CHAPTER 13

Chapter Summary

WHAT did you learn?	*WHY did you learn it?*
Evaluate trigonometric functions.	
• of acute angles (13.1)	Find the altitude of a kite. (p. 771)
• of any angle (13.3)	Find the horizontal distance traveled by a golf ball. (p. 787)
Find the sides and angles of a triangle.	
• solve right triangles (13.1)	Find the length of a zip-line at a ropes course. (p. 774)
• use the law of sines (13.5)	Find the distance between two buildings. (p. 805)
• use the law of cosines (13.6)	Find the angle at which two trapeze artists meet. (p. 811)
Measure angles using degree measure and radian measure. (13.2)	Find the angle generated by a figure skater performing a jump. (p. 781)
Find arc lengths and areas of sectors. (13.2)	Find the area irrigated by a rotating sprinkler. (p. 781)
Evaluate inverse trigonometric functions. (13.4)	Find the angle at which to set the arm of a crane. (p. 794)
Find the area of a triangle.	
• using two sides and the included angle (13.5)	Find the amount of paint needed for the side of a house. (p. 806)
• using Heron's formula (13.6)	Find the area of the Dinosaur Diamond. (p. 812)
Use parametric equations to model linear or projectile motion. (13.7)	Model the path of a leaping dolphin. (p. 818)
Use trigonometric and inverse trigonometric functions to solve real-life problems. (13.1, 13.3–13.7)	Find distances for a marching band on a football field. (p. 787)

How does Chapter 13 fit into the BIGGER PICTURE of algebra?

Trigonometry is closely tied to both algebra and geometry. In this chapter you studied trigonometric functions of *angles*, defined by ratios of side lengths of right triangles.

In the next chapter you will study trigonometric functions of *real numbers*, used to model periodic behavior. You will see even more connections between trigonometry and algebra as you graph trigonometric functions in a coordinate plane.

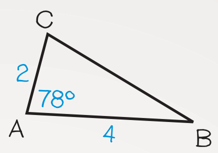
STUDY STRATEGY

How did you draw diagrams?

Here is an example of a diagram drawn for Exercise 22 on page 810, following the **Study Strategy** on page 768.

Draw Diagrams

Find the remaining angle measures and side lengths of $\triangle ABC$: $A = 78°$, $b = 2$, $c = 4$.



CHAPTER 13

Chapter Review

VOCABULARY

- sine, p. 769
- cosine, p. 769
- tangent, p. 769
- cosecant, p. 769
- secant, p. 769
- cotangent, p. 769
- solving a right triangle, p. 770
- angle of elevation, p. 771
- angle of depression, p. 771
- initial side of an angle, p. 776
- terminal side of an angle, p. 776
- standard position, p. 776
- coterminal angles, p. 777
- radian, p. 777
- sector, p. 779
- central angle, p. 779
- quadrantal angle, p. 785
- reference angle, p. 785
- inverse sine, p. 792
- inverse cosine, p. 792
- inverse tangent, p. 792
- law of sines, p. 799
- law of cosines, p. 807
- parametric equations, p. 813
- parameter, p. 813

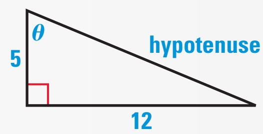
13.1 RIGHT TRIANGLE TRIGONOMETRY

Examples on pp. 769–771

EXAMPLE You can evaluate the six trigonometric functions of θ for the triangle shown. First find the hypotenuse length: $\sqrt{5^2 + 12^2} = \sqrt{169} = 13$.

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{12}{13}$ $\quad\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{5}{13}$ $\quad\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{12}{5}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{13}{12}$ $\quad\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{13}{5}$ $\quad\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{5}{12}$



Evaluate the six trigonometric functions of θ.

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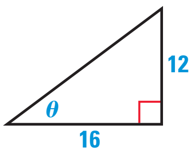

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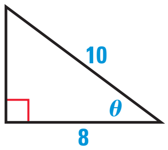

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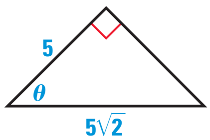

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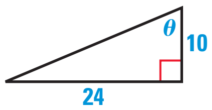


13.2 GENERAL ANGLES AND RADIAN MEASURE

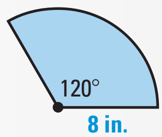
Examples on pp. 776–779

EXAMPLES You can measure angles using degree measure or radian measure.

$20^\circ = 20^\circ\left(\frac{\pi \text{ radians}}{180^\circ}\right) = \frac{\pi}{9} \text{ radians}$ $\qquad \frac{7\pi}{6} \text{ radians} = \left(\frac{7\pi}{6} \text{ radians}\right)\left(\frac{180^\circ}{\pi \text{ radians}}\right) = 210^\circ$

Arc length of the sector at the right: $s = r\theta = 8\left(\frac{2\pi}{3}\right) = \frac{16\pi}{3}$ inches

Area of the sector at the right: $A = \frac{1}{2}r^2\theta = \frac{1}{2}(8^2)\left(\frac{2\pi}{3}\right) = \frac{64\pi}{3}$ square inches



Rewrite each degree measure in radians and each radian measure in degrees.

5. 30° **6.** 225° **7.** -15° **8.** $\frac{3\pi}{4}$ **9.** $\frac{5\pi}{3}$ **10.** $\frac{\pi}{3}$

Find the arc length and area of a sector with the given radius r and central angle θ.

11. $r = 5$ ft, $\theta = \frac{\pi}{2}$ **12.** $r = 12$ in., $\theta = 25°$ **13.** $r = 16$ cm, $\theta = 210°$

13.3 TRIGONOMETRIC FUNCTIONS OF ANY ANGLE

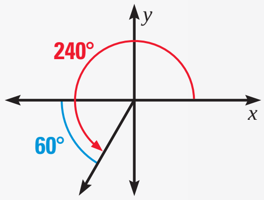
Examples on pp. 784–787

EXAMPLE You can evaluate the six trigonometric functions of $\theta = 240°$ using a reference angle: $\theta' = \theta - 180° = 240° - 180° = 60°$.

$\sin 240° = -\sin 60° = -\frac{\sqrt{3}}{2}$ $\csc 240° = -\csc 60° = -\frac{2\sqrt{3}}{3}$

$\cos 240° = -\cos 60° = -\frac{1}{2}$ $\sec 240° = -\sec 60° = -2$

$\tan 240° = +\tan 60° = \sqrt{3}$ $\cot 240° = +\cot 60° = \frac{\sqrt{3}}{3}$



Evaluate the function without using a calculator.

14. $\tan \frac{11\pi}{4}$ **15.** $\cos \frac{11\pi}{6}$ **16.** $\sec 225°$ **17.** $\sin 390°$ **18.** $\csc(-120°)$

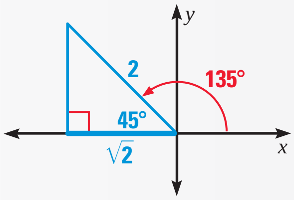
13.4 INVERSE TRIGONOMETRIC FUNCTIONS

Examples on pp. 792–794

EXAMPLE You can find an angle within a certain range that corresponds to a given value of a trigonometric function.

To find $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$, find θ so that $\cos\theta = -\frac{\sqrt{2}}{2}$ and $0° \le \theta \le 180°$.

So, $\theta = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = 135° \left(\text{or } \frac{3\pi}{4} \text{ radians}\right)$.



Evaluate the expression without using a calculator. Give your answer in both radians and degrees.

19. $\sin^{-1}\frac{\sqrt{2}}{2}$ **20.** $\tan^{-1}\frac{\sqrt{3}}{3}$ **21.** $\cos^{-1} 0$ **22.** $\tan^{-1}(-1)$ **23.** $\cos^{-1}\left(-\frac{1}{2}\right)$

13.5 THE LAW OF SINES

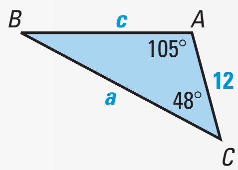
Examples on pp. 799–802

EXAMPLE You can solve the triangle shown using the law of sines.

The measure of the third angle is: $B = 180° - 105° - 48° = 27°$.

$$\frac{a}{\sin 105°} = \frac{12}{\sin 27°} \qquad \frac{c}{\sin 48°} = \frac{12}{\sin 27°}$$

$$a = \frac{12 \sin 105°}{\sin 27°} \approx 25.5 \qquad c = \frac{12 \sin 48°}{\sin 27°} \approx 19.6$$



Area of this triangle $= \frac{1}{2}bc \sin A = \frac{1}{2}(12)(19.6)\sin 105° \approx 114$ square units

13.5 continued

Solve △*ABC*. (*Hint:* Some of the "triangles" may have no solution and some may have two.)

24. $A = 45°, B = 60°, c = 44$ **25.** $B = 18°, b = 12, a = 19$ **26.** $C = 140°, c = 40, b = 20$

Find the area of the triangle with the given side lengths and included angle.

27. $C = 35°, b = 10, a = 22$ **28.** $A = 110°, b = 8, c = 7$ **29.** $B = 25°, a = 15, c = 31$

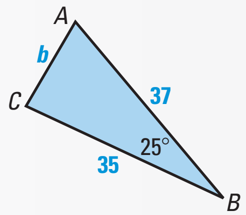
13.6 THE LAW OF COSINES

Examples on pp. 807–809

EXAMPLE You can solve the triangle below using the law of cosines.

Law of cosines: $b^2 = 35^2 + 37^2 - 2(35)(37) \cos 25° \approx 247$

$b \approx 15.7$



Law of sines: $\frac{\sin A}{35} \approx \frac{\sin 25°}{15.7}$, $\sin A \approx \frac{35 \sin 25°}{15.7}$, $A \approx 70.4°$

$C \approx 180° - 25° - 70.4° = 84.6°$

You can use Heron's formula to find the area of this triangle:

$s \approx \frac{1}{2}(35 + 15.7 + 37) \approx 44$, so area $\approx \sqrt{44(44 - 35)(44 - 15.7)(44 - 37)} \approx 280$ square units

Solve △*ABC*.

30. $a = 25, b = 18, c = 28$ **31.** $a = 6, b = 11, c = 14$ **32.** $B = 30°, a = 80, c = 70$

Find the area of △*ABC* having the given side lengths.

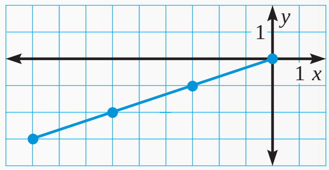
33. $a = 11, b = 2, c = 12$ **34.** $a = 4, b = 24, c = 26$ **35.** $a = 15, b = 8, c = 21$

13.7 PARAMETRIC EQUATIONS AND PROJECTILE MOTION

Examples on pp. 813–815

EXAMPLE You can graph the parametric equations $x = -3t$ and $y = -t$ for $0 \le t \le 3$. Make a table of values, plot the points (x, y), and connect the points.

t	0	1	2	3
x	0	−3	−6	−9
y	0	−1	−2	−3



To write an *xy*-equation for these parametric equations, solve the first equation for *t*: $t = -\frac{1}{3}x$. Substitute into the second equation: $y = \frac{1}{3}x$. The domain is $-9 \le x \le 0$.

Graph the parametric equations.

36. $x = 3t + 1$ and $y = 3t + 6$ for $0 \le t \le 5$ **37.** $x = 2t + 4$ and $y = -4t + 2$ for $2 \le t \le 5$

Write an *xy*-equation for the parametric equations. State the domain.

38. $x = 5t$ and $y = t + 7$ for $0 \le t \le 20$ **39.** $x = 2t - 3$ and $y = -4t + 5$ for $0 \le t \le 8$

CHAPTER 13

Chapter Test

Evaluate the six trigonometric functions of θ.

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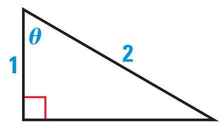

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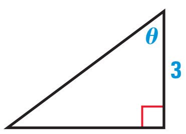

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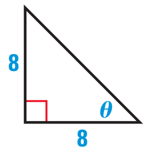

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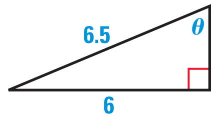


Rewrite each degree measure in radians and each radian measure in degrees.

5. $120°$ **6.** $360°$ **7.** $-60°$ **8.** $\frac{\pi}{9}$ **9.** 5π **10.** $-\frac{5\pi}{4}$

Find the arc length and area of a sector with the given radius r and central angle θ.

11. $r = 4$ ft, $\theta = 240°$ **12.** $r = 20$ cm, $\theta = 45°$ **13.** $r = 12$ in., $\theta = 150°$

Evaluate the function without using a calculator.

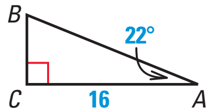
14. $\cos 180°$ **15.** $\sec(-30°)$ **16.** $\cot 495°$ **17.** $\sin \frac{7\pi}{6}$ **18.** $\tan\left(-\frac{\pi}{4}\right)$ **19.** $\csc\left(-\frac{7\pi}{4}\right)$

Evaluate the expression without using a calculator. Give your answer in both radians and degrees.

20. $\sin^{-1} 1$ **21.** $\tan^{-1} \sqrt{3}$ **22.** $\cos^{-1} \frac{\sqrt{3}}{2}$ **23.** $\tan^{-1} 0$ **24.** $\cos^{-1} 1$ **25.** $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

Solve $\triangle ABC$.

26.



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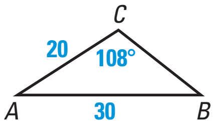

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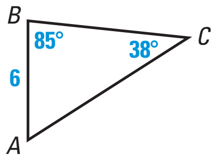

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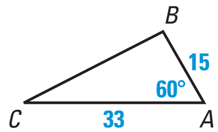
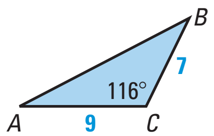


30. $A = 120°, a = 14, b = 10$ **31.** $B = 40°, a = 7, c = 10$ **32.** $C = 105°, a = 4, b = 3$

Find the area of $\triangle ABC$.

33.



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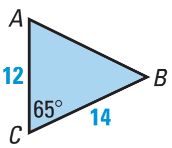

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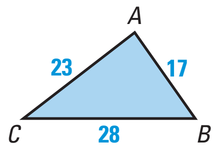

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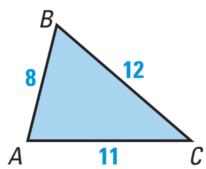

Graph the parametric equations. Then write an *xy*-equation and state the domain.

37. $x = 2t - 3$ and $y = -5t + 6$ for $1 \le t \le 4$ **38.** $x = t - 4$ and $y = -t + 6$ for $0 \le t \le 6$

39. **BOAT RIDE** A boat travels 50 miles due west before adjusting its course 25° north of west and traveling an additional 35 miles. How far is the boat from its point of departure?

40. **PROJECTILE MOTION** You throw a ball at an angle of 50°, from a height of 6 feet, and with an initial speed of 25 feet per second. Write a set of parametric equations for the path of the ball. How far from you does the ball land?