13.1 Right Triangle Trigonometry

What you should learn

GOAL 1 Use trigonometric relationships to evaluate trigonometric functions of acute angles.

GOAL 2 Use trigonometric functions to solve **real-life** problems, such as finding the altitude of a kite in **Example 4**.

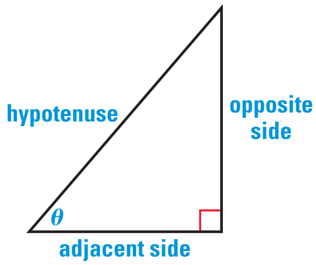
Why you should learn it

▼ To solve **real-life** problems, such as finding the length of a zip-line at a ropes course in **Ex. 50**.

REAL LIFE

GOAL 1 EVALUATING TRIGONOMETRIC FUNCTIONS

Consider a right triangle, one of whose acute angles is θ (the Greek letter *theta*). The three sides of the triangle are the *hypotenuse,* the side *opposite* θ, and the side *adjacent* to θ.



Ratios of a right triangle's three sides are used to define the six trigonometric functions: **sine**, **cosine**, **tangent**, **cosecant**, **secant**, and **cotangent**. These six functions are abbreviated sin, cos, tan, csc, sec, and cot, respectively.

RIGHT TRIANGLE DEFINITION OF TRIGONOMETRIC FUNCTIONS

Let θ be an acute angle of a right triangle. The six trigonometric functions of θ are defined as follows.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

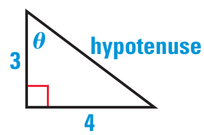
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

The abbreviations *opp, adj,* and *hyp* represent the lengths of the three sides of the right triangle. Note that the ratios in the second row are the reciprocals of the ratios in the first row. That is:

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

EXAMPLE 1 *Evaluating Trigonometric Functions*

Evaluate the six trigonometric functions of the angle θ shown in the right triangle.



SOLUTION

From the Pythagorean theorem, the length of the hypotenuse is:

$$\sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

Using adj = 3, opp = 4, and hyp = 5, you can write the following.

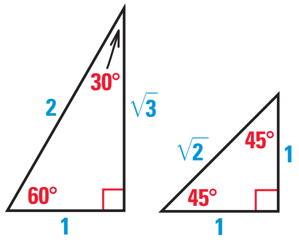
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}$$

STUDENT HELP

Skills Review
For help with the Pythagorean theorem, see p. 917.

The angles 30°, 45°, and 60° occur frequently in trigonometry. The table below gives the values of the six trigonometric functions for these angles. To remember these values, you may find it easier to draw the triangles shown, rather than memorize the table.



θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

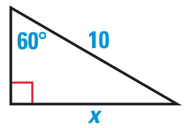
Trigonometric functions can be used to find a missing side length or angle measure of a right triangle. Finding *all* missing side lengths and angle measures is called **solving a right triangle**.

EXAMPLE 2 *Finding a Missing Side Length of a Right Triangle*

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

Find the value of x for the right triangle shown.



SOLUTION

Write an equation using a trigonometric function that involves the ratio of x and 10. Solve the equation for x.

$$\sin 60^\circ = \frac{\text{opp}}{\text{hyp}} \qquad \textbf{Write trigonometric equation.}$$

$$\frac{\sqrt{3}}{2} = \frac{x}{10} \qquad \textbf{Substitute.}$$

$$5\sqrt{3} = x \qquad \textbf{Multiply each side by 10.}$$

▶ The length of the side is $x = 5\sqrt{3} \approx 8.66$.

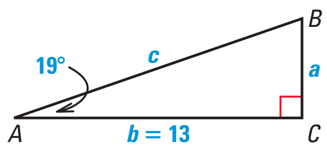
You can use a calculator to evaluate trigonometric functions of *any* angle, not just 30°, 45°, and 60°. Use the keys SIN, COS, and TAN for sine, cosine, and tangent. Use these keys and the reciprocal key for cosecant, secant, and cotangent. Before using the calculator be sure it is set in degree mode.

STUDENT HELP

Study Tip
In Example 3, *B* is used to represent both the angle and its measure. Throughout this chapter, a capital letter is used to denote a vertex of a triangle and the same letter in lowercase is used to denote the side opposite that angle.

EXAMPLE 3 *Using a Calculator to Solve a Right Triangle*

Solve $\triangle ABC$.



SOLUTION

Because the triangle is a right triangle, A and B are complementary angles, so $B = 90^\circ - 19^\circ = 71^\circ$.

$$\frac{a}{13} = \tan 19^\circ \approx 0.3443 \qquad \frac{c}{13} = \sec 19^\circ = \frac{1}{\cos 19^\circ} \approx 1.058$$

$$a \approx 4.48 \qquad c \approx 13.8$$

FOCUS ON APPLICATIONS

KITE FLYING In the late 1800s and early 1900s, kites were used to lift weather instruments. In 1919 the German Weather Bureau set a kite-flying record. Eight kites on a single line, like those pictured above, were flown at an altitude of 9740 meters.

GOAL 2 USING TRIGONOMETRY IN REAL LIFE

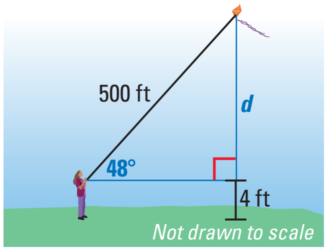
EXAMPLE 4 *Finding the Altitude of a Kite*

KITE FLYING Wind speed affects the angle at which a kite flies. The table at the right shows the angle the kite line makes with a line parallel to the ground for several different wind speeds. You are flying a kite 4 feet above the ground and are using 500 feet of line. At what altitude is the kite flying if the wind speed is 35 miles per hour?

Wind speed (miles per hour)	Angle of kite line (degrees)
25	70
30	60
35	48
40	29
45	0

SOLUTION

At a wind speed of 35 miles per hour, the angle the kite line makes with a line parallel to the ground is 48°. Write an equation using a trigonometric function that involves the ratio of the distance d and 500.

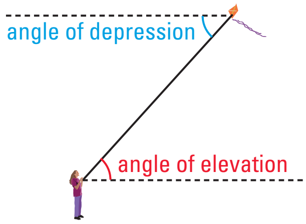


$\sin 48° = \frac{d}{500}$ **Write trigonometric equation.**

$0.7431 \approx \frac{d}{500}$ **Simplify.**

$372 \approx d$ **Solve for *d*.**

▶ When you add 4 feet for the height at which you are holding the kite line, the kite's altitude is about 376 feet.



In Example 4 the angle the kite line makes with a line parallel to the ground is the **angle of elevation**. At the height of the kite, the angle from a line parallel to the ground to the kite line is the **angle of depression**. These two angles have the same measure.

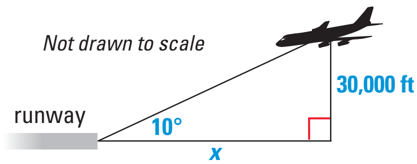
REAL LIFE *Aviation*

EXAMPLE 5 *Finding the Distance to an Airport*

An airplane flying at an altitude of 30,000 feet is headed toward an airport. To guide the airplane to a safe landing, the airport's landing system sends radar signals from the runway to the airplane at a 10° angle of elevation. How far is the airplane (measured along the ground) from the airport runway?

SOLUTION

Begin by drawing a diagram.



$$\frac{x}{30{,}000} = \cot 10° = \frac{1}{\tan 10°} \approx 5.671$$

$$x \approx 170{,}100$$

▶ The plane is about 170,100 feet (or 32.2 miles) from the airport.

GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

1. Explain what it means to solve a right triangle.

2. Given a 30°-60°-90° triangle with only the measures of the angles labeled, can you find the lengths of any of the sides? Explain.

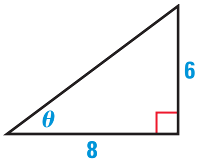
3. If you are given a right triangle with an acute angle θ, what two trigonometric functions of θ can you calculate using the lengths of the hypotenuse and the side opposite θ?

4. For which acute angle θ is $\cos\theta = \frac{\sqrt{3}}{2}$?

Skill Check ✓

Evaluate the six trigonometric functions of the angle θ.

5.



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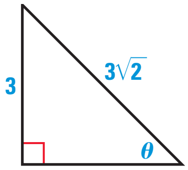

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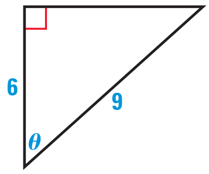
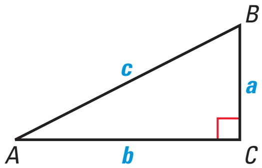


Solve $\triangle ABC$ using the diagram at the right and the given measurements.



8. $A = 20°, a = 12$

9. $A = 75°, c = 20$

10. $B = 40°, c = 5$

11. $A = 62°, b = 30$

12. $B = 63°, a = 15$

13. $B = 15°, b = 42$

14. **KITE FLYING** Look back at Example 4 on page 771. Suppose you are flying a kite 4 feet above the ground on a line that is 300 feet long. If the wind speed is 30 miles per hour, what is the altitude of the kite?

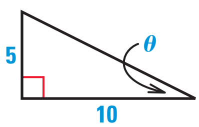
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 957.

EVALUATING FUNCTIONS **Evaluate the six trigonometric functions of the angle θ.**

15.



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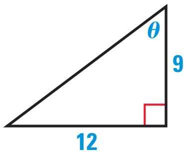

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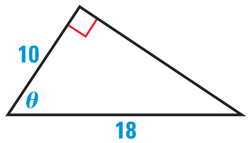

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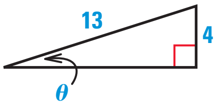

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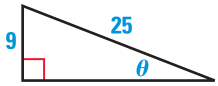

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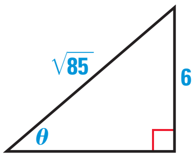


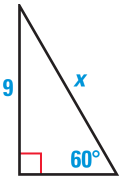
STUDENT HELP

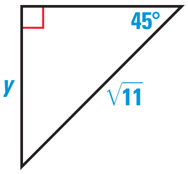
HOMEWORK HELP

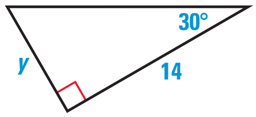
Example 1: Exs. 15–21
Example 2: Exs. 22–24
Example 3: Exs. 25–40
Examples 4, 5: Exs. 43–50

21. **VISUAL THINKING** The lengths of the sides of a right triangle are 5 centimeters, 12 centimeters, and 13 centimeters. Sketch the triangle. Let θ represent the angle that is opposite the side whose length is 5 centimeters. Evaluate the six trigonometric functions of θ.

FINDING SIDE LENGTHS Find the missing side lengths x and y.

22. 

23. 

24. 

EVALUATING FUNCTIONS Use a calculator to evaluate the trigonometric function. Round the result to four decimal places.

25. $\sin 14°$ **26.** $\cos 31°$ **27.** $\tan 59°$ **28.** $\sec 23°$

29. $\csc 80°$ **30.** $\cot 36°$ **31.** $\csc 6°$ **32.** $\cot 11°$

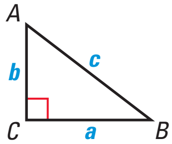
SOLVING TRIANGLES Solve $\triangle ABC$ using the diagram and the given measurements.

33. $B = 24°, a = 8$ **34.** $A = 37°, c = 22$

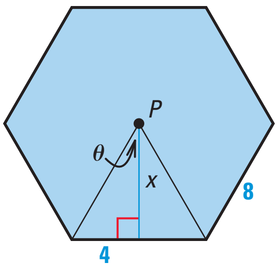
35. $A = 19°, b = 4$ **36.** $B = 41°, c = 18$

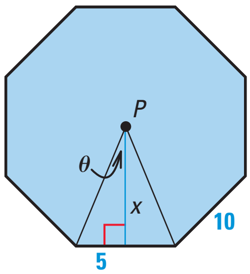
37. $A = 29°, b = 21$ **38.** $B = 56°, a = 6.8$

39. $B = 65°, c = 12$ **40.** $A = 70°, c = 30$



GEOMETRY CONNECTION Find the area of the regular polygon with point P at its center.

41. 

42. 

FOCUS ON APPLICATIONS

DUQUESNE INCLINE Built in Pittsburgh in 1877, the Duquesne Incline transports people up and down the side of a mountain in cable cars. In 1877 the cost of a one-way trip was $.05. Today the cost is $1.

DUQUESNE INCLINE In Exercises 43 and 44, use the following information. The track of the Duquesne Incline is about 800 feet long and the angle of elevation is 30°. The average speed of the cable cars is about 320 feet per minute.

43. How high does the Duquesne Incline rise?

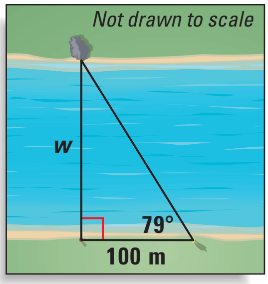
44. What is the vertical speed of the cable cars (in feet per minute)?

45. **SKI SLOPE** A ski slope at a mountain has an angle of elevation of 25.2°. The vertical height of the slope is 1808 feet. How long is the ski slope?

46. **BOARDING A SHIP** A gangplank is a narrow ramp used for boarding or leaving a ship. The maximum safe angle of elevation for a gangplank is 20°. Suppose a gangplank is 10 feet long. What is the closest a ship can come to the dock for the gangplank to be used?

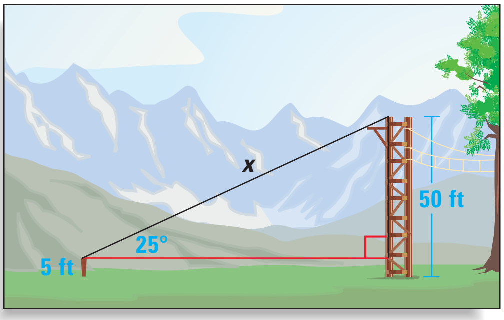
47. **JIN MAO BUILDING** You are standing 75 meters from the base of the Jin Mao Building in Shanghai, China. You estimate that the angle of elevation to the top of the building is 80°. What is the approximate height of the building? Suppose one of your friends is at the top of the building. What is the distance between you and your friend?

48. **MEASURING RIVER WIDTH** To measure the width of a river you plant a stake on one side of the river, directly across from a boulder. You then walk 100 meters to the right of the stake and measure a 79° angle between the stake and the boulder. What is the width *w* of the river?



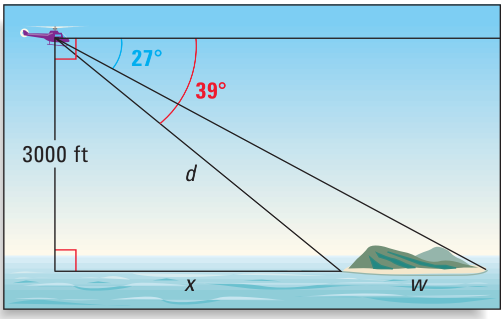
49. **MOUNT COOK** You are climbing Mount Cook in New Zealand. You are below the mountain's peak at an altitude of 8580 feet. Using surveying instruments, you measure the angle of elevation to the peak to be 30.5°. The distance (along the face of the mountain) between you and the peak is 7426 feet. What is the altitude of the peak?

50. **ROPES COURSE** You are designing a zip-line for a ropes course at a summer camp. A zip-line is a cable to which people can attach their safety harnesses and slide down to the ground. You want to attach one end of the cable to a pole 50 feet high and the other end to a pole 5 feet high. The maximum safe angle of elevation for the zip-line is 25°. Calculate the minimum length *x* of cable needed.



Test Preparation

51. **MULTI-STEP PROBLEM** You are a surveyor in a helicopter and are trying to determine the width of an island, as illustrated at the right.



a. What is the shortest distance *d* the helicopter would have to travel to land on the island?

b. What is the horizontal distance *x* that the helicopter has to travel before it is directly over the nearer end of the island?

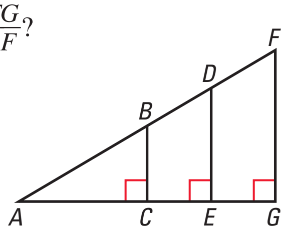
c. *Writing* Find the width *w* of the island. Explain the process you used to find your answer.

★ **Challenge**

ANALYZING SIMILAR TRIANGLES **In Exercises 52–54, use the diagram below.**

52. Explain why $\triangle ABC$, $\triangle ADE$, and $\triangle AFG$ are similar triangles.

53. What does similarity imply about the ratios $\frac{BC}{AB}$, $\frac{DE}{AD}$, and $\frac{FG}{AF}$? Does the value of sin *A* depend on which triangle from Exercise 52 is used to calculate it? Would the value of sin *A* change if it were found using a different right triangle that is similar to the three given triangles?



54. Do your observations about sin *A* also apply to the other five trigonometric functions? Explain.

STUDENT HELP

Skills Review
For help with similar triangles, see p. 923.

MIXED REVIEW

UNIT ANALYSIS **Find the product. Give the answer with the appropriate unit of measure.** (Review 1.1 for 13.2)

55. $(3.5 \text{ hours}) \cdot \dfrac{45 \text{ miles}}{1 \text{ hour}}$

56. $(500 \text{ dollars}) \cdot \dfrac{12.2 \text{ schillings}}{1 \text{ dollar}}$

57. $\dfrac{3 \text{ dollars}}{1 \text{ square foot}} \cdot (1222 \text{ square feet})$

58. $(12 \text{ seconds}) \cdot \dfrac{254 \text{ feet}}{1 \text{ second}}$

CLASSIFYING **Classify the conic section.** (Review 10.6)

59. $y^2 - 16x - 14y + 17 = 0$

60. $25x^2 + y^2 - 100x - 2y + 76 = 0$

61. $x^2 + y^2 = 25$

62. $x^2 - y^2 = 100$

63. **ESSAY TOPICS** For a homework assignment you have to choose from 15 possible topics on which to write an essay. If all of the topics are equally interesting, what is the probability that you and your five friends will all choose different topics? (Review 12.5)

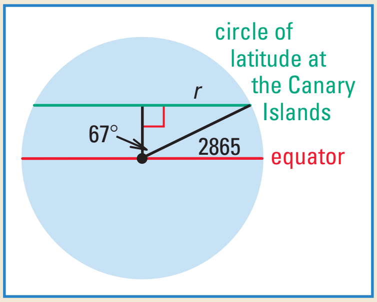
MATH & History

Columbus's Voyage

APPLICATION LINK www.mcdougallittell.com

THEN

IN 1492 Christopher Columbus set sail west from the Canary Islands intending to reach Japan. Due to miscalculations of Earth's circumference and the relative location of Japan, he instead sailed to the New World.



Columbus believed the distance west from the Canary Islands to Japan to be $\frac{1}{6}$ the circumference of Earth at that latitude. He supposed Earth's radius at the equator to be about 2865 miles.

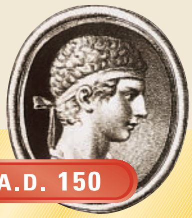
1. Use the diagram at the right to calculate what Columbus believed to be the radius r of Earth at the latitude of the Canary Islands.
2. Use your answer to Exercise 1 to calculate the distance west from the Canary Islands that Columbus believed he would find Japan.
3. Use reference materials to find the true distance west from the Canary Islands to Japan. How far off were Columbus's calculations?

NOW

TODAY aerial photography and computers are used to make maps. Accurate maps in combination with satellite-based navigation make travel a more exact science.

The oldest existing map was made on a clay tablet in Babylonia.

2500 B.C.



A.D. 150

Influential map maker Claudius Ptolemy wrote his eight-volume *Geography*.

1492

Columbus sails to the Bahama Islands and Cuba, intending to reach Japan.

1999

The Landsat 7 satellite was launched.