must be non-negative?

   A $x^2 - 2ax + a^2$
   B $-2ax$
   C $2ax$
   D $x^2 - a^2$
   E $a^2 - x^2$

10. **Grid-In** Points E, F, G, and H lie on a line in that order. If $EG = \frac{5}{3}EF$ and $HF = 5FG$, then what is $\frac{EF}{HG}$?

interNET CONNECTION **SAT/ACT Practice** For additional test practice questions, visit: **www.amc.glencoe.com**