7.5 Graphing Square Root and Cube Root Functions

What you should learn

GOAL 1 Graph square root and cube root functions.

GOAL 2 Use square root and cube root functions to find **real-life** quantities, such as the power of a race car in **Ex. 48**.

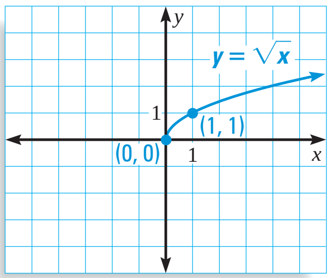
Why you should learn it

▼ To solve **real-life** problems, such as finding the age of an African elephant in **Example 6**.

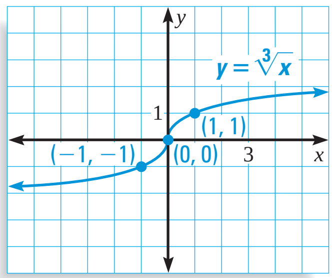
REAL LIFE

GOAL 1 GRAPHING RADICAL FUNCTIONS

In Lesson 7.4 you saw the graphs of $y = \sqrt{x}$ and $y = \sqrt[3]{x}$. These are examples of **radical functions**.



Domain: $x \geq 0$, Range: $y \geq 0$



Domain and range: all real numbers

In this lesson you will learn to graph functions of the form $y = a\sqrt{x - h} + k$ and $y = a\sqrt[3]{x - h} + k$.

ACTIVITY Developing Concepts — Investigating Graphs of Radical Functions

1. Graph $y = a\sqrt{x}$ for $a = 2, \frac{1}{2}, -3$, and -1. Use the graph of $y = \sqrt{x}$ shown above and the labeled points on the graph as a reference. Describe how a affects the graph.

2. Graph $y = a\sqrt[3]{x}$ for $a = 2, \frac{1}{2}, -3$, and -1. Use the graph of $y = \sqrt[3]{x}$ shown above and the labeled points on the graph as a reference. Describe how a affects the graph.

In the activity you may have discovered that the graph of $y = a\sqrt{x}$ starts at the origin and passes through the point $(1, a)$. Similarly, the graph of $y = a\sqrt[3]{x}$ passes through the origin and the points $(-1, -a)$ and $(1, a)$. The following describes how to graph more general radical functions.

GRAPHS OF RADICAL FUNCTIONS

To graph $y = a\sqrt{x - h} + k$ or $y = a\sqrt[3]{x - h} + k$, follow these steps.

STEP 1 Sketch the graph of $y = a\sqrt{x}$ or $y = a\sqrt[3]{x}$.

STEP 2 Shift the graph h units horizontally and k units vertically.

EXAMPLE 1 *Comparing Two Graphs*

Describe how to obtain the graph of $y = \sqrt{x + 1} - 3$ from the graph of $y = \sqrt{x}$.

SOLUTION

Note that $y = \sqrt{x + 1} - 3 = \sqrt{x - (-1)} + (-3)$, so $h = -1$ and $k = -3$. To obtain the graph of $y = \sqrt{x + 1} - 3$, shift the graph of $y = \sqrt{x}$ left 1 unit and down 3 units.

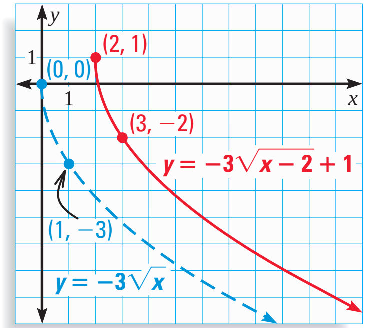
EXAMPLE 2 *Graphing a Square Root Function*

Graph $y = -3\sqrt{x - 2} + 1$.

STUDENT HELP

Skills Review
For help with transformations, see p. 921.

SOLUTION



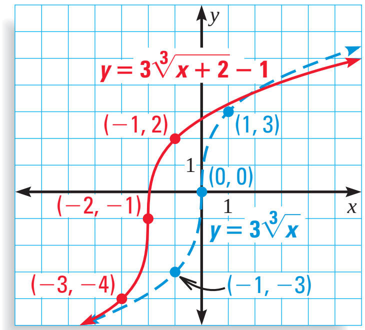
1. Sketch the graph of $y = -3\sqrt{x}$ (shown dashed). Notice that it begins at the origin and passes through the point $(1, -3)$.

2. Note that for $y = -3\sqrt{x - 2} + 1$, $h = 2$ and $k = 1$. So, shift the graph right 2 units and up 1 unit. The result is a graph that starts at $(2, 1)$ and passes through the point $(3, -2)$.

EXAMPLE 3 *Graphing a Cube Root Function*

Graph $y = 3\sqrt[3]{x + 2} - 1$.

SOLUTION



1. Sketch the graph of $y = 3\sqrt[3]{x}$ (shown dashed). Notice that it passes through the origin and the points $(-1, -3)$ and $(1, 3)$.

2. Note that for $y = 3\sqrt[3]{x + 2} - 1$, $h = -2$ and $k = -1$. So, shift the graph left 2 units and down 1 unit. The result is a graph that passes through the points $(-3, -4)$, $(-2, -1)$, and $(-1, 2)$.

EXAMPLE 4 *Finding Domain and Range*

State the domain and range of the function in (**a**) Example 2 and (**b**) Example 3.

SOLUTION

a. From the graph of $y = -3\sqrt{x - 2} + 1$ in Example 2, you can see that the domain of the function is $x \geq 2$ and the range of the function is $y \leq 1$.

b. From the graph of $y = 3\sqrt[3]{x + 2} - 1$ in Example 3, you can see that the domain and range of the function are both all real numbers.

FOCUS ON CAREERS

REAL LIFE

AMUSEMENT RIDE DESIGNER

An amusement ride designer uses math and science to ensure the safety of the rides. Most amusement ride designers have a degree in mechanical engineering.

INTERNET CAREER LINK
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GOAL 2 USING RADICAL FUNCTIONS IN REAL LIFE

When you use radical functions in real life, the domain is understood to be restricted to the values that make sense in the real-life situation.

EXAMPLE 5 *Modeling with a Square Root Function*

AMUSEMENT PARKS At an amusement park a ride called the *rotor* is a cylindrical room that spins around. The riders stand against the circular wall. When the rotor reaches the necessary speed, the floor drops out and the centrifugal force keeps the riders pinned to the wall.

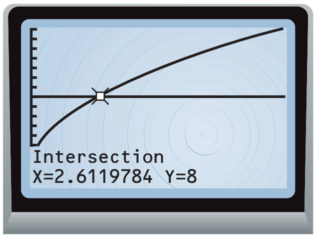
The model that gives the speed s (in meters per second) necessary to keep a person pinned to the wall is

$$s = 4.95\sqrt{r}$$

where r is the radius (in meters) of the rotor. Use a graphing calculator to graph the model. Then use the graph to estimate the radius of a rotor that spins at a speed of 8 meters per second.

SOLUTION

Graph $y = 4.95\sqrt{x}$ and $y = 8$. Choose a viewing window that shows the point where the graphs intersect. Then use the *Intersect* feature to find the x-coordinate of that point. You get $x \approx 2.61$.



▶ The radius is about 2.61 meters.

REAL LIFE Biology

EXAMPLE 6 *Modeling with a Cube Root Function*

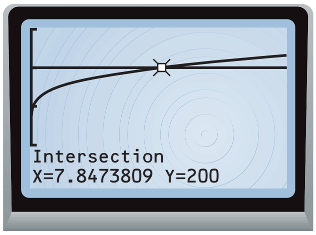
Biologists have discovered that the shoulder height h (in centimeters) of a male African elephant can be modeled by

$$h = 62.5\sqrt[3]{t} + 75.8$$

where t is the age (in years) of the elephant. Use a graphing calculator to graph the model. Then use the graph to estimate the age of an elephant whose shoulder height is 200 centimeters. ▶ Source: *Elephants*

SOLUTION

Graph $y = 62.5\sqrt[3]{x} + 75.8$ and $y = 200$. Choose a viewing window that shows the point where the graphs intersect. Then use the *Intersect* feature to find the x-coordinate of that point. You get $x \approx 7.85$.



▶ The elephant is about 8 years old.

Guided Practice

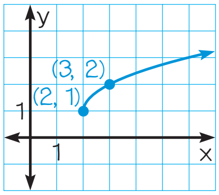
Vocabulary Check ✓

1. Complete this statement: Square root functions and cube root functions are examples of _?_ functions.

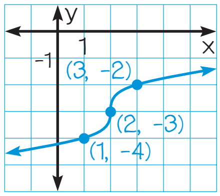
Concept Check ✓

2. **Error Analysis** Explain why the graph shown at the near right is not the graph of $y = \sqrt{x - 1} + 2$.

3. **Error Analysis** Explain why the graph shown at the far right is not the graph of $y = \sqrt[3]{x + 2} - 3$.



Ex. 2



Ex. 3

Skill Check ✓

Describe how to obtain the graph of *g* from the graph of *f*.

4. $g(x) = \sqrt{x + 5},\ f(x) = \sqrt{x}$

5. $g(x) = \sqrt[3]{x} - 10,\ f(x) = \sqrt[3]{x}$

Graph the function. Then state the domain and range.

6. $y = -\sqrt{x}$

7. $y = \sqrt{x + 1}$

8. $y = \sqrt{x - 2}$

9. $y = 2\sqrt{x + 3} - 1$

10. $y = \frac{2}{3}\sqrt[3]{x}$

11. $y = \sqrt[3]{x} - 6$

12. $y = \sqrt[3]{x + 5}$

13. $y = -3\sqrt[3]{x - 7} - 4$

14. **Elephants** Look back at Example 6. Use a graphing calculator to graph the model. Then use the graph to estimate the age of an elephant whose shoulder height is 250 centimeters.

Practice and Applications

Student Help

Extra Practice to help you master skills is on p. 950.

Comparing Graphs **Describe how to obtain the graph of *g* from the graph of *f*.**

15. $g(x) = \sqrt{x + 14},\ f(x) = \sqrt{x}$

16. $g(x) = 5\sqrt{x - 10} - 3,\ f(x) = 5\sqrt{x}$

17. $g(x) = -\sqrt[3]{x} - 10,\ f(x) = -\sqrt[3]{x}$

18. $g(x) = \sqrt[3]{x + 6} - 5,\ f(x) = \sqrt[3]{x}$

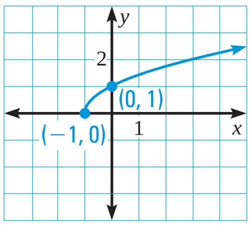
Matching Graphs **Match the function with its graph.**

19. $y = \sqrt{x} - 1$

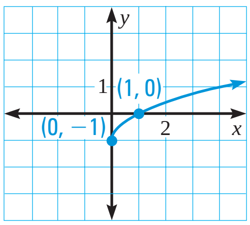
20. $y = \sqrt{x + 1}$

21. $y = \sqrt{x + 1} - 1$

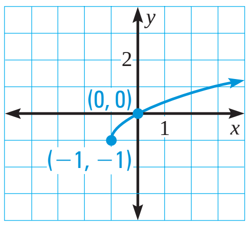
A.



B.



C.



Student Help

Homework Help

Example 1: Exs. 15–18
Example 2: Exs. 19–30
Example 3: Exs. 31–39
Example 4: Exs. 22–45
Example 5: Exs. 46, 47
Example 6: Exs. 48, 49

Square Root Functions **Graph the function. Then state the domain and range.**

22. $y = -5\sqrt{x}$

23. $y = \frac{1}{3}\sqrt{x}$

24. $y = x^{1/2} + \frac{1}{4}$

25. $y = x^{1/2} - 2$

26. $y = \sqrt{x + 6}$

27. $y = (x - 7)^{1/2}$

28. $y = (x - 1)^{1/2} + 7$

29. $y = 2\sqrt{x + 5} - 1$

30. $y = -\frac{2}{5}\sqrt{x - 3} - 2$

CUBE ROOT FUNCTIONS **Graph the function. Then state the domain and range.**

31. $y = \frac{1}{2}\sqrt[3]{x}$

32. $y = -2x^{1/3}$

33. $y = \sqrt[3]{x} - 7$

34. $y = \sqrt[3]{x} + \frac{3}{4}$

35. $y = \sqrt[3]{x - 5}$

36. $y = \left(x + \frac{2}{3}\right)^{1/3}$

37. $y = \frac{1}{5}x^{1/3} - 2$

38. $y = -3\sqrt[3]{x + 4}$

39. $y = 2\sqrt[3]{x - 4} + 3$

CRITICAL THINKING **Find the domain and range of the function without graphing. Explain how you found your solution.**

40. $y = \sqrt{x - 13}$

41. $y = 2\sqrt{x} - 2$

42. $y = -\sqrt{x - 3} - 7$

43. $y = \sqrt[3]{x + 8}$

44. $y = -\frac{2}{3}\sqrt[3]{x} - 5$

45. $y = 4\sqrt[3]{x + 4} + 7$

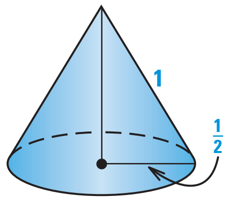
STUDENT HELP

HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with problem solving in Exs. 40–45.

GRAPHING MODELS **In Exercises 46–49, use a graphing calculator to graph the models. Then use the *Intersect* feature to solve the problems.**

46. **OCEAN DISTANCES** When you look at the ocean, the distance d (in miles) you can see to the horizon can be modeled by $d = 1.22\sqrt{a}$ where a is your altitude (in feet above sea level). Graph the model. Then determine at what altitude you can see 10 miles. ▶ Source: *Mathematics in Everyday Things*

47. **GEOMETRY CONNECTION** In a right circular cone with a slant height of 1 unit, the radius r of the cone is given by $r = \frac{1}{\sqrt{\pi}}\sqrt{S + \frac{\pi}{4}} - \frac{1}{2}$ where S is the surface area of the cone. Graph the model. Then find the surface area of a right circular cone with a slant height of 1 unit and a radius of $\frac{1}{2}$ unit.



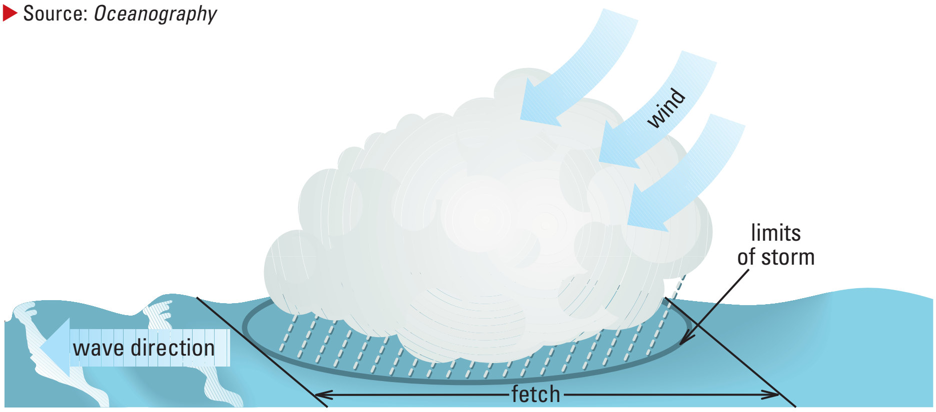
48. **RACING** Drag racing is an acceleration contest over a distance of a quarter mile. For a given total weight, the speed of a car at the end of the race is a function of the car's power. For a total weight of 3500 pounds, the speed s (in miles per hour) can be modeled by $s = 14.8\sqrt[3]{p}$ where p is the power (in horsepower). Graph the model. Then determine the power of a car that reaches a speed of 100 miles per hour. ▶ Source: *The Physics of Sports*

FOCUS ON CAREERS

COAST GUARD Members of the Coast Guard have a variety of responsibilities. Some participate in search and rescue missions that involve rescuing people caught in storms at sea, discussed in Ex. 49.

CAREER LINK www.mcdougallittell.com

49. **STORMS AT SEA** The fetch f (in nautical miles) of the wind at sea is the distance over which the wind is blowing. The minimum fetch required to create a fully developed storm can be modeled by $s = 3.1\sqrt[3]{f + 10} + 11.1$ where s is the speed (in knots) of the wind. Graph the model. Then determine the minimum fetch required to create a fully developed storm if the wind speed is 25 knots.
▶ Source: *Oceanography*



Test Preparation

50. MULTI-STEP PROBLEM Follow the steps below to graph radical functions of the form $y = f(-x)$.

a. Graph $f_1(x) = \sqrt{x}$ and $f_2(x) = \sqrt{-x}$. How are the graphs related?

b. Graph $g_1(x) = \sqrt[3]{x}$ and $g_2(x) = \sqrt[3]{-x}$. How are the graphs related?

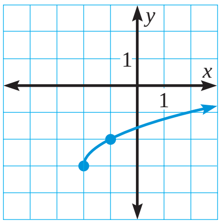
c. Graph $f_3(x) = \sqrt{-(x-2)} - 4$ and $g_3(x) = 2\sqrt[3]{-(x+1)} + 5$ using what you learned from parts (a) and (b) and what you know about the effects of a, h, and k on the graphs of $y = a\sqrt{x-h} + k$ and $y = a\sqrt[3]{x-h} + k$.

d. *Writing* Describe the steps for graphing a function of the form $f(x) = a\sqrt{-(x-h)} + k$ or $g(x) = a\sqrt[3]{-(x-h)} + k$.

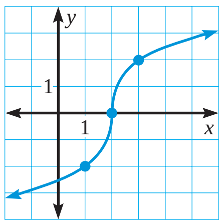
★ Challenge

ANALYZING GRAPHS **Write an equation for the function whose graph is shown.**

51.



52.



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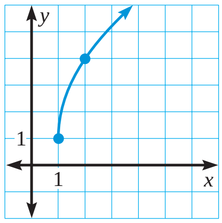

EXTRA CHALLENGE
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MIXED REVIEW

SOLVING EQUATIONS **Solve the equation.** (Review 5.3 for 7.6)

54. $2x^2 = 32$ **55.** $(x+7)^2 = 10$ **56.** $9x^2 + 3 = 5$

57. $\frac{1}{2}x^2 - 5 = 13$ **58.** $\frac{1}{4}(x+6)^2 = 22$ **59.** $2(x - 0.25)^2 = 16.5$

SPECIAL PRODUCTS **Find the product.** (Review 6.3 for 7.6)

60. $(x+4)^2$ **61.** $(x-9y)^2$ **62.** $(2x^3+7)^2$

63. $(-3x+4y^4)^2$ **64.** $(6-5x)^2$ **65.** $(-1-2x^2)^2$

COMPOSITION OF FUNCTIONS **Find $f(g(x))$ and $g(f(x))$.** (Review 7.3)

66. $f(x) = x + 7,\ g(x) = 2x$ **67.** $f(x) = 2x + 1,\ g(x) = x - 3$

68. $f(x) = x^2 - 1,\ g(x) = x + 2$ **69.** $f(x) = x^2 + 7,\ g(x) = 3x - 3$

70. MANDELBROT SET To determine whether a complex number c belongs to the Mandelbrot set, consider the function $f(z) = z^2 + c$ and the infinite list of complex numbers $z_0 = 0$, $z_1 = f(z_0)$, $z_2 = f(z_1)$, $z_3 = f(z_2)$, ...

- If the absolute values $|z_0|, |z_1|, |z_2|, |z_3|, \ldots$ are all less than some fixed number N, then c belongs to the Mandelbrot set.
- If the absolute values $|z_0|, |z_1|, |z_2|, |z_3|, \ldots$ become infinitely large, then c does not belong to the Mandelbrot set.

Tell whether the complex number c belongs to the Mandelbrot set. (Review 5.4)

a. $c = 3i$ **b.** $c = 2 + 2i$ **c.** $c = 6$